USN	1	CR	1	8	ME			

Sub:	Fluid Mechanics					Sub Code:	18ME43	Branch:
Date:	08/08/2022	Duration:	90 min's	Max Marks:	50	Sem / Sec:	IV

Note: Answer any 3 questions from part A and any 1 question from Part B	MARKS

Part A

1. Define the terms: a. Buoyancy, b. Centre of buoyancy, c. Meta-center, d. Meta-centric height — **2.5*4**

2. Distinguish between :
   a. Steady flow and unsteady flow,
   b. Uniform and non-uniform flow,
   c. Compressible and incompressible flow,
   d. Laminar and turbulent flow. — **2.5*4**

3. A rectangular pontoon is 5m long, 3 *m* wide, and 1.30 *m* high. The depth of immersion of the pontoon is 0.80 *m* in sea water. If the center of gravity is 0.6 *m* above the bottom of the pontoon, determine the meta-centric height. The density for sea water = 1025 kg/m^3 — **6+4**

4. A pipe, through which water is flowing, is having diameters, 20 *cm* and 10 *cm* at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 *m/s*. find the velocity head at sections 1 and 2 and also rate of discharge. — **6+4**

Part B

5. Obtain the Euler's equation of motion along a stream line and hence derive Bernoulli's equation for a steady incompressible fluid flow. Also mention the assumptions made. — **12+4+4**

6. Derive an expression for the meta-centric height of a floating body. Show that the distance between the meta-center and center of buoyancy is given by $GM = \frac{I}{\forall}$ — 12+8

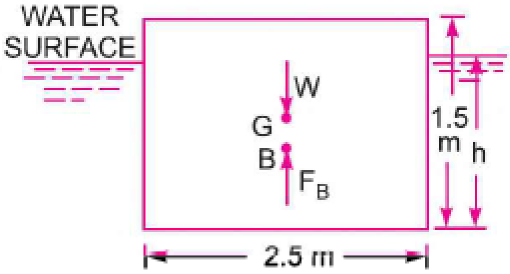
   Where = Moment of inertia of the plan of the floating body at water surface about longitudinal axis, $\forall$ = Volume of the body sub-merged in liquid.

Solution

1. Define the terms: a. Buoyancy, b. Centre of buoyancy, c. Meta-center, d. Meta-centric height

a.

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.



b.

It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

c.

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

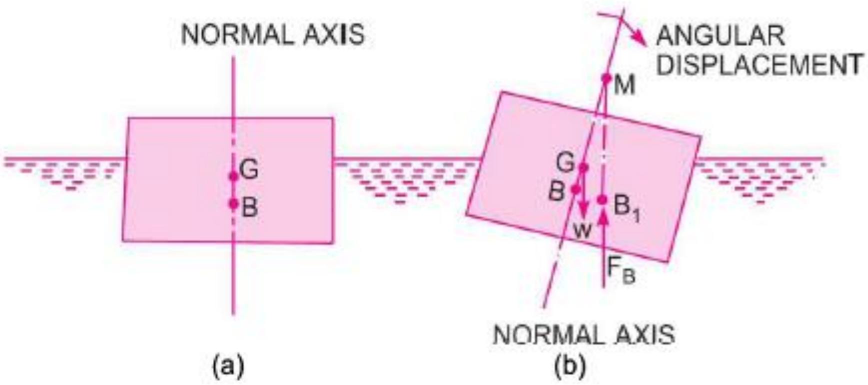


Fig. 4.5 *Meta-centre*

d.

The distance *MG*, *i.e.*, the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

2 .Distinguish between:

a. Steady flow and unsteady flow,

5.3.1 Steady and Unsteady Flows. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0,\, y_0,\, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0,\, y_0,\, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0,\, y_0,\, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0,\, y_0,\, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0,\, y_0,\, z_0} \neq 0 \text{ etc.}$$

b. Uniform and non-uniform flow,

5.3.2 Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} = 0$$

where ∂V = Change of velocity

∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t\,=\,\text{constant}} \neq 0.$$

c. Compressible and incompressible flow,

5.3.4 Compressible and Incompressible Flows. Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

$$\rho = \text{Constant.}$$

d. Laminar and turbulent flow.

5.3.3 Laminar and Turbulent Flows. Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a *zig-zag* way. Due to the movement of fluid particles in a *zig-zag* way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number,

where D = Diameter of pipe

V = Mean velocity of flow in pipe

and ν = Kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

3. A rectangular pontoon is 5m long, 3 *m* wide, and 1.30 *m* high. The depth of immersion of the pontoon is 0.80 *m* in sea water. If the center of gravity is 0.6 *m* above the bottom of the pontoon, determine the meta-centric height. The density for sea water = 1025 kg/m^3

Solution. Given :

Dimension of pontoon = 5 m × 3 m × 1.20 m

Depth of immersion = 0.8 m

Distance $AG = 0.6$ m

Distance $AB = \frac{1}{2} \times$ Depth of immersion

$= \frac{1}{2} \times .8 = 0.4$ m

Density for sea water = 1025 kg/m^3

Meta-centre height GM, given by equation (4.4) is

$$GM = \frac{I}{\forall} - BG$$

where I = M.O. Inertia of the plan of the pontoon about Y-Y axis

$$= \frac{1}{12} \times 5 \times 3^3 \text{ m}^4 = \frac{45}{4} \text{ m}^4$$

$\forall$ = Volume of the body sub-merged in water

$= 3 \times 0.8 \times 5.0 = 12.0 \text{ m}^3$

$BG = AG - AB = 0.6 - 0.4 = 0.2$ m

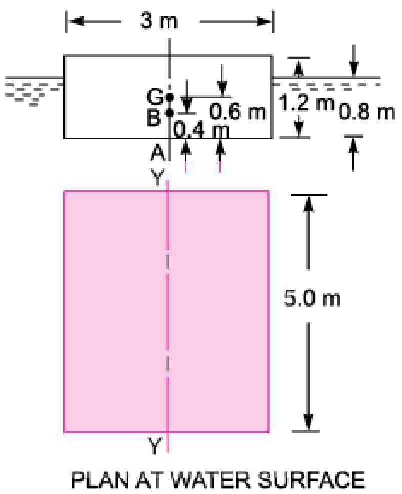


Fig. 4.7

$$\therefore \quad GM = \frac{45}{4} \times \frac{1}{12.0} - 0.2 = \frac{45}{48} - 0.2 = 0.9375 - 0.2 = \mathbf{0.7375 \text{ m.}} \quad \textbf{Ans.}$$

4. A pipe, through which water is flowing, is having diameters, 20 *cm* and 10 *cm* at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 *m/s*. find the velocity head at sections 1 and 2 and also rate of discharge.

Solution. Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}$$

$$V_1 = 4.0 \text{ m/s}$$
$$D_2 = 0.1 \text{ m}$$

$_2$ = 10 cm

$\therefore$ $$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

Fig. 6.2

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \textbf{0.815 m. Ans.}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

$\therefore$ $$A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$\therefore$ Velocity head at section 2 $$= \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \textbf{83.047 m. Ans.}$$

(*iii*) Rate of discharge $= A_1V_1$ or A_2V_2

$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3/\text{s}$

= **125.6 litres/s. Ans.** $\{\because 1 \text{ m}^3 = 1000 \text{ litres}\}$

5. Obtain the Euler's equation of motion along a stream line and hence derive Bernoulli's equation for a steady incompressible fluid flow. Also mention the assumptions made.

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in *s*-direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section *dA* and length *ds*. The forces acting on the cylindrical element are:

1. Pressure force *pdA* in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of *s* must be equal to the mass of fluid element $\times$ acceleration in the direction *s*.

$$\therefore \qquad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta$$

$$= \rho dA ds \times a_s \qquad \text{...(6.2)}$$

where a_s is the acceleration in the direction of s.

Now $a_s = \frac{dv}{dt}$, where v is a function of s and t.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \qquad a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} dsdA - \rho g\, dAds \cos\theta = \rho dAds \times \frac{v\partial v}{\partial s}$$

Dividing by $\rho dsdA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

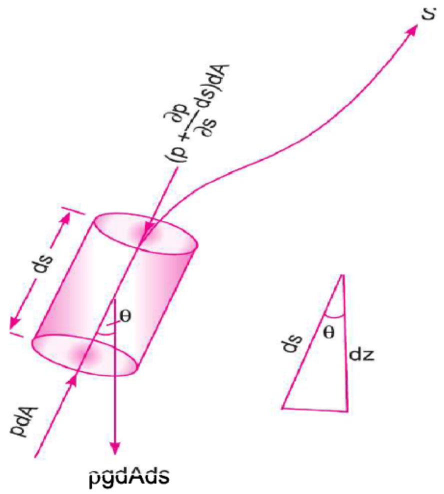


Fig. 6.1 *Forces on a fluid element.*

$$\text{or} \qquad \frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

But from Fig. 6.1 (*b*), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \qquad \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + gdz + vdv = 0$$

$$\text{or} \qquad \frac{dp}{\rho} + gdz + vdv = 0 \qquad \text{...(6.3)}$$

Equation (6.3) is known as Euler's equation of motion.

▶ 6.4 BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int gdz + \int vdv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \text{...(6.4)}$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

▶ 6.5 ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

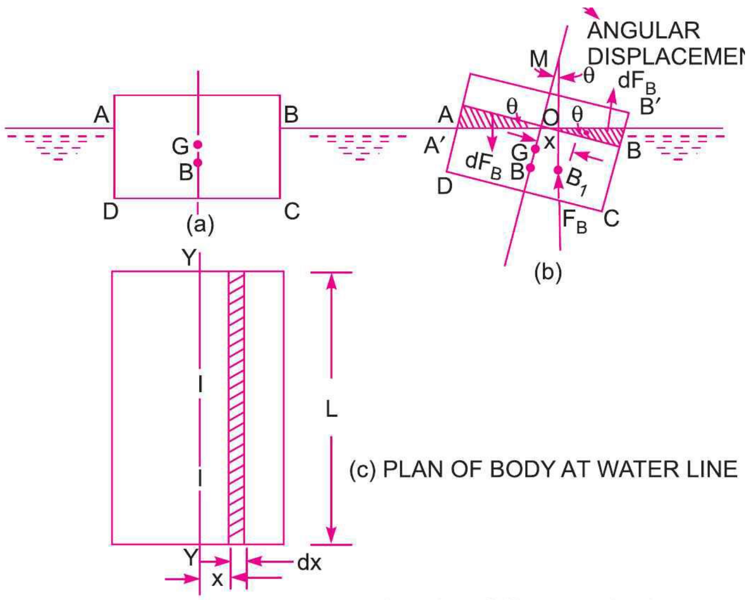
(*i*) The fluid is ideal, *i.e.*, viscosity is zero (*ii*) The flow is steady

(*iii*) The flow is incompressible (*iv*) The flow is irrotational.

6. How will you determine the meta-centric height of a floating body experimentally? Explain with a neat sketch.

▶ 4.6 ANALYTICAL METHOD FOR META-CENTRE HEIGHT

Fig. 4.6 (*a*) shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at G and B. The floating body is given a small angular displacement in the clockwise direction. This is shown in Fig. 4.6 (*b*). The new centre of buoyancy is at B_1. The vertical line through B_1 cuts the normal axis at M. Hence M is the meta-centre and GM is meta-centric height.



The angular displacement of the body in the clockwise direction causes the wedge-shaped prism BOB' on the right of the axis to go inside the water while the identical wedge-shaped prism represented by AOA' emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force dF_B acting through the $C.G.$ of the prism BOB' while the loss is represented by an equal and opposite force dF_B acting vertically downward through the centroid of AOA'. The couple due to these buoyant forces dF_B tends to rotate the ship in the counterclockwise direction. Also the moment caused by the displacement of the centre of buoyancy from B to B_1 is also in the counterclockwise direction. Thus these two couples must be equal.

Couple Due to Wedges. Consider towards the right of the axis a small strip of thickness dx at a distance x from O as shown in Fig. 4.5 (b). The height of strip $x \times \angle BOB' = x \times \theta$.

$$\{\because \angle BOB' = \angle AOA' = BMB_1' = \theta\}$$

$\therefore$ Area of strip $= \text{Height} \times \text{Thickness} = x \times \theta \times dx$

If L is the length of the floating body, then

Volume of strip $= \text{Area} \times L$

$= x \times \theta \times L \times dx$

$\therefore$ Weight of strip $= \rho g \times \text{Volume} = \rho g x\ \theta L\ dx$

Similarly, if a small strip of thickness dx at a distance x from O towards the left of the axis is considered, the weight of strip will be $\rho g x \theta L\, dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple = Weight of each strip × Distance between these two weights

$$= \rho g x\, \theta L\, dx\, [x + x]$$

$$= \rho g x\, \theta L\, dx \times 2x = 2\rho g x^2\, \theta L\, dx$$

∴ Moment of the couple for the whole wedge

$$= \int 2\rho g x^2\, \theta L\, dx \qquad ...(4.1)$$

Moment of couple due to shifting of centre of buoyancy from B to B_1

$$= F_B \times BB_1$$

$$= F_B \times BM \times \theta \qquad \{\because BB_1 = BM \times \theta \text{ if } \theta \text{ is very small}\}$$

$$= W \times BM \times \theta \qquad \{\because F_B = W\} \quad ...(4.2)$$

But these two couples are the same. Hence equating equations (4.1) and (4.2), we get

$$W \times BM \times \theta = \int 2\rho g x^2\, \theta\, L dx$$

$$W \times BM \times \theta = 2\rho g \theta \int x^2 L dx$$

$$W \times BM = 2\rho g \int x^2 L dx$$

Now Ldx = Elemental area on the water line shown in Fig. 4.6 (c) and = dA

∴ $$W \times BM = 2\rho g \int x^2 dA.$$

But from Fig. 4.5 (c) it is clear that $2 \int x^2\, dA$ is the second moment of area of the plan of the body at water surface about the axis Y-Y. Therefore

$$W \times BM = \rho g I \qquad \{\text{where } I = 2 \int x^2\, dA\}$$

∴ $$BM = \frac{\rho g I}{W}$$

But W = Weight of the body

= Weight of the fluid displaced by the body

= ρg × Volume of the fluid displaced by the body

= ρg × Volume of the body sub-merged in water

$= \rho g \times \forall$

∴ $$BM = \frac{\rho g \times I}{\rho g \times \forall} = \frac{I}{\forall} \qquad ...(4.3)$$

$$GM = BM - BG = \frac{I}{\forall} - BG$$

∴ Meta-centric height $$= GM = \frac{I}{\forall} - BG. \qquad ...(4.4)$$