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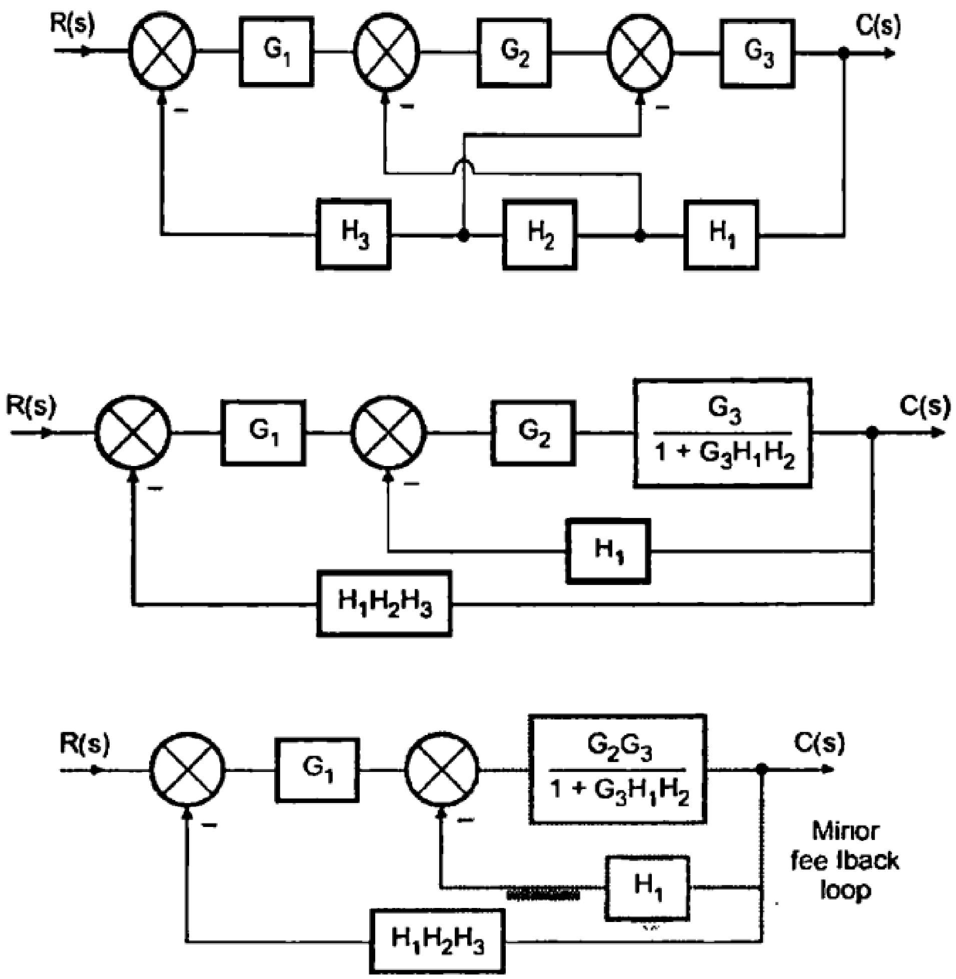
Internal Assessment Test – II - Solutions

Sub:	**Control Systems**							Code:	18EE61
Date:	08.06.2022	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	Determine C(S)/R(S) using the block diagram reduction technique.	10	CO2	L3



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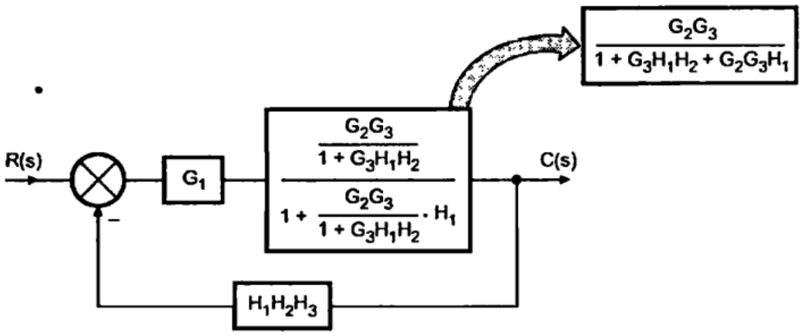

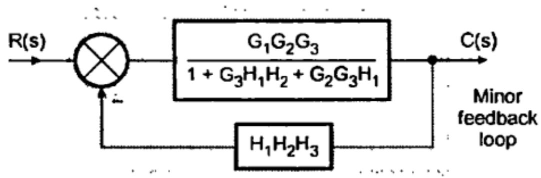

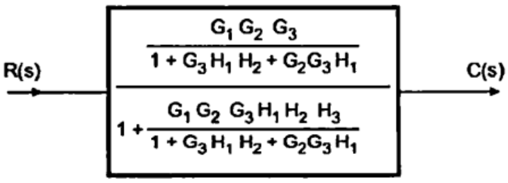

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_3 H_1 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 H_1 H_2 H_3}$$

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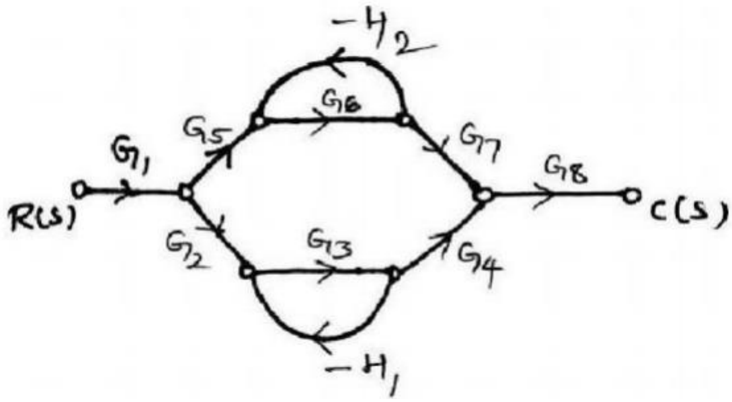
<tr><td>2</td><td>

Determine C(S)/R(S) using Mason’s gain formula.



Solution : Number of forward paths = K = 2

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\sum_{K=1}^{2} T_K \Delta_K}{\Delta} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta} \quad \text{... Mason's gain formula}$$

$\therefore \quad T_1 = G_1 G_5 G_6 G_7 G_8 \quad T_2 = G_1 G_2 G_3 G_4 G_8$

Individual loops are,

$L_1 = -G_6 H_6 \qquad L_2 = -G_3 H_3$

Both L_1 and L_2 are non touching to each other.

$$\therefore \quad \Delta = 1 - [L_1 + L_2] + [L_1 L_2]$$
$$= 1 + G_6 H_6 + G_3 H_3 + G_3 G_6 H_3 H_6$$

</td><td>10</td><td>CO2</td><td>L3</td></tr>
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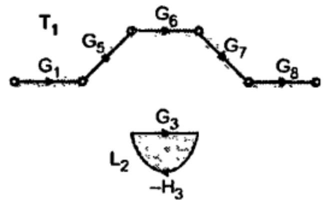
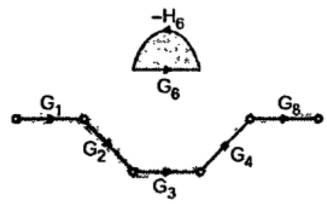
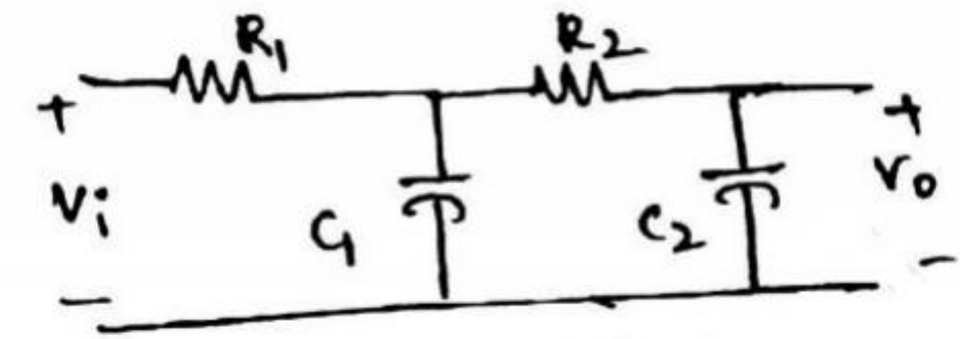
	Consider T_1 , L_2 is non touching  $\therefore \quad \Delta_1 = 1 - L_2 = 1 + G_3 H_3$ Consider T_2 , L_1 is non touching  $\therefore \quad \Delta_2 = 1 - L_1 = 1 + G_6 H_6$ $\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta}$ $= \frac{G_1 G_5 G_6 G_7 G_8 (1 + G_3 H_3) + G_1 G_2 G_3 G_4 G_8 (1 + G_6 H_6)}{\Delta}$ $\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_5 G_6 G_7 G_8 (1 + G_3 H_3) + G_1 G_2 G_3 G_4 G_8 (1 + G_6 H_6)}{1 + G_6 H_6 + G_3 H_3 + G_3 G_6 H_3 H_6}}$			
3	For the network shown in the figure, Construct the signal flowgraph and find the transfer function using mason's gain formula. Given R_1=100KΩ, R_2=1MΩ, C_1=10µf and C_2=1µf  	10	CO2	L3

Writing down equations for I_1, V_1, I_2, V_o we get,

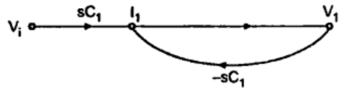
$$I_1 = \frac{(V_i - V_1)}{\frac{1}{sC_1}} = sC_1 (V_i - V_o) \quad \text{... (I)}$$

$$V_1 = (I_1 - I_2) R_1 \quad \text{... (II)}$$

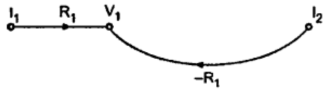
$$I_2 = \frac{(V_1 - V_o)}{\frac{1}{sC_2}} = sC_2 (V_1 - V_o) \quad \text{... (III)}$$

$$V_o = I_2 R_2 \quad \text{... (IV)}$$

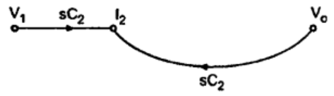
Simulating above equations by signal flow graph.



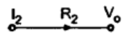
For equation (I)



For equation (II)

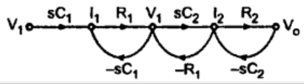


For equation (III)



For equation (IV)

Combining we get signal flow graph for given network.



To find T.F. apply Mason's gain formula

$$\text{T.F.} = \frac{\Sigma T_K \Delta_1}{\Delta} \qquad \text{Number of forward path} = K = 1$$

$$\therefore \quad \text{T.F.} = \frac{T_1 \Delta_1}{\Delta}$$

$$T_1 = sC_1 R_1 sC_2 R_2 = s^2 R_1 R_2 C_1 C_2$$

Individual loops :

$$L_1 = -R_1 C_1 s, \quad L_2 = -sR_1 C_2 \quad L_3 = -R_2 sC_2$$

Out of three, L_1 and L_3 are nontouching

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$$

$$= 1 - [-sR_1 C_1 - sR_1 C_2 - sR_2 C_2] + [s^2 R_1 C_1 R_2 C_2]$$

$$= 1 + s[R_1 C_1 + R_1 C_2 + R_2 C_2] + s^2 [R_1 C_1 R_2 C_2]$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{T_1 \Delta_1}{\Delta}$$

All loops are touching to T_1, $\therefore \Delta_1 = 1$

$$\therefore \quad \boxed{\frac{V_o(s)}{V_i(s)} = \frac{s^2 R_1 C_1 R_2 C_2}{1 + s[R_1 C_1 + R_1 C_2 + R_2 C_2] + s^2 [R_1 C_1 R_2 C_2]}}$$

4	For the system shown in figure 1, Draw the signal flowgraph and find the transfer function using mason's gain formula.	10	CO2	L3

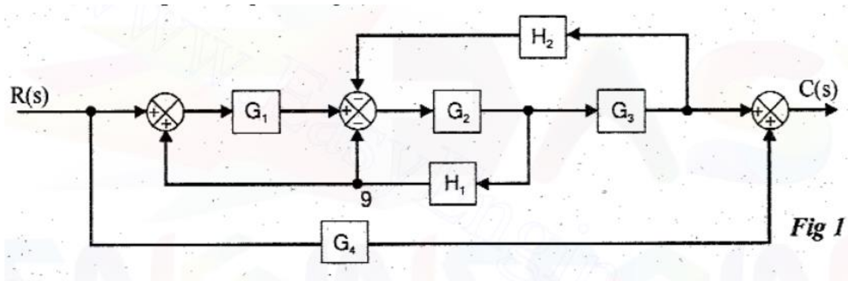


Fig 1

The signal flow graph for the above block diagram is shown in fig 3.

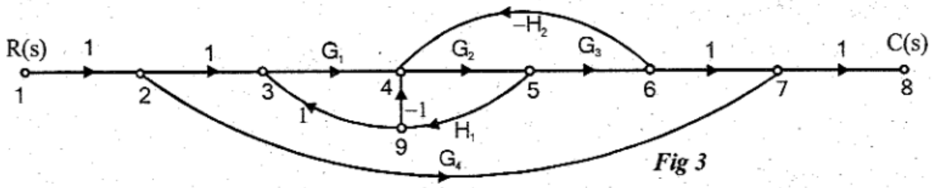


Fig 3

Forward Path Gains

There are two forward path, ∴ K=2.

Let the forward path gains be P_1 and P_2.

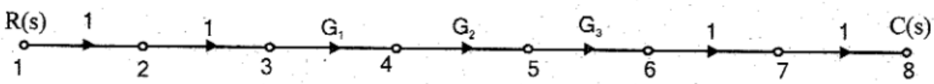


Fig 4 : Forward path-1.

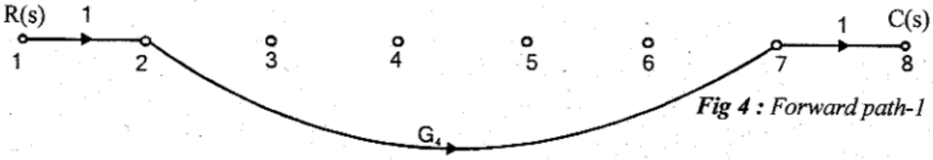


Fig 4 : Forward path-1

Gain of forward path-1, $P_1 = G_1G_2G_3$

Gain of forward path-2, $P_2 = G_4$

II. Individual Loop Gain

There are three individual loops with gains P_{11}, P_{21} and P_{31}.

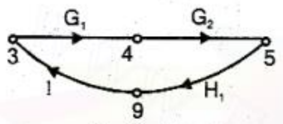


Fig 6 : loop-1.

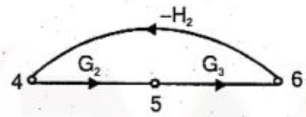


Fig 7 : loop-2.

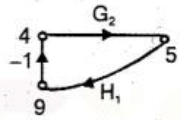


Fig 8 : loop-3.

Gain of individual loop-1, $P_{11} = G_1G_2H_1$

Gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Gain of individual loop-3, $P_{31} = -G_2H_1$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two-non touching loops, three non-touching loops, etc.,.

IV. Calculation of Δ and Δ_K

$$\Delta = 1-[P_{11}+P_{21}+P_{31}] = 1 - G_1G_2H_1 + G_2G_3H_2 + G_2H_1$$

Since no part of graph touches forward path-1, $\Delta_1 = 1$.

The part of graph non touching forward path-2 is shown in fig 9.

$$\therefore \Delta_2 = 1-[G_1G_2H_1 - G_2G_3H_2 - G_2H_1]$$
$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_2H_1$$

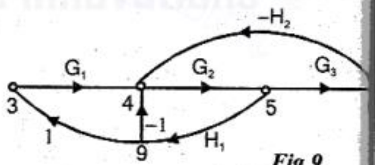


Fig 9

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

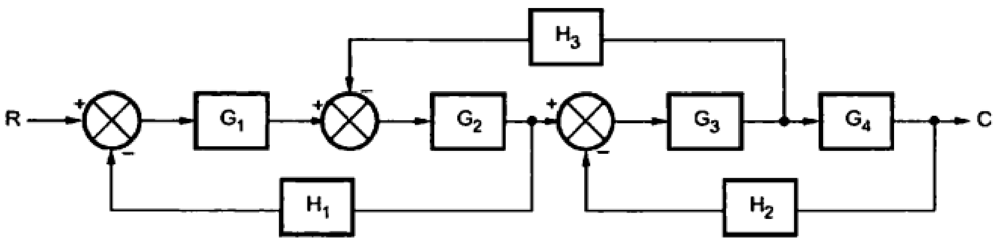
$$T = \frac{1}{\Delta}\sum_k P_k\Delta_k = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2]$$ (Number of forward paths is 2 and so K = 2)

$$= \frac{1}{\Delta}[G_1G_2G_3 + G_4(1 - G_1G_2H_1 + G_2G_3H_2 + G_2H_1)]$$

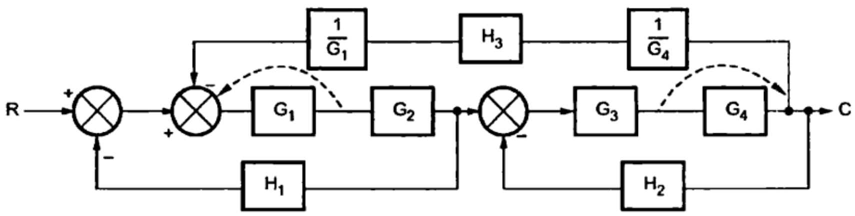
$$= \frac{1}{\Delta}[G_1G_2G_3 + G_4 - G_1G_2G_4H_1 + G_2G_3G_4H_2 + G_2G_4H_1]$$

$$= \frac{G_1G_2G_3 + G_4 - G_1G_2G_4H_1 + G_2G_3G_4H_2 + G_2G_4H_1}{1 - G_1G_2H_1 + G_2G_3H_2 + G_2H_1}$$

5 | Determine C/R using the block diagram reduction technique. | CO2 | 10 | L3

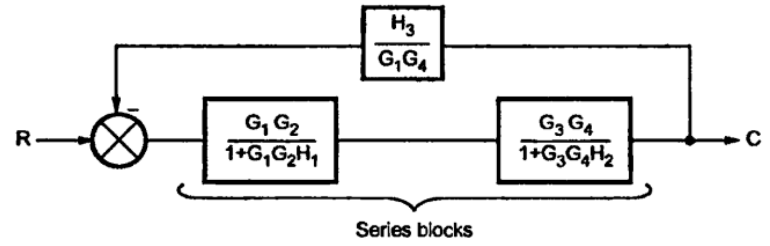


Solution : Shifting summing point before G_1 and takeoff point after G_4 we get,



Exchanging summing points and takeoff points using associative law and combining series blocks we get,

Eliminating minor feedback loops we get,

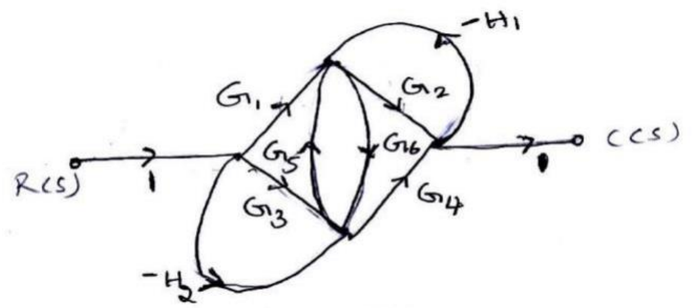


$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\left[\frac{G_1G_2}{1+G_1G_2H_1}\right]\left[\frac{G_3G_4}{1+G_3G_4H_2}\right]}{1+\left[\frac{G_1G_2}{1+G_1G_2H_1}\right]\left[\frac{G_3G_4}{1+G_3G_4H_2}\right]\left[\frac{H_3}{G_1G_4}\right]}$$

$$= \frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)+G_2G_3H_3}$$

$$\therefore \quad \boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_1G_2H_1+G_3G_4H_2+G_1G_2G_3G_4H_1H_2+G_2G_3H_3}}$$

6 Determine C(S)/R(S) using Mason's gain formula. | CO2 | 10 | L3



Solution : Number of forward paths K = 4

$$T_1 = 1 \cdot G_1 \cdot G_2 \cdot 1 = G_1 G_2$$
$$T_2 = 1 \cdot G_3 \cdot G_4 \cdot 1 = G_3 G_4$$
$$T_3 = 1 \cdot G_1 \cdot G_6 \cdot G_4 \cdot 1 = G_1 G_6 G_4$$
$$T_4 = 1 \cdot G_3 \cdot G_5 \cdot G_2 \cdot 1 = G_2 G_3 G_5$$

Individual loops are

$L_1 = -G_2 H_1$, $L_2 = -G_3 H_2$, $L_3 = G_5 G_6$

$L_4 = -G_4 H_1 G_6$, $L_5 = -G_1 G_6 H_2$

Combinations of two non touching loops are

i) L_1 and L_2

ii) No combination of three non touching loops

$$\Delta = 1-(L_1+L_2+L_3+L_4+L_5)+(L_1 L_2)$$

$$= 1-[(-G_2 H_1 - G_3 H_2 + G_5 G_6 - G_4 G_6 H_1 - G_1 G_6 H_2)]+[(-G_2 H_1)(-G_3 H_2)]$$

$$\therefore \quad \Delta = 1+G_2 H_1+G_3 H_2-G_5 G_6+G_4 G_6 H_1+G_1 G_6 H_2+G_2 G_3 H_1 H_2$$

For all the forward paths all the loops are touching hence

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$$

Hence $\frac{C(s)}{R(s)} = \frac{\Sigma T_K \Delta_K}{\Delta} = \frac{T_1\Delta_1+T_2\Delta_2+\Gamma_3\Delta_3+T_4\Delta_4}{\Delta}$

$$\therefore \quad \boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2+G_3 G_4+G_1 G_6 G_4+G_2 G_3 G_5}{1+G_2 H_1+G_3 H_2-G_5 G_6+G_5 G_6 H_1+G_1G_6H_2+G_2G_3H_1H_2}} \cdots$$

7	Derive the expression for unit step response of the second-order underdamped system.	CO3	10	L3

The standard form of closed loop transfer function of second order system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For underdamped system, $0 < \zeta < 1$ and roots of the denominator (characteristic equation) are complex conjugate.

The roots of the denominator are, $s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

Since $\zeta < 1$, ζ^2 is also less then 1, and so $1 - \zeta^2$ is always positive.

$$\therefore\ s = -\zeta\omega_n \pm \omega_n\sqrt{(-1)(1-\zeta^2)} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

The damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\therefore s = -\zeta\omega_n \pm j\omega_d$$

The response in s-domain, $C(s) = R(s)\,\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

For unit step input, r(t) = 1 and R(s) = 1/s.

$$\therefore C(s) = \frac{\omega_n^2}{s\,(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

A is obtained by multiplying C(s) by s and letting s = 0.

$$\therefore\ A = s \times C(s)\big|_{s=0} = s \times \frac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}\Bigg|_{s=0} = \frac{\omega_n^2}{\omega_n^2} = 1$$

To solve for B and C, cross multiply equation (2.25) and equate like power of s.

On cross multiplication equation (2.25) after substituting A = 1, we get,

$$\omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 + (Bs + C)s$$

$$\omega_n^2 = s^2 + 2\zeta\omega_n s + \omega_n^2 + Bs^2 + Cs$$

Equating coefficients of s^2 we get, $0 = 1 + B$ $\quad \therefore B = -1$

Equating coefficient of s we get, $0 = 2\zeta\omega_n + C$ $\quad \therefore C = -2\zeta\omega_n$

$$\therefore\ C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad .....(2.26)$$

Let us add and subtract $\zeta^2\omega_n^2$ to the denominator of second term in the equation (2.26).

$$\therefore\ C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2 + \zeta^2\omega_n^2 - \zeta^2\omega_n^2} = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2) + (\omega_n^2 - \zeta^2\omega_n^2)}$$

$$= \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)} = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

$$\boxed{\omega_d = \omega_n\sqrt{1-\zeta^2}}$$

$$= \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} \quad .....(2.27)$$

Let us multiply and divide by ω_d in the third term of the equation (2.27).

$$\therefore\ C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{\omega_d}\,\frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

The response in time domain is given by,

$$\boxed{\mathcal{L}\{e^{-at}\sin\omega t\} = \frac{\omega}{(s+a)^2 + \omega^2}}$$

$$\therefore\ C(s)=\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{\omega_d}\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$\mathcal{L}\{1\}=\frac{1}{s}$

$\mathcal{L}\{e^{-at}\sin\omega t\}=\frac{\omega}{(s+a)^2+\omega^2}$

$\mathcal{L}\{e^{-at}\cos\omega t\}=\frac{s+a}{(s+a)^2+\omega^2}$

The response in time domain is given by,

$$c(t)=\mathcal{L}^{-1}\{C(s)\}=\mathcal{L}^{-1}\left\{\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{\omega_d}\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right\}$$

$$=1-e^{-\zeta\omega_n t}\cos\omega_d t-\frac{\zeta\omega_n}{\omega_d}e^{-\zeta\omega_n t}\sin\omega_d t=1-e^{-\zeta\omega_n t}\left(\cos\omega_d t+\frac{\zeta\omega_n}{\omega_n\sqrt{1-\zeta^2}}\sin\omega_d t\right)$$

$\omega_d=\omega_n\sqrt{1-\zeta^2}$

$$=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sqrt{1-\zeta^2}\ \cos\omega_d t+\zeta\ \sin\omega_d t\right)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\sin\omega_d t\times\zeta+\cos\omega_d t\times\sqrt{1-\zeta^2}\right)$$

Let us express c(t) in a standard form as shown below.

$$c(t)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(\sin\omega_d t\times\cos\theta+\cos\omega_d t\times\sin\theta)$$

$$=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\theta) \qquad .....(2.28)$$

$$\text{where, }\left(\theta=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

***Note :** On constructing right angle triangle with ζ and $\sqrt{1-\zeta^2}$, we get*

$\sin\theta=\sqrt{1-\zeta^2}$

$\cos\theta=\zeta$

$\tan\theta=\frac{\sqrt{1-\zeta^2}}{\zeta}$