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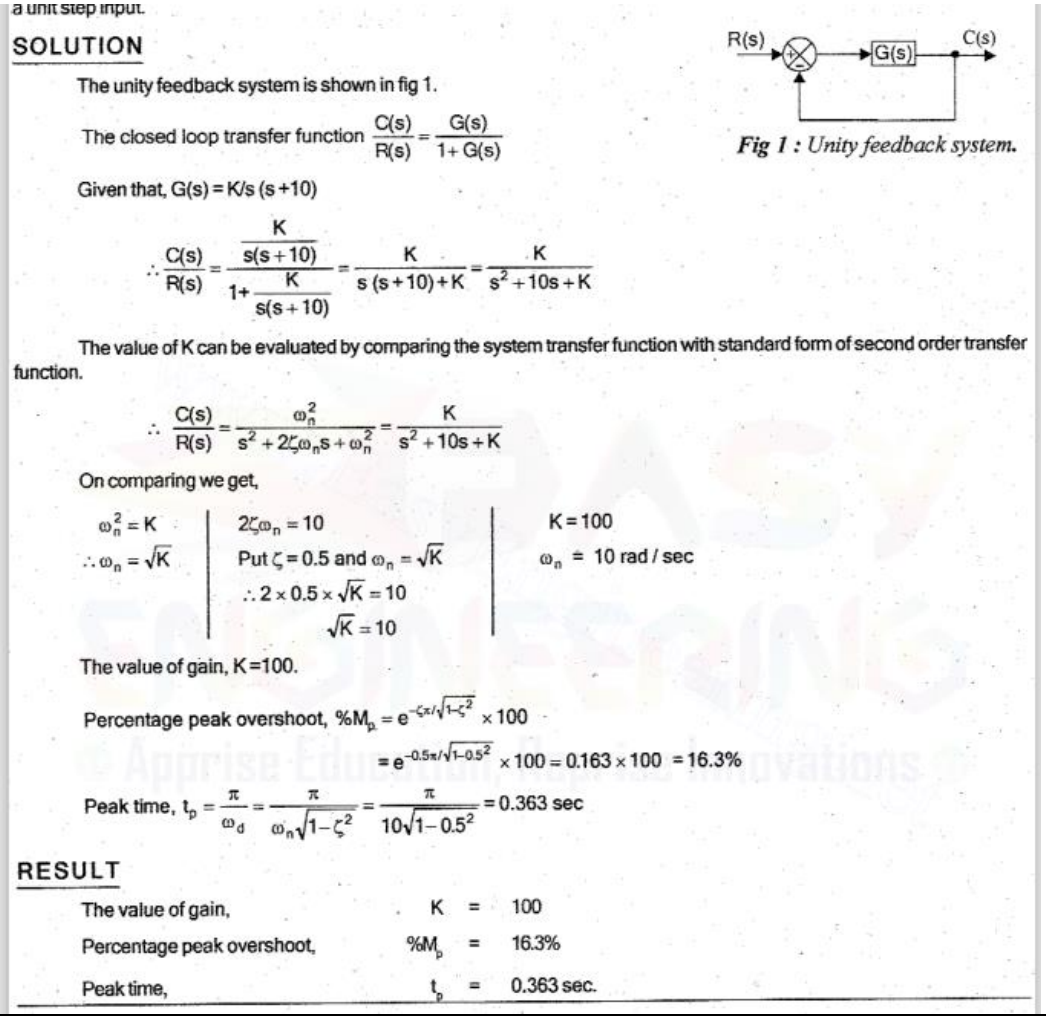
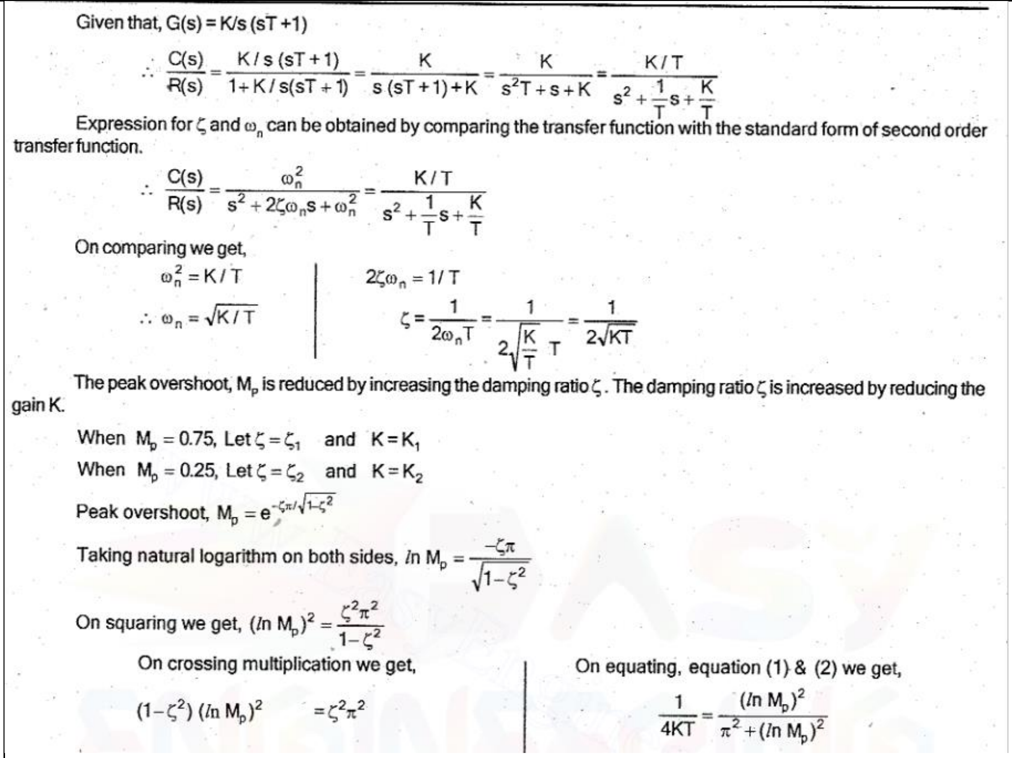
CMR
INSTITUTE OF
TECHNOLOGY

Internal Assessment Test - III

Sub:	**Control Systems – A Section**							Code:	18EE61
Date:	08.07.2022	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	EEE

Answer Any FIVE FULL Questions

<table>
<tr><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2">Marks</td><td colspan="2">OBE</td></tr>
<tr><td>CO</td><td>RBT</td></tr>
<tr><td>1</td><td>To determine the rise time, Peak time, Peak overshoot and settling time of a unity feedback system with open loop transfer function $G(s) = \frac{K}{s(s+10)}$ so that the system will have a damping ratio of 0.5. Compare theoretical and Practical values.

a unit step input.
SOLUTION
The unity feedback system is shown in fig 1.
The closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$
Fig 1 : Unity feedback system.
Given that, G(s) = K/s (s +10)
$\therefore \frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}} = \frac{K}{s\,(s+10)+K} = \frac{K}{s^2+10s+K}$
The value of K can be evaluated by comparing the system transfer function with standard form of second order transfer function.
$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{K}{s^2+10s+K}$
On comparing we get,
$\omega_n^2 = K$
$\therefore \omega_n = \sqrt{K}$
$2\zeta\omega_n = 10$
Put $\zeta = 0.5$ and $\omega_n = \sqrt{K}$
$\therefore 2 \times 0.5 \times \sqrt{K} = 10$
$\sqrt{K} = 10$
$K = 100$
ω_n = 10 rad / sec
The value of gain, K=100.
Percentage peak overshoot, $\%M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$
$= e^{-0.5\pi/\sqrt{1-0.5^2}} \times 100 = 0.163 \times 100 = 16.3\%$
Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{10\sqrt{1-0.5^2}} = 0.363$ sec
RESULT
The value of gain, K = 100
Percentage peak overshoot, $\%M_p$ = 16.3%
Peak time, t_p = 0.363 sec.</td><td>10</td><td>CO3</td><td>L3</td></tr>
<tr><td>2</td><td>The open-loop transfer function of a unity feedback control system is given by $G(s) = \frac{K}{s(sT+1)}$. By what factor should the amplifier gain K be reduced, so that the peak overshoot of unit step response of the system is reduced from 70% to 20%.</td><td>10</td><td>CO3</td><td>L3</td></tr>
</table>



Given that, G(s) = K/s (sT +1)

$$\therefore \frac{C(s)}{R(s)} = \frac{K/s(sT+1)}{1+K/s(sT+1)} = \frac{K}{s(sT+1)+K} = \frac{K}{s^2T+s+K} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

Expression for ζ and ω_n can be obtained by comparing the transfer function with the standard form of second order transfer function.

$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

On comparing we get,

$$\omega_n^2 = K/T \qquad \therefore \omega_n = \sqrt{K/T}$$

$$2\zeta\omega_n = 1/T \qquad \zeta = \frac{1}{2\omega_n T} = \frac{1}{2\sqrt{\frac{K}{T}}\,T} = \frac{1}{2\sqrt{KT}}$$

The peak overshoot, M_p is reduced by increasing the damping ratio ζ. The damping ratio ζ is increased by reducing the gain K.

When $M_p = 0.75$, Let $\zeta = \zeta_1$ and $K = K_1$

When $M_p = 0.25$, Let $\zeta = \zeta_2$ and $K = K_2$

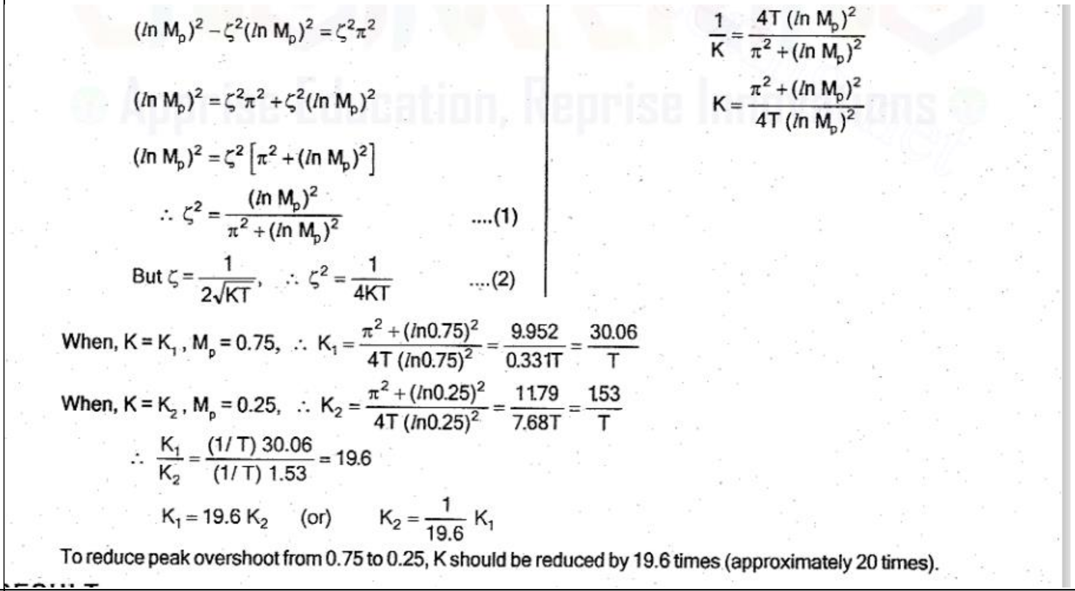
Peak overshoot, $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

Taking natural logarithm on both sides, $\ln M_p = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$

On squaring we get, $(\ln M_p)^2 = \frac{\zeta^2\pi^2}{1-\zeta^2}$

On crossing multiplication we get,

$$(1-\zeta^2)\,(\ln M_p)^2 = \zeta^2\pi^2$$



$$(\ln M_p)^2 - \zeta^2(\ln M_p)^2 = \zeta^2\pi^2$$

$$(\ln M_p)^2 = \zeta^2\pi^2 + \zeta^2(\ln M_p)^2$$

$$(\ln M_p)^2 = \zeta^2\left[\pi^2 + (\ln M_p)^2\right]$$

$$\therefore \zeta^2 = \frac{(\ln M_p)^2}{\pi^2+(\ln M_p)^2} \qquad ....(1)$$

$$\text{But } \zeta = \frac{1}{2\sqrt{KT}}, \quad \therefore \zeta^2 = \frac{1}{4KT} \qquad ....(2)$$

On equating, equation (1) & (2) we get,

$$\frac{1}{4KT} = \frac{(\ln M_p)^2}{\pi^2+(\ln M_p)^2}$$

$$\frac{1}{K} = \frac{4T\,(\ln M_p)^2}{\pi^2+(\ln M_p)^2}$$

$$K = \frac{\pi^2+(\ln M_p)^2}{4T\,(\ln M_p)^2}$$

When, $K = K_1$, $M_p = 0.75$, $\therefore K_1 = \frac{\pi^2+(\ln 0.75)^2}{4T\,(\ln 0.75)^2} = \frac{9.952}{0.331T} = \frac{30.06}{T}$

When, $K = K_2$, $M_p = 0.25$, $\therefore K_2 = \frac{\pi^2+(\ln 0.25)^2}{4T\,(\ln 0.25)^2} = \frac{11.79}{7.68T} = \frac{1.53}{T}$

$$\therefore \frac{K_1}{K_2} = \frac{(1/T)\,30.06}{(1/T)\,1.53} = 19.6$$

$$K_1 = 19.6\,K_2 \quad \text{(or)} \quad K_2 = \frac{1}{19.6}K_1$$

To reduce peak overshoot from 0.75 to 0.25, K should be reduced by 19.6 times (approximately 20 times).

3	A unity feedback system with open loop transfer function $G(s) = \frac{0.4s+1}{s(s+0.6)}$. Determine its transient response for unit step input. Evaluate the maximum overshoot and the corresponding peak time, Rise time, Settling time.	10	CO3	L3

The closed loop transfer function, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)}$

Given that, G(s) = (0.4 s +1)/s(s +0.6)

For unity feedback system, H(s) = 1.

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{0.4\,s+1}{s(s+0.6)}}{1+\frac{0.4\,s+1}{s(s+0.6)}} = \frac{0.4\,s+1}{s\,(s+0.6)+0.4\,s+1}$$

$$= \frac{0.4\,s+1}{s^2+0.6s+0.4s+1} = \frac{0.4\,s+1}{s^2+s+1}$$

The s - domain response, $C(s) = R(s) \times \frac{0.4\,s+1}{s^2+s+1}$

For step input, R(s) = 1/s.

$$\therefore\ C(s) = \frac{1}{s}\ \frac{0.4\,s+1}{s^2+s+1} = \frac{0.4\,s+1}{s(s^2+s+1)}$$

By partial fraction expansion C(s) can be expressed as,

$$C(s) = \frac{0.4\,s+1}{s\,(s^2+s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+s+1}$$

The residue A is solved by multiplying C(s) by s and letting s = 0.

$$\therefore\ A = C(s)\ \times\ s\Big|_{s=0} = \frac{0.4\,s+1}{s^2+s+1}\Bigg|_{s=0} = 1$$

The residues B and C are solved by cross multiplying the following equation and equating the coefficients of like powers of s.

$$\frac{0.4\,s+1}{s(s^2+s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+s+1}$$

On cross multiplication we get,

$$0.4\,s+1 = A(s^2+s+1)+(Bs+C)\,s$$

$$0.4\,s+1 = As^2+As+A+Bs^2+Cs$$

On equating coefficients of s^2 we get, $0 = A+B$ $\quad\therefore B = -A = -1$

On equating coefficients of s we get, $0.4 = A+C$ $\quad\therefore C = 0.4-A = -0.6$

$$\therefore\ C(s) = \frac{1}{s} + \frac{-s-0.6}{s^2+s+1} = \frac{1}{s} - \frac{s+0.6}{s^2+s+0.25+0.75} = \frac{1}{s} - \frac{s+0.6}{\left(s^2+2\times0.5s+0.5^2\right)+0.75}$$

The transient response of $c(t) = e^{-0.5t}\left[0.1155\ \sin(\sqrt{0.75}\ t)+\cos(\sqrt{0.75}\ t)\right]$

The value of ζ and ω_n can be estimated by comparing the characteristic equation of the system with standard form of econd order characteristic equation.

$$\therefore s^2+2\zeta\omega_n s+\omega_n{}^2 = s^2+s+1$$

On comparing we get,

$\omega_n^2 = 1$ | $2\zeta\omega_n = 1$

$\therefore\ \omega_n = 1$ rad / sec | $\therefore\ \zeta = \frac{1}{2\omega_n} = \frac{1}{2} = 0.5$

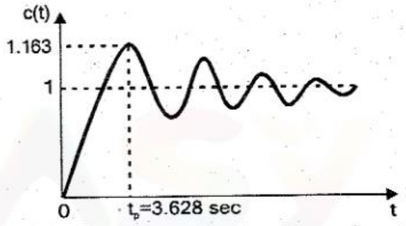


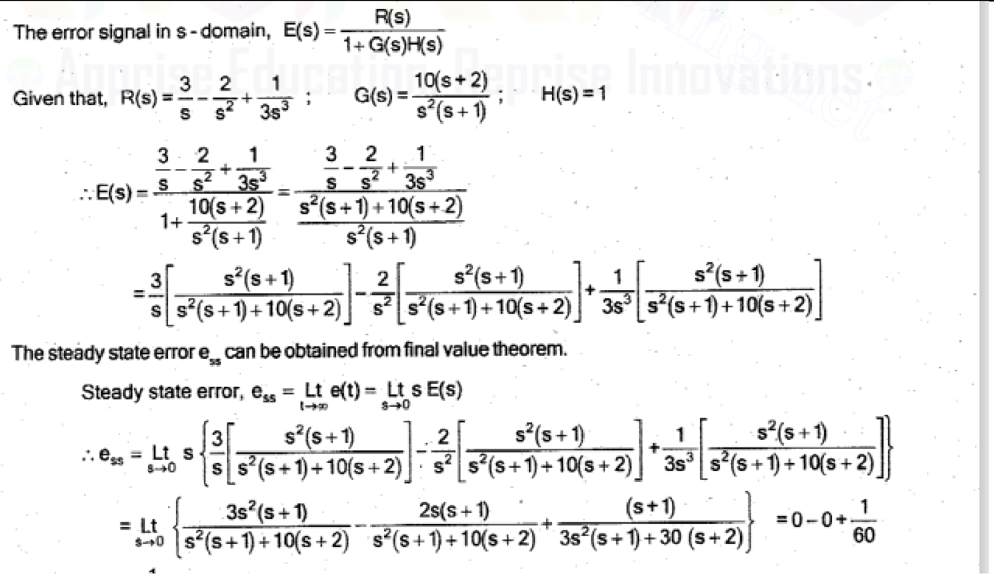
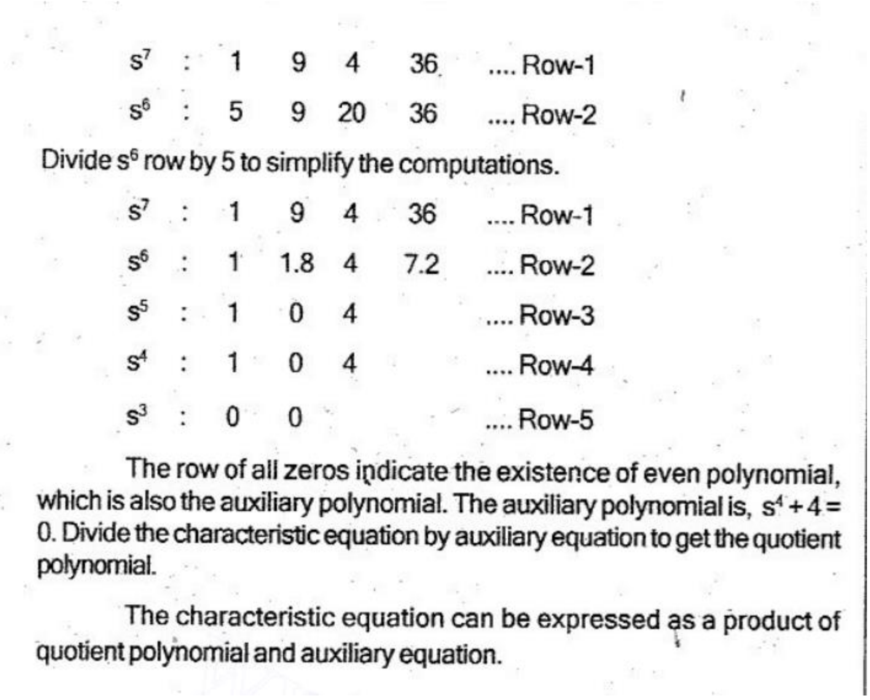
Fig 1 : Response of under damped system.

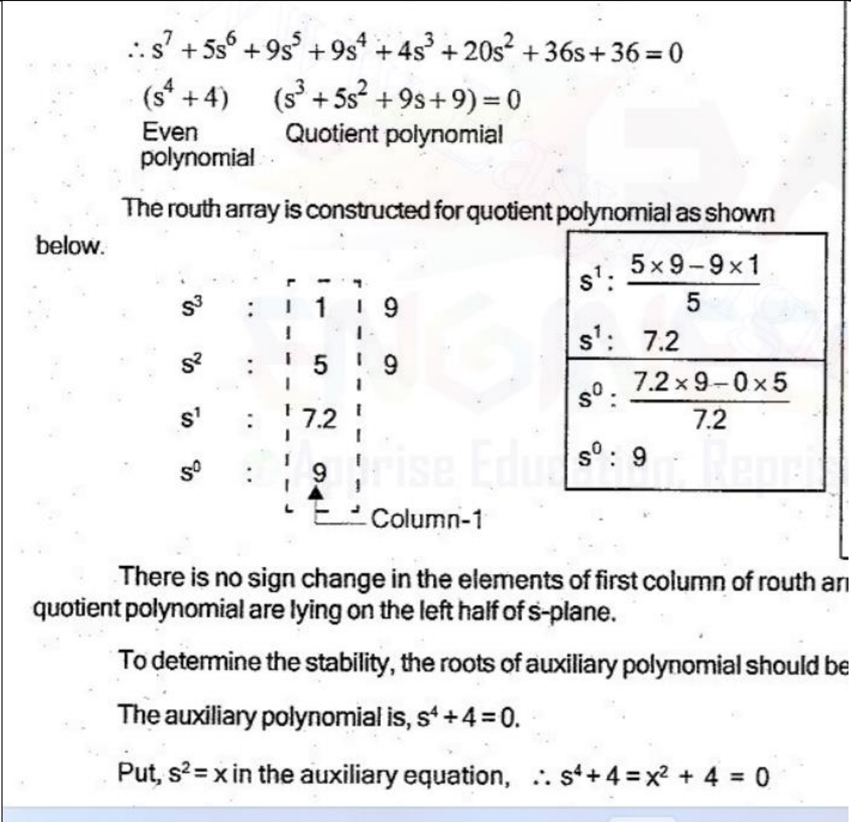
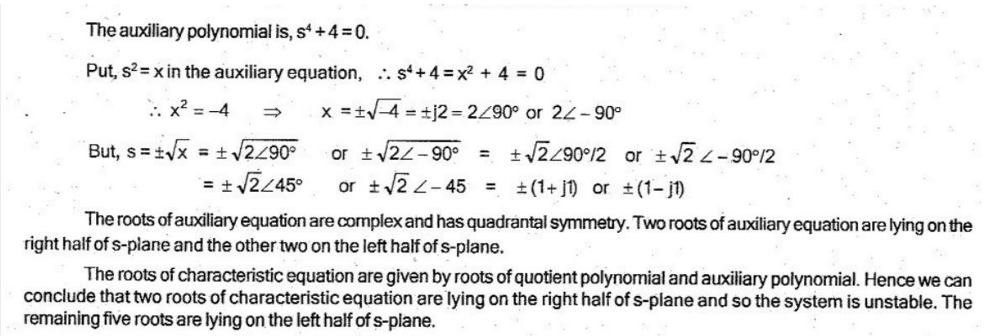
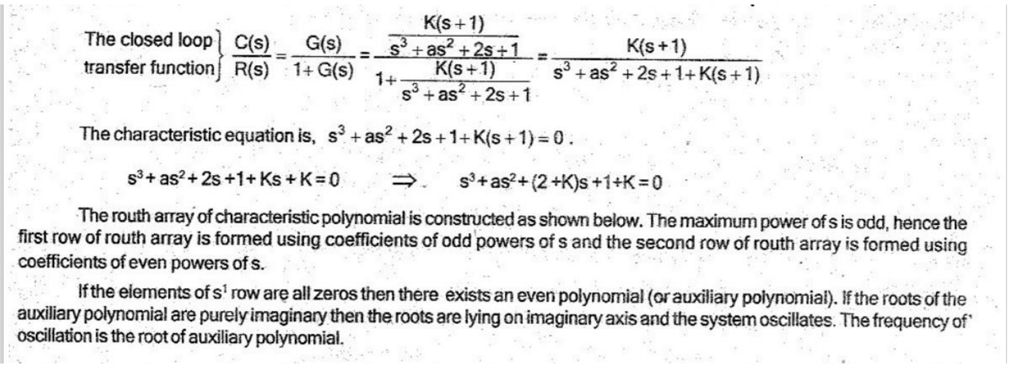
Maximum overshoot, $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{\frac{-0.5\pi}{\sqrt{1-0.5^2}}} = 0.163$

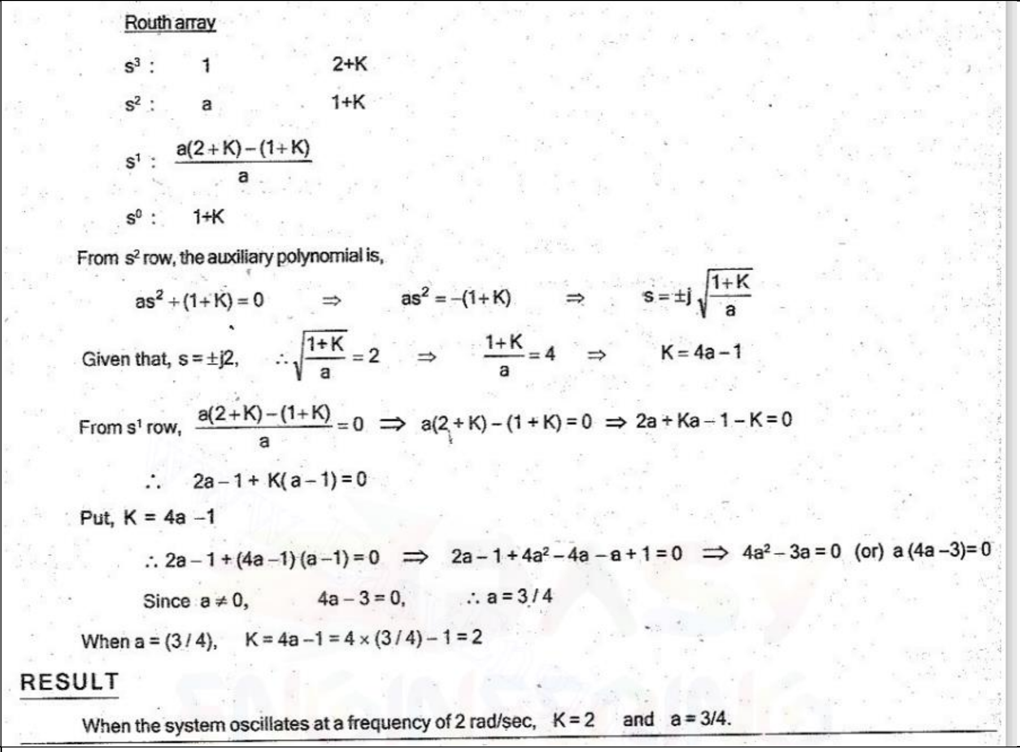
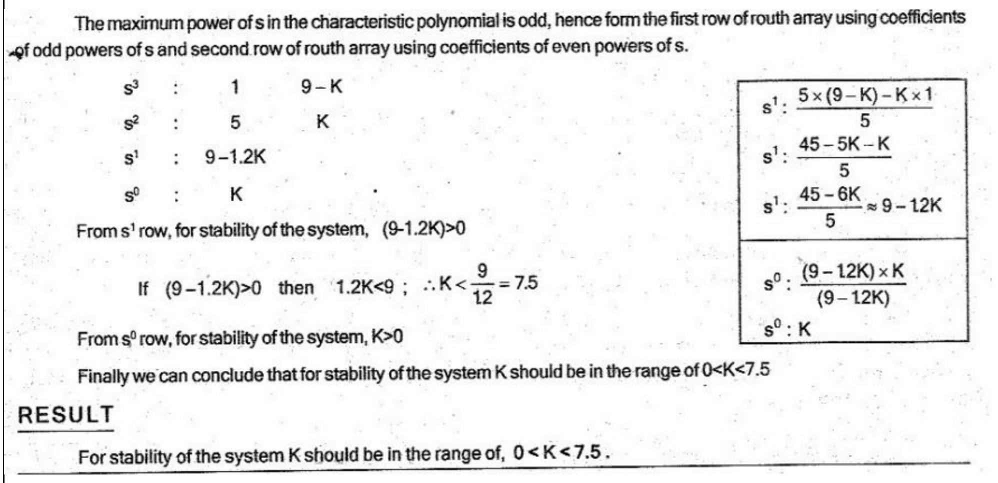
% Maximum overshoot, $\%M_p = M_p \times 100 = 0.163 \times 100 = 16.3\%$

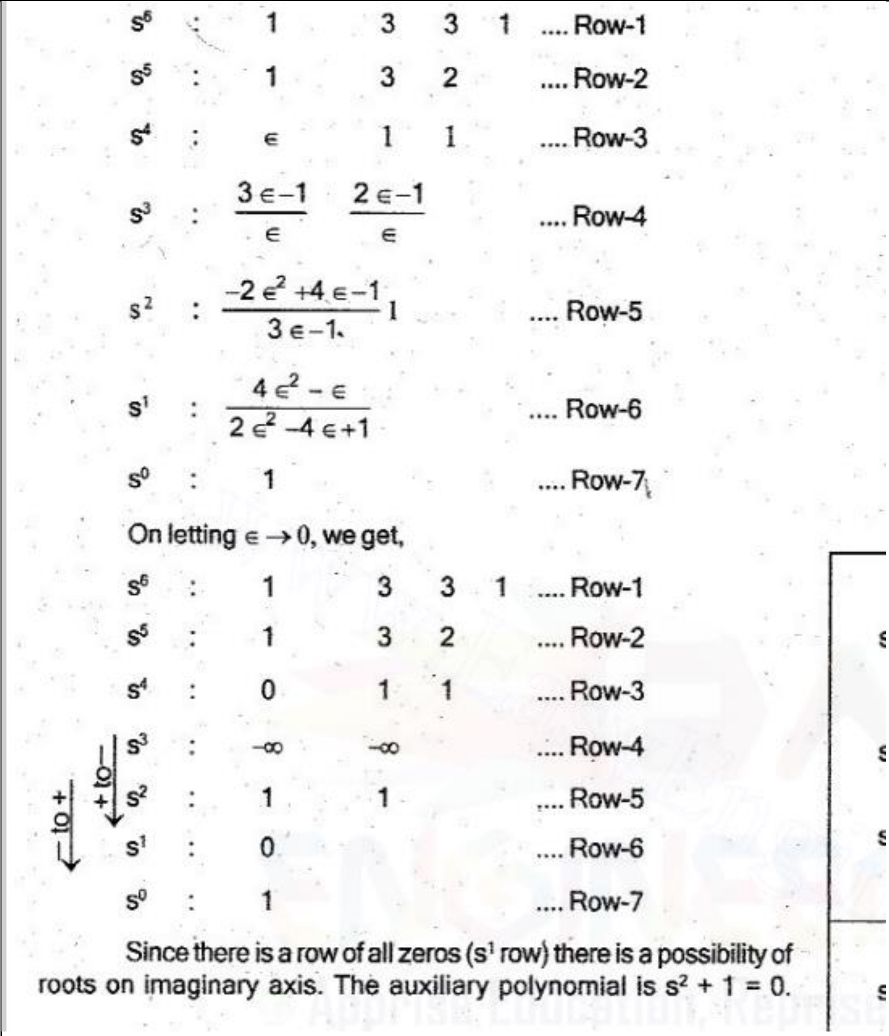
Peak time, $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{1\times\sqrt{1-0.5^2}} = 3.628$ sec

4	The forward path transfer function of a unity feedback type-1, second order system has a pole at -2. The nature of gain K is so adjusted that damping ratio is 0.4. Evaluate the steady state error when the input is $r(t) = 1 + 4t$.	10	CO3	L3

	 The error signal in s - domain, $E(s)=\frac{R(s)}{1+G(s)H(s)}$ Given that, $R(s)=\frac{3}{s}-\frac{2}{s^2}+\frac{1}{3s^3}$; $G(s)=\frac{10(s+2)}{s^2(s+1)}$; $H(s)=1$ $\therefore E(s)=\frac{\frac{3}{s}-\frac{2}{s^2}+\frac{1}{3s^3}}{1+\frac{10(s+2)}{s^2(s+1)}}=\frac{\frac{3}{s}-\frac{2}{s^2}+\frac{1}{3s^3}}{\frac{s^2(s+1)+10(s+2)}{s^2(s+1)}}$ $=\frac{3}{s}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]-\frac{2}{s^2}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]+\frac{1}{3s^3}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]$ The steady state error e_{ss} can be obtained from final value theorem. Steady state error, $e_{ss}=\underset{t\to\infty}{Lt}\ e(t)=\underset{s\to0}{Lt}\ s\,E(s)$ $\therefore e_{ss}=\underset{s\to0}{Lt}\ s\left\{\frac{3}{s}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]-\frac{2}{s^2}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]+\frac{1}{3s^3}\left[\frac{s^2(s+1)}{s^2(s+1)+10(s+2)}\right]\right\}$ $=\underset{s\to0}{Lt}\left\{\frac{3s^2(s+1)}{s^2(s+1)+10(s+2)}-\frac{2s(s+1)}{s^2(s+1)+10(s+2)}+\frac{(s+1)}{3s^2(s+1)+30\ (s+2)}\right\}\quad=0-0+\frac{1}{60}$			
5	Use the Routh-Hurwitz stability criterion to determine the location of roots on the S-plane and hence the stability of the system represented by the characteristic polynomial $s^7+5s^6+9s^5+9s^4+4s^3+20s^2+36s+36=0$.  s^7 : 1 9 4 36 .... Row-1 s^6 : 5 9 20 36 .... Row-2 Divide s^6 row by 5 to simplify the computations. s^7 : 1 9 4 36 .... Row-1 s^6 : 1 1.8 4 7.2 .... Row-2 s^5 : 1 0 4 .... Row-3 s^4 : 1 0 4 .... Row-4 s^3 : 0 0 .... Row-5 The row of all zeros indicate the existence of even polynomial, which is also the auxiliary polynomial. The auxiliary polynomial is, $s^4+4=0$. Divide the characteristic equation by auxiliary equation to get the quotient polynomial. The characteristic equation can be expressed as a product of quotient polynomial and auxiliary equation.	10	CO3	L3

	 $\therefore s^7+5s^6+9s^5+9s^4+4s^3+20s^2+36s+36=0$ (s^4+4) $(s^3+5s^2+9s+9)=0$ Even polynomial Quotient polynomial The routh array is constructed for quotient polynomial as shown below. s^3 : 1 9 s^2 : 5 9 s^1 : 7.2 s^0 : 9 Column-1 $s^1 : \frac{5\times 9-9\times 1}{5}$ s^1 : 7.2 $s^0 : \frac{7.2\times 9-0\times 5}{7.2}$ s^0 : 9 There is no sign change in the elements of first column of routh arr quotient polynomial are lying on the left half of s-plane. To determine the stability, the roots of auxiliary polynomial should be The auxiliary polynomial is, $s^4+4=0$. Put, $s^2=x$ in the auxiliary equation, $\therefore s^4+4=x^2+4=0$  The auxiliary polynomial is, $s^4+4=0$. Put, $s^2=x$ in the auxiliary equation, $\therefore s^4+4=x^2+4=0$ $\therefore x^2=-4 \quad \Rightarrow \quad x=\pm\sqrt{-4}=\pm j2=2\angle 90° \text{ or } 2\angle -90°$ But, $s=\pm\sqrt{x}=\pm\sqrt{2\angle 90°} \text{ or } \pm\sqrt{2\angle -90°} = \pm\sqrt{2}\angle 90°/2 \text{ or } \pm\sqrt{2}\angle -90°/2$ $=\pm\sqrt{2}\angle 45° \text{ or } \pm\sqrt{2}\angle -45 = \pm(1+j1) \text{ or } \pm(1-j1)$ The roots of auxiliary equation are complex and has quadrantal symmetry. Two roots of auxiliary equation are lying on the right half of s-plane and the other two on the left half of s-plane. The roots of characteristic equation are given by roots of quotient polynomial and auxiliary polynomial. Hence we can conclude that two roots of characteristic equation are lying on the right half of s-plane and so the system is unstable. The remaining five roots are lying on the left half of s-plane.			
6	The open loop transfer function of a unity feedback system is given by $G(s)=\frac{K(s+1)}{s^3+as^2+2s+1}$. Determine the value of **K** and **a** so that the system oscillates at a frequency of 3rad/sec.  The closed loop transfer function $\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)}=\frac{\frac{K(s+1)}{s^3+as^2+2s+1}}{1+\frac{K(s+1)}{s^3+as^2+2s+1}}=\frac{K(s+1)}{s^3+as^2+2s+1+K(s+1)}$ The characteristic equation is, $s^3+as^2+2s+1+K(s+1)=0$. $s^3+as^2+2s+1+Ks+K=0 \quad \Rightarrow \quad s^3+as^2+(2+K)s+1+K=0$ The routh array of characteristic polynomial is constructed as shown below. The maximum power of s is odd, hence the first row of routh array is formed using coefficients of odd powers of s and the second row of routh array is formed using coefficients of even powers of s. If the elements of s^1 row are all zeros then there exists an even polynomial (or auxiliary polynomial). If the roots of the auxiliary polynomial are purely imaginary then the roots are lying on imaginary axis and the system oscillates. The frequency of oscillation is the root of auxiliary polynomial.	10	CO3	L3

	 Routh array s^3 : 1 2+K s^2 : a 1+K s^1 : $\frac{a(2+K)-(1+K)}{a}$ s^0 : 1+K From s^2 row, the auxiliary polynomial is, $as^2+(1+K)=0 \Rightarrow as^2=-(1+K) \Rightarrow s=\pm j\sqrt{\frac{1+K}{a}}$ Given that, $s=\pm j2$, $\therefore \sqrt{\frac{1+K}{a}}=2 \Rightarrow \frac{1+K}{a}=4 \Rightarrow K=4a-1$ From s^1 row, $\frac{a(2+K)-(1+K)}{a}=0 \Rightarrow a(2+K)-(1+K)=0 \Rightarrow 2a+Ka-1-K=0$ $\therefore 2a-1+K(a-1)=0$ Put, $K = 4a - 1$ $\therefore 2a-1+(4a-1)(a-1)=0 \Rightarrow 2a-1+4a^2-4a-a+1=0 \Rightarrow 4a^2-3a=0$ (or) $a(4a-3)=0$ Since $a \neq 0$, $4a-3=0$, $\therefore a=3/4$ When $a=(3/4)$, $K=4a-1=4\times(3/4)-1=2$ **RESULT** When the system oscillates at a frequency of 2 rad/sec, $K=2$ and $a=3/4$.			
7a	The characteristic equation of a feedback control system is given by $s^4+20s^3+15s^2+2s+K=0$. (a) Determine the range of values of K for the system to be stable. (b) Can the system be marginally stable? If so, find the required value of K and the frequency of sustained oscillations.  The maximum power of s in the characteristic polynomial is odd, hence form the first row of routh array using coefficients of odd powers of s and second row of routh array using coefficients of even powers of s. s^3 : 1 $9-K$ s^2 : 5 K s^1 : $9-1.2K$ s^0 : K $s^1: \frac{5\times(9-K)-K\times 1}{5}$ $s^1: \frac{45-5K-K}{5}$ $s^1: \frac{45-6K}{5} \approx 9-1.2K$ $s^0: \frac{(9-1.2K)\times K}{(9-1.2K)}$ s^0 : K From s^1 row, for stability of the system, $(9-1.2K)>0$ If $(9-1.2K)>0$ then $1.2K<9$; $\therefore K<\frac{9}{1.2}=7.5$ From s^0 row, for stability of the system, $K>0$ Finally we can conclude that for stability of the system K should be in the range of $0<K<7.5$ **RESULT** For stability of the system K should be in the range of, $0<K<7.5$.	5	CO3	L3
7b	Use the Routh-Hurwitz stability criterion to determine the location of roots on the s-plane and hence the stability of the system represented by the characteristic polynomial $s^6+2s^5+s^4+2s^3+3s^2+4s+5=0$.	5	CO3	L3



s^6	:	1	3	3	1	.... Row-1
s^5	:	1	3	2		.... Row-2
s^4	:	$\in$	1	1		.... Row-3
s^3	:	$\frac{3\in-1}{\in}$	$\frac{2\in-1}{\in}$			.... Row-4
s^2	:	$\frac{-2\in^2+4\in-1}{3\in-1}$	1			.... Row-5
s^1	:	$\frac{4\in^2-\in}{2\in^2-4\in+1}$				.... Row-6
s^0	:	1				.... Row-7

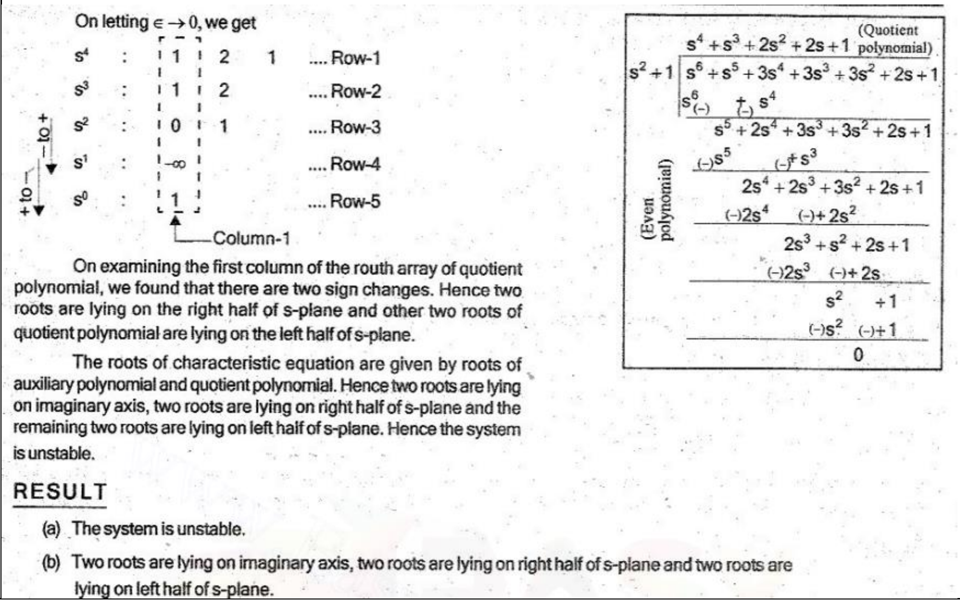
On letting $\in \to 0$, we get,

s^6	:	1	3	3	1	.... Row-1
s^5	:	1	3	2		.... Row-2
s^4	:	0	1	1		.... Row-3
s^3	:	$-\infty$	$-\infty$			.... Row-4
s^2	:	1	1			.... Row-5
s^1	:	0				.... Row-6
s^0	:	1				.... Row-7

+ to −

− to +

Since there is a row of all zeros (s^1 row) there is a possibility of roots on imaginary axis. The auxiliary polynomial is $s^2 + 1 = 0$.



On letting $\in \to 0$, we get

s^4	:	1	2	1	.... Row-1
s^3	:	1	2		.... Row-2
s^2	:	0	1		.... Row-3
s^1	:	$-\infty$			.... Row-4
s^0	:	1			.... Row-5

− to +

+ to −

Column-1

(Quotient polynomial)

$$s^4+s^3+2s^2+2s+1$$

$$s^2+1 \overline{\big) s^6+s^5+3s^4+3s^3+3s^2+2s+1}$$

$$\underline{s^6_{(-)} \quad \overset{+}{_{(-)}} s^4}$$

$$s^5+2s^4+3s^3+3s^2+2s+1$$

$$\underline{_{(-)}s^5 \quad _{(-)}+s^3}$$

$$2s^4+2s^3+3s^2+2s+1$$

$$\underline{_{(-)}2s^4 \quad _{(-)}+2s^2}$$

$$2s^3+s^2+2s+1$$

$$\underline{_{(-)}2s^3 \quad _{(-)}+2s}$$

$$s^2 \quad +1$$

$$\underline{_{(-)}s^2 \quad _{(-)}+1}$$

$$0$$

(Even polynomial)

On examining the first column of the routh array of quotient polynomial, we found that there are two sign changes. Hence two roots are lying on the right half of s-plane and other two roots of quotient polynomial are lying on the left half of s-plane.

The roots of characteristic equation are given by roots of auxiliary polynomial and quotient polynomial. Hence two roots are lying on imaginary axis, two roots are lying on right half of s-plane and the remaining two roots are lying on left half of s-plane. Hence the system is unstable.

RESULT

(a) The system is unstable.

(b) Two roots are lying on imaginary axis, two roots are lying on right half of s-plane and two roots are lying on left half of s-plane.

CI **CCI** **HOD**