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Internal Assesment Test - I								
Sub:	ELECTROMAGNETIC FIELD THEORY						Code:	18EE45
Date:	11/07/2022	Duration:	90 mins	Max Marks:	50	Sem: 4	Branch:	EEE
Answer FIVE FULL Questions. Mention units wherever necessary.								

		Marks	OBE CO	OBE RBT
1 (a)	Transform to cylindrical co-ordinates, the vector $\boldsymbol{F} = 10\,\boldsymbol{a_x} - 8\boldsymbol{a_y} + 6\boldsymbol{a_z}$ at point P(10,-8,6)	[05]	CO1	L3
1 (b)	Obtain the formula to convert a vector from cylindrical system of co-ordinate to rectangular system of co-ordinate.	[05]	CO1	L3
2 (a)	State and explain Coulomb's law of force between two charges.	[05]	CO1	L1
2 (b)	Define electric field intensity. Deduce the expression for electric field intensity due to a system of N charges.	[05]	CO1	L2
3	Derive Maxwell's first equation of electrostatics in point form. Also state and prove Gauss's divergence theorem.	[10]	CO1	L2
4 (a)	Determine electric flux density caused at P(6,8,-10) due to i) a point charge of 30 mC at origin. ii) a surface charge with $\rho_s = 57.2\ \mu C/m^2$ on a plane Z = - 9 m.	[05]	CO2	L3
4 (b)	Derive the expression for potential due to a system of N number of charges.	[05]	CO2	L3
5	Given that $\boldsymbol{D} = z\rho\ (\cos\phi)^2\ \boldsymbol{a_z}$ C/m^2, calculate charge density at $(1, \frac{\pi}{4}, 3)$ and the total charge enclosed by the cylinder of radius 1 m with $-2m \le z \le 2m$ using volume integral. Verify the result using Divergence theorem.	[10]	CO2	L3
6 (a)	For a potential $= 2x^2y - 5z$, determine the electric field intensity and electric flux density at (-4, 3, 6).	[03]	CO2	L3
6 (b)	Define Potential difference and Potential. Also Establish the relationship between electric field intensity and electric scalar potential.	[07]	CO2	L2
7 (a)	Determine the work done in carrying a charge of $-2\ C$ form $(2,1,-1)$ to $(8,2,-1)$ in the electric field $\boldsymbol{E} = y\ \boldsymbol{a_x} + x\boldsymbol{a_y}\ V/m$ through the path given by the parabola $x = 2\ y^2$.	[06]	CO2	L2
7 (b)	Derive an expression for work done in moving a point charge Q in the presence of an electric field **E**.	[04]	CO2	L3
8	Derive the expression for potential and electric field due to an electric dipole.	[10]	CO3	L2

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	Marks	OBE CO	OBE RBT
1 (a) Transform to cylindrical co-ordinates, the vector $\boldsymbol{F} = 10\,a_x - 8a_y + 6a_z$ at point P(10,-8,6)	[05]	CO1	L3

$\rho = 12.806,\ \phi = 321.34^\circ,\ z = 6$ – (2)

$$\begin{bmatrix} F_\rho \\ F_\phi \\ F_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ -8 \\ 6 \end{bmatrix}$$ –(2)

$\boxed{\vec{F} = 12.806\,\vec{a_\rho} + 4.2\times10^{-5}\,\vec{a_\phi} + 6\vec{a_z}}$ –(1)

	Marks	CO	RBT
1 (b) Obtain the formula to convert a vector from cylindrical system of co-ordinate to rectangular system of co-ordinate.	[05]	CO1	L3

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$ (5)

	Marks	CO	RBT
2 (a) State and explain Coulomb's law of force between two charges.	[05]	CO1	L1

Statement : (2)
Explanation : (3)

	Marks	CO	RBT
2 (b) Define electric field intensity. Deduce the expression for electric field intensity due to a system of N charges.	[05]	CO1	L2

Definition : (2)

$\vec{E}$ due to N charges : (3) $\vec{E} = \sum_{i=1}^{N} \frac{Q}{4\pi\epsilon_0 |R_i|^3} \vec{R_i}$

	Marks	CO	RBT
3 Derive Maxwell's first equation of electrostatics in point form. Also state and prove Gauss's divergence theorem.	[10]	CO1	L2

$\vec{\nabla}\cdot\vec{D} = \rho_v$ –

$\oint_S \vec{D}\cdot\vec{ds} = \iiint_V \vec{\nabla}\cdot\vec{D}\,dv$ –

Gauss's law –(2)
Definition of Divergence –(2)
$\vec{\nabla}\cdot\vec{D} = \rho_v$ – (2)
Divergence theorem – (4)

	Marks	CO	RBT
4 (a) Determine electric flux density caused at P(6,8,-10) due to	[05]	CO2	L3

i) a point charge of 30 mC at origin.

$\vec{R} = 6\vec{a_x} + 8\vec{a_y} - 10\vec{a_z}$ (1)

$\vec{D} = 5.06\vec{a_x} + 6.75\vec{a_y} - 8.44\vec{a_z}\ \mu C/m^2$ (2)

ii) a surface charge with $\rho_s = 57.2\ \mu C/m^2$ on a plane Z = - 9 m.

$\vec{D} = -28.6\,\vec{a_z}\ \mu C/m^2$; $\vec{a_n} = -\vec{a_z}$ (1)

$\vec{D} = \frac{\rho_s}{2}\vec{a_n}$ (1)

4 (b) Derive the expression for potential due to a system of N number of charges. [05] CO2 L3

Diagram / V expression -(5)

5 Given that $D = z\rho(\cos\phi)^2 \, a_z$ C/m², calculate charge density at $(1, \frac{\pi}{4}, 3)$ and the total charge enclosed by the cylinder of radius 1 m with $-2m \leq z \leq 2m$ using volume integral. Verify the result using Divergence theorem. [10] CO2 L3

LHS -(5) RHS -(5)

$\oint_S D_s \cdot ds = \frac{4\pi}{3}$

$\rho_v = \rho \cos^2\phi$

RHS $= \frac{4\pi}{3}$ C

6 (a) For a potential = $2x^2y - 5z$, determine the electric field intensity and electric flux density at (-4, 3, 6). [03] CO2 L3

$\vec{E} = -\nabla V$ $\vec{E} = -[4xy \, \vec{a_x} + 2x^2 \vec{a_y} - 5\vec{a_z}] = 48\vec{a_x} - 32\vec{a_y} + 5\vec{a_z}$ V/m -(2)

$\vec{D} = 424.99\vec{a_x} - 283.33\vec{a_y} + 44.27\vec{a_z}$ pC/m² -(1)

6 (b) Define Potential difference and Potential. Also Establish the relationship between electric field intensity and electric scalar potential. [07] CO2 L2

Definition V -(1)

V_{AB} -(1)

$\boxed{\vec{E} = -\nabla V}$ -(5)

7 (a) Determine the work done in carrying a charge of $-2\ C$ form $(2,1,-1)$ to $(8,2,-1)$ in the electric field $\boldsymbol{E} = y\,\boldsymbol{a_x} + x\boldsymbol{a_z}\ V/m$ through the path given by the parabola $x = 2\,y^2$. [06] CO2 L2

$W = -2\left\{\int y\,dx + \int x\,dy\right\} = -2\left\{\int_2^8 \sqrt{\frac{x}{2}}\,dx + \int_1^2 2y^2\,dy\right\}$ -(4)

$W = -28$ J -(1)

7 (b) Derive an expression for work done in moving a point charge Q in the presence of an electric field **E**. [04] CO2 L3

Definition -(1)

Derivation -(3)

8 Derive the expression for potential and electric field due to an electric dipole. [10] CO3 L2

Dipole -(2)

$V = \frac{Q}{4\pi\epsilon}\left[\frac{R_2 - R_1}{R_1R_2}\right]$ -(1)

$V = \frac{Qd\cos\theta}{4\pi\epsilon r^2} \rightarrow V = \frac{\vec{p}\cdot\vec{a_r}}{4\pi\epsilon r^2}$ -(3)

$\vec{E} = \frac{Qd\cos\theta}{2\pi\epsilon r^3}\vec{a_r} + \frac{Qd\sin\theta}{4\pi\epsilon r^3}\vec{a_\theta}$ -(4)

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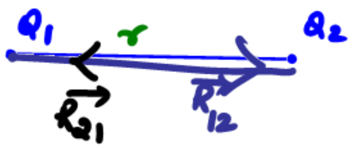
2.a

Coulomb's law:

The force between two very small objects separated in Vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them.

$$|\vec{F}| \propto \frac{Q_1 Q_2}{r^2}$$



$$\vec{F} = k \frac{Q_1 Q_2}{|\vec{R}|^2} \hat{a}_R$$

where

$$\boxed{k = 9\times10^9 = \frac{1}{4\pi\varepsilon_0}}$$ in vacuum (or) free space

$$\varepsilon_0 = \frac{1}{4\pi\times9\times10^9} = \frac{10^{-9}}{36\pi}$$

permittivity of freespace, $\varepsilon_0 = 8.854\times10^{-12}$ F/m

$$\boxed{\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{|\vec{R}|^2}\cdot\vec{a}_R}\ N$$

unit vector in the direction of $\vec{R}$

$$\vec{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

$$\boxed{\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{|\vec{R}|^3}\cdot\vec{R}}\ N$$

Force exerted on Q_2 by Q_1

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{|\vec{R}_{12}|^3}\cdot\vec{R}_{12}$$

Force exerted on Q_1 by Q_2

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{|\vec{R}_{21}|^3}\cdot\vec{R}_{21}$$

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

3.

Derive Maxwell's First Equation of Electrostatics from Gauss's Law: (Differential form of Gauss's law)

(or)

(Point form of Gauss's law)

Gauss's law:

$$\boxed{\psi = Q_{enc}} \Rightarrow \boxed{\oint_S \vec{D} \cdot \vec{ds} = Q_{enc}} \rightarrow (1)$$

Integral form of Gauss's law:

$$\boxed{\oint_S \vec{D} \cdot \vec{ds} = \iiint_V \rho_v \, dv} \rightarrow (2)$$

Divergence of a vector $\vec{A}$:

The divergence of the vector flux density **A** *is the outflow of flux from a small closed surface per unit volume as the volume shrinks to zero.*

$$\boxed{\vec{\nabla} \cdot \vec{A} = \lim_{\Delta v \to 0} \frac{\text{outward flux}}{\Delta v} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{A} \cdot \vec{ds}}{\Delta v}}$$

For electric flux density vector $\vec{D}$, by definition of divergence,

$$\boxed{\vec{\nabla} \cdot \vec{D} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{D} \cdot \vec{ds}}{\Delta v}} \rightarrow (3)$$

By definition of volume charge density,

$$\boxed{\rho_v = \lim_{\Delta v \to 0} \frac{Q}{\Delta v}} \rightarrow (4)$$

From ①, ③ & ④

$$\vec{\nabla}\cdot\vec{D} = \lim_{\Delta V \to 0} \frac{Q}{\Delta V} = \rho_v$$

$$\therefore \boxed{\vec{\nabla}\cdot\vec{D} = \rho_v} \rightarrow ⑤$$

Differential form (or) Point form of Gauss's law

Maxwell's first equation of Electrostatics

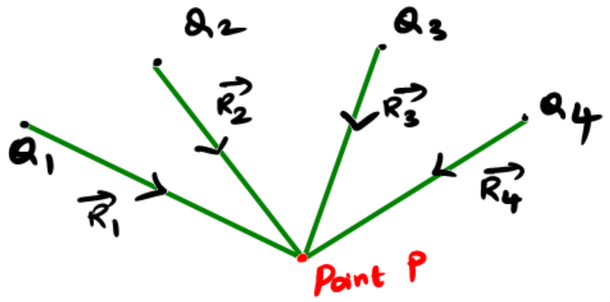
Sub ⑤ in ②

$$② \Rightarrow \oint\!\!\oint_S \vec{D}\cdot\vec{ds} = \iiint_V \rho_v\, dv = \iiint_V \vec{\nabla}\cdot\vec{D}\, dv$$

$$\Rightarrow \boxed{\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = \iiint_V \vec{\nabla}\cdot\vec{D}\, dv} \rightarrow ⑥$$

Gauss's Di...

4.b.Potential due to a system of 'N' number of point charges:



$R_1 = |\vec{R_1}|$

$R_2 = |\vec{R_2}|$

$R_3 = |\vec{R_3}|$

$R_4 = |\vec{R_4}|$

Potential at point P,

$$V = V_1 + V_2 + V_3 + V_4$$

$$V = \frac{Q_1}{4\pi\varepsilon R_1} + \frac{Q_2}{4\pi\varepsilon R_2} + \frac{Q_3}{4\pi\varepsilon R_3} + \frac{Q_4}{4\pi\varepsilon R_4}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{Q_i}{R_i}} \; V$$

6.b.

The potential difference V is defined as the work done (by an external source) in moving a unit positive charge from one point to another in an electric field.

The potential difference VAB may be regarded as the potential at B with reference to A

$$\boxed{V_{AB} = -\int_B^A \vec{E}\cdot\vec{dl}} \quad V$$

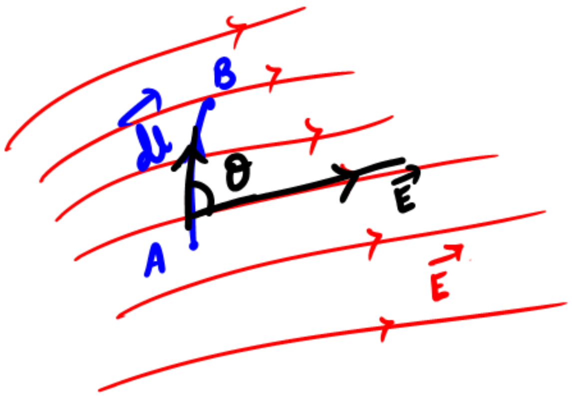
The potential at any point is the potential difference between that point and a chosen point (or reference point) at which the potential is zero.

VB and VA are the potentials (or absolute potentials) at B and A, respectively.

$$V_A = V_{A\infty} = -\int_\infty^A \vec{E}\cdot\vec{dl}$$

$$V_B = V_{B\infty} = -\int_\infty^B \vec{E}\cdot\vec{dl}$$

Show that electric field is the negative gradient of potential for electrostatics
(or) Derive the relation between electric field intensity and electric potential



$$\boxed{V_{BA} = -\int_A^B \vec{E}\cdot\vec{dl}}$$

(or)

$$\boxed{V = -\int_L \vec{E}\cdot\vec{dl}}$$

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

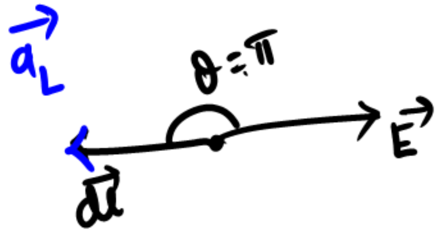
$$dV = -\vec{E}\cdot\vec{dl}$$

$$dV = -|\vec{E}||\vec{dl}|\cos\theta$$

$$dV = -E\,dl\cos\theta$$

$$E\cos\theta = -\frac{dV}{dl}$$

For $\theta = \pi$, $\boxed{E_{max} = \left.\frac{dV}{dl}\right|_{max}}$



$$\boxed{\text{Direction of } \vec{E},\ \vec{a}_E = -\vec{a}_L}$$

$$\vec{E} = \frac{dV}{dl}\cdot(-\vec{a}_L)$$

$$\boxed{\vec{E} = -\frac{dV}{dl}\vec{a}_L}$$

Rect.

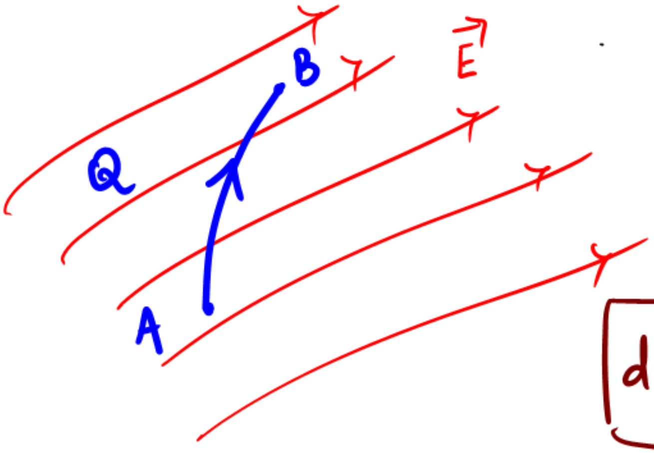
$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{a}_x + \frac{\partial}{\partial y}\vec{a}_y + \frac{\partial}{\partial z}\vec{a}_z$$

(or)

$$\boxed{\vec{E} = -\vec{\nabla}V = -\text{grad } V}$$

7.b.

Energy expended in moving a point charge in an electric field -
(Work Done)



Work done = Force x Displacement

Work done through displacement $\vec{dl}$

$$\boxed{dW = \vec{F}_{applied} \cdot \vec{dl}}$$

Force on Q due to $\vec{E}$, $\boxed{\vec{F} = Q\vec{E}}$ $\left[\because \vec{E} = \frac{\vec{F}}{Q}\right]$

$$\vec{F}_{applied} = -\vec{F} = -Q\vec{E}$$

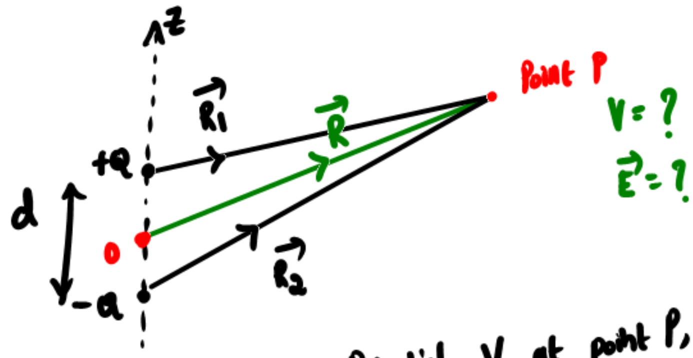
Work done through $\vec{dl}$, $dW = -Q\vec{E}\cdot\vec{dl}$

Work done through length L $\boxed{W = \int_L dW = \int_L -Q\,\vec{E}\cdot\vec{dl} = -Q\int_{initial}^{final} \vec{E}\cdot\vec{dl}}$ J

8.

Potential and Electric Field intensity due to Electric Dipole:

An electric dipole is formed when two point charges of equal magnitude but opposite sign are separated by a small distance.



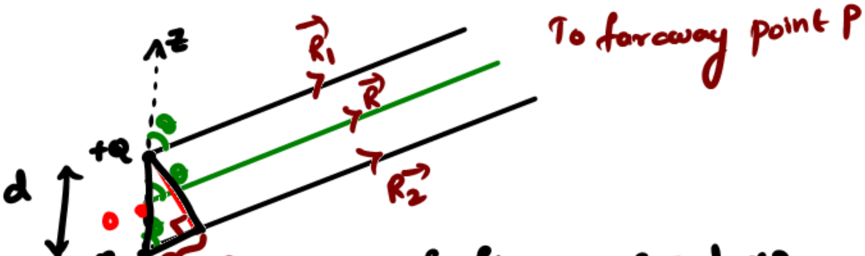
Potential V at point P,

$$V = V_{+Q} + V_{-Q}$$

$$V = \frac{Q}{4\pi\varepsilon R_1} + \frac{-Q}{4\pi\varepsilon R_2}$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon}\left[\frac{R_2 - R_1}{R_1 R_2}\right]}$$ V



$R_2 - R_1$ $\quad \cos\theta = \frac{R_2 - R_1}{d} \Rightarrow R_2 - R_1 = d\cos\theta$

$R_1 \simeq R_2 \simeq R \qquad R_1 R_2 = R^2$

$$\boxed{V = \frac{Q}{4\pi\varepsilon}\left[\frac{d\cos\theta}{R^2}\right]}$$

In spherical system $R = r$

$$\boxed{V = \frac{Q\,d\cos\theta}{4\pi\varepsilon\, r^2}}$$

Potential field due to electric dipole

Electric field due to electric dipole, $\vec{E} = -\vec{\nabla} V$

In spherical system,

$$\vec{E} = -\left[\frac{\partial V}{\partial r}\vec{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{a_\phi}\right]$$

$$\vec{E} = -\left[\frac{Qd\cos\theta}{4\pi\varepsilon}\left(\frac{-2}{r^3}\right)\vec{a_r} + \frac{1}{r}\frac{Qd}{4\pi\varepsilon r^2}(-\sin\theta)\vec{a_\theta} + 0\vec{a_\phi}\right]$$

Electric field due to electric dipole

$$\boxed{\vec{E} = \frac{Qd\cos\theta}{2\pi\varepsilon r^3}\vec{a_r} + \frac{Qd\sin\theta}{4\pi\varepsilon r^3}\vec{a_\theta}} \quad V/m$$

Dipole Moment

Dipole moment, $\boxed{\vec{p} = Q\vec{d}}$ Cm $\rightarrow \boxed{p = Qd}$

$$\vec{d}\cdot\vec{a_r} = |\vec{d}||\vec{a_r}|\cos\theta$$

$$\vec{d}\cdot\vec{a_r} = d\times 1\times\cos\theta = d\cos\theta$$

Potential V due to dipole in terms of Dipole moment

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon r^2} = \frac{Q\vec{d}\cdot\vec{a_r}}{4\pi\varepsilon r^2}$$

$$\boxed{V = \frac{\vec{p}\cdot\vec{a_r}}{4\pi\varepsilon r^2}} \quad V$$

Electric Field Intensity due to dipole in terms of Dipole moment

$$\vec{E} = \frac{Qd\cos\theta}{2\pi\varepsilon r^3}\vec{a_r} + \frac{Qd\sin\theta}{4\pi\varepsilon r^3}\vec{a_\theta}$$

$$\boxed{\vec{E} = \frac{p\cos\theta}{2\pi\varepsilon r^3}\vec{a_r} + \frac{p\sin\theta}{4\pi\varepsilon r^3}\vec{a_\theta}} \quad V/m$$