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Internal Assessment Test 2 – May. 2022

Sub:	HEAT TRANSFER					Sub Code:	18ME63	Branch:	ME
Date:	09.06.2022	Duration:	90 min	Max Marks:	50	Sem / Sec:	VI/A&B		OBE

<u>**Answer All the Questions**</u>

<u>**Use of Heat Transfer Data handbook is permitted**</u>

		MARKS	CO	RBT
1	What do you mean by critical radius of insulation? Derive an expression for critical radius of insulation for cylinders.	[10]	CO2	L2 L3
2	Derive an expression for the temperature distribution and heat flow for a pin fin when the end of the fin is insulated.	[10]	CO2	L3
3	A rod (k =200 W/mk) 5mm in diameter and 5 cm long has its one end maintained at 100°C. The surface of the rod is exposed to ambient air at 25°C with convection heat transfer coefficient 100 W/m^{2}K. Assuming other end is insulated, determine: (i) the temperature of rod at 20mm distance from the end at 100°C (ii) Heat dissipation rate from the surface of rod (iii) Effectiveness	[10]	CO2	L3
4	A longitudinal copper fin (k=380 W/mK) 600 mm long and 5 mm diameter is exposed to air stream at 20°C. The convective heat transfer coefficient is 20 W/m^{2}K. If the fin base temperature is 150°C, determine: (i)The rate of heat transfer (ii)Fin efficiency (iii)Fin effectiveness	[10]	CO2	L3
5	A short fin with insulated tip of 0.08 m length and diameter 12 mm is exposed to air at 30°C. Thermal conductivity is 15 W/mK. The base temperature is 280°C. The heat dissipated by the fin is 7W. Determine the value of convection coefficient. Also determine the tip temperature.	[10]	CO2	L3

CI CCI HOD

1)

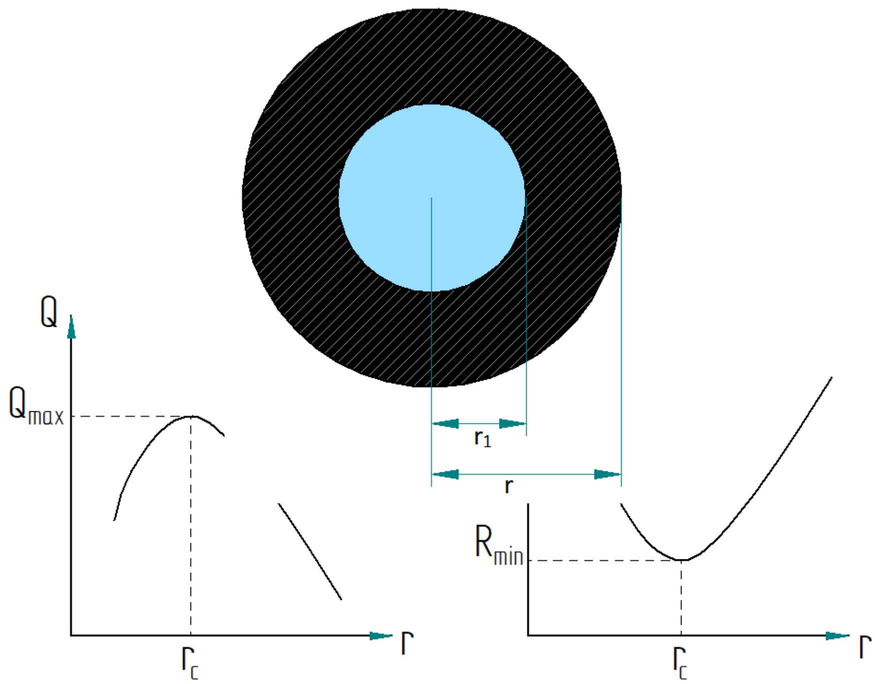
CRITICAL THICKNESS / RADIUS OF INSULATION

*In **cylinders and spheres**, the additional insulation increases the conduction resistance of insulation and decreases the convection resistance of the surface and thus net resistance (and thus the rate heat transfer) may increase or decrease with the variation of insulation thickness.*

The radius of insulation at which the rate of heat transfer is maximum (net thermal resistance is minimum) is called critical radius of insulation. The thickness of insulation corresponding to critical radius of insulation is called as critical thickness of insulation.

If the thickness of insulation is lower than the critical thickness of insulation, rate of heat transfer increases on increasing the insulation thickness up to critical thickness. Further increase in thickness of insulation results in decrease in rate of heat transfer.

The thickness of insulation up to which heat flow increases and after which heat flow decreases is called as critical thickness of insulation.

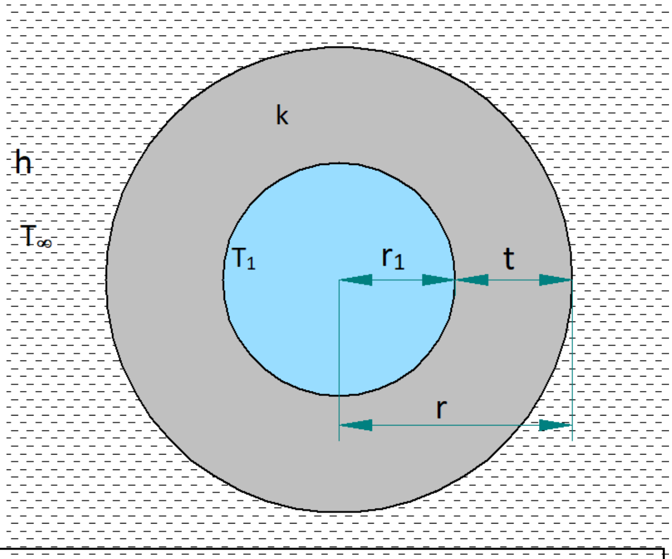


CRITICAL THICKNESS / RADIUS OF INSULATION (For Cylinders)

Consider a cylinder at a temperature T_1, radius r_1 and length L to be insulated with a material with thermal conductivity k.

Let the material be surrounded by ambient at temperature T_∞ and heat transfer coefficient be h for convective heat transfer from the outer surface to the surroundings.

Let the thickness of insulation be t and outer radius of insulation be r.



The net thermal resistance between T_1 and T_∞ can be given by:

$$\mathbf{R_{th} = \frac{\ln\left(\frac{r}{r_1}\right)}{2\pi kl} + \frac{1}{h \times 2\pi rl}}$$

$$\mathbf{At\ r = r_c, Q = Q_{max}\ and\ R_{th} = R_{min}}$$

$$\therefore \frac{dR_{th}}{dr} = 0$$

$$\frac{d}{dr}\left(\frac{\ln\left(\frac{r}{r_1}\right)}{2\pi kl} + \frac{1}{h \times 2\pi rl}\right) = 0$$

$$\frac{1}{r \times 2\pi kl} + \frac{-1}{r^2 \times 2\pi hl} = 0$$

$$\frac{1}{kr} - \frac{1}{hr^2} = 0$$

$$\mathbf{r = \frac{k}{h}}$$

Checking for minima at r = k/h:

$$\frac{d^2R_{th}}{dr} = \frac{d}{dr}\left(\frac{1}{r \times 2\pi kl} + \frac{-1}{r^2 \times 2\pi hl}\right)$$

$$\frac{d^2R_{th}}{dr} = \left(\frac{-1}{r^2 \times 2\pi kl} + \frac{-1 \times -2}{r^3 \times 2\pi hl}\right)$$

$$At\ r = \frac{k}{h}:$$

$$\frac{d^2R_{th}}{dr} = \left(\frac{-1}{\left(\frac{k}{h}\right)^2 \times 2\pi kl} + \frac{2}{\left(\frac{k}{h}\right)^3 \times 2\pi hl}\right)$$

$$\frac{d^2R_{th}}{dr} = \left(\frac{-h^2}{2\pi lk^3} + \frac{2h^2}{2\pi lk^3}\right)$$

$$\frac{d^2R_{th}}{dr} = \frac{h^2}{2\pi lk^3} > 0$$

$$\therefore R_{th} = R_{min}\ at\ r = \frac{k}{h}$$

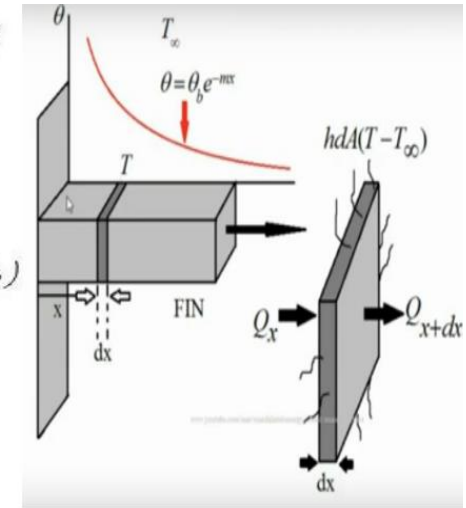
For a cylinder,

Critical radius of insulation, $\mathbf{r_c = \frac{k}{h}}$

Critical thickness of insulation, $\mathbf{t_c = (r_c - r_1) = \left(\frac{k}{h} - r_1\right)}$

2)

Conductive heat entering at x = Conductive heat leaving at $x+dx$ + Heat leaving by convection



$$\dot{Q}_x = \dot{Q}_{x+dx} + \dot{Q}_{convection}$$

$$\dot{Q}_x = -KA_c \frac{dT}{dx}$$

$$\dot{Q}_{x+dx} = \dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x} dx$$

$$\dot{Q}_{convection} = h\, dA_s (T-T_\infty) = h(P.dx)(T-T_\infty)$$

since

$$\dot{Q}_x = \dot{Q}_{x+dx} + \dot{Q}_{convection}$$

$$\dot{Q}_x = \dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x} dx + h(P.dx)(T-T_\infty)$$

$$0 = \frac{\partial}{\partial x}\left\{-KA_c \frac{\partial T}{\partial x}\right\} + h(P)(T-T_\infty)$$

or

$$\frac{d}{dx}\left\{KA_c \frac{dT}{dx}\right\} - hP(T-T_\infty) = 0$$

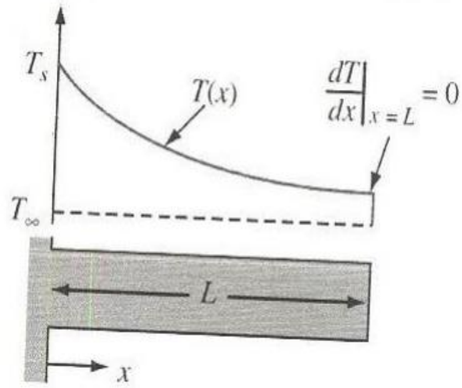
$$\frac{d^2T}{dx^2} - \frac{hP}{KA_c}(T-T_\infty) = 0$$

Taking $\frac{hP}{KA_c} = m^2$ and $T-T_\infty = \theta \Rightarrow \frac{\partial^2\theta}{\partial x^2} = \frac{\partial^2 T}{\partial x^2} \Rightarrow \frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$

$\frac{d^2\theta}{dx^2} - m^2\theta = 0$, this is a differential equation of second order which has solution as: $\boxed{\theta = C_1 e^{-mx} + C_2 e^{mx}}$

The general equation for temperature distribution for a fin is:

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$



Boundary conditions possible with the fins with end insulated:

(i) At $x=0$, $T=T_b$ or $\theta = \theta_b = T_b - T_\infty$

(ii) at $x=L$, $\frac{dT}{dx}=0$ or $\frac{d\theta}{dx}=0$

General equation: $\theta = C_1 e^{-mx} + C_2 e^{mx}$

$$\therefore \frac{d\theta}{dx} = -C_1 m e^{-mx} + C_2 m e^{mx}$$

using (ii) boundary condition,

$$0 = -C_1 m e^{-mL} + C_2 m e^{mL}$$

$$\therefore \boxed{C_1 = C_2 e^{2mL}} \quad \ldots.$$

using (i) boundary condition

$$\boxed{\theta_b = C_1 + C_2} \quad \ldots.$$

$$0 = -C_1 m e^{-ml} + C_2 m e^{ml}$$

$$0 = \frac{-C_1}{e^{ml}} + C_2 e^{ml}$$

$$0 = \frac{-C_1 + C_2 e^{2ml}}{e^{ml}}$$

$$\boxed{C_1 = C_2 e^{2mL}} \qquad \boxed{\theta_b = C_1 + C_2}$$

$$\theta_b = C_2 e^{2ml} + C_2$$
$$\theta_b = C_2 (e^{2ml} + 1)$$

Solving these equations, we get:

$$\boxed{C_1 = \frac{\theta_b}{1+e^{-2mL}}} \qquad \boxed{C_2 = \frac{\theta_b}{1+e^{2mL}}}$$

$$C_2 = \frac{\theta_b}{1+e^{2ml}}$$
$$C_1 = \frac{\theta_b e^{2ml}}{1+e^{2ml}}$$
$$C_1 = \frac{\theta_b}{(1+e^{2ml})e^{-2ml}}$$

The general equation for temperature distribution for a fin is:

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$

$$\theta = \theta_b \left[\frac{e^{-mx}}{1+e^{-2mL}} + \frac{e^{mx}}{1+e^{2mL}}\right] = \theta_b \left[\frac{e^{-mx}}{1+e^{-2mL}} \times \frac{e^{mL}}{e^{mL}} + \frac{e^{mx}}{1+e^{2mL}} \times \frac{e^{-mL}}{e^{-mL}}\right]$$
$$= \theta_b \left[\frac{e^{m(L-x)}}{e^{mL}+e^{-mL}} + \frac{e^{m(x-L)}}{e^{-mL}+e^{mL}}\right] = \theta_b \left[\frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{mL}+e^{-mL}}\right] = \theta_b \frac{\cosh m(L-x)}{\cosh mL}$$

Now, temperature distribution is given by

$$\theta = \theta_b \frac{\cosh\{m(L-x)\}}{\cosh(mL)} \Rightarrow \boxed{\frac{T-T_\infty}{T_b-T_\infty} = \frac{\cosh\{m(L-x)\}}{\cosh\{mL\}} = \frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{mL}+e^{-mL}}}$$

$$\theta = \theta_b \frac{\cosh\{m(L-x)\}}{\cosh(mL)}$$

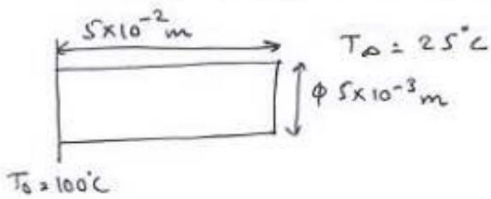
The rate of heat transfer from the fin base:

$$\dot{Q}_{fin} = -KA_c \left.\frac{d\theta}{dx}\right|_{x=0} = -KA_c\theta_b \left.\frac{\sinh\{m(L-x)\}(-m)}{\cosh\{mL\}}\right|_{x=0} = KA_c m\theta_b \tanh(mL)$$

$$\therefore \dot{Q}_{fin} = \dot{Q}_{x=0} = KA_c\sqrt{\frac{hP}{KA_c}} \cdot \theta_b \cdot \tanh(mL) = \sqrt{hPKA_c}\,\theta_b \tanh(mL)$$

$$\therefore \boxed{\dot{Q}_{fin} = \sqrt{hPKA_c}\,(T_b - T_\infty)\tanh(mL)}$$

3)



Given: $k = 200 \ W/mK$

$h = 100 \ W/m^2K$

$l = 5\times10^{-2} m$

$d = 5\times10^{-3} m$

Now: $m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{100 \times \pi d \times 4}{200 \times \pi d^2}} = \sqrt{\frac{100\times 4}{200 \times 5\times10^{-3}}}$

$\Rightarrow \boxed{m = 20}$

i) $\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(l-x)]}{\cosh(ml)}$

$x = 20\times10^{-3} m = 0.02$

$\therefore \frac{T-25}{100-25} = \frac{\cosh[20(0.05-0.02)]}{\cosh[20\times0.05]}$

$\Rightarrow \frac{T-25}{75} = 0.768$

$\Rightarrow \boxed{T = 82.6°C}$

ii) $Q = (hPkA_c)^{1/2} \theta_0 \tanh(ml)$

$\Rightarrow Q = \left(100\times\pi\times5\times10^{-3}\times200\times\pi\times\frac{(5\times10^{-3})^2}{4}\right)^{1/2}\times(100-25) \times \tanh(20\times0.05)$

$\Rightarrow \boxed{Q = 4.48 \ W.}$

iii) $\epsilon = \frac{Q_w}{Q_{w/o}} = \frac{4.48}{h \times \frac{\pi d^2}{4} \times \theta_0} = \frac{4.48\times4}{100\times3.14\times(5\times10^{-3})^2 \times (100-25)}$

$\Rightarrow \boxed{\epsilon = 30.43}$

4)

$k = 380\ W/mK$

$l = 600mm = 0.6m$

$d = 5mm = 0.005m$

$h = 20\ W/m^2K$

$T_\infty = 20^\circ C$

$T_b = 150^\circ C$

$$Q_{fin} = \sqrt{hPkA_c}\,(T_b - T_\infty)\,\frac{\tanh ml + \frac{h}{mk}}{1 + \frac{h}{mk}\tanh ml}$$

$P = \pi d = \pi \times 0.005 = 0.0157\,m$

$A_c = \frac{\pi d^2}{4} = \frac{\pi \times 0.005^2}{4} = 1.963 \times 10^{-5}\ m^2$

$\frac{P}{A_c} = \frac{4\pi d}{\pi d^2} = \frac{4}{d}$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 20}{380 \times 0.005}} = 6.488$$

$$Q_{fin} = \sqrt{20 \times 380 \times 0.0157 \times 1.963 \times 10^{-5}}\,(150-20)\,\frac{\tanh(6.488 \times 0.6) + \frac{20}{6.488 \times 380}}{1 + \frac{20}{6.488 \times 380}\tanh(6.488 \times 0.6)}$$

$$\boxed{Q_{fin} = 6.2864\ W}$$

Fin efficiency
$$\eta_{conv} = \frac{\dfrac{\tanh(ml) + \frac{h}{mk}}{1 + \frac{h}{mk}\tanh(ml)}}{ml} = \frac{\dfrac{\tanh(6.488 \times 0.6) + \frac{20}{6.488 \times 380}}{1 + \frac{20}{6.488 \times 380}\tanh(6.488 \times 0.6)}}{6.488 \times 0.6}$$

$= 0.25667$

$$\boxed{\eta_{conv} = 25.667\%}$$

Fin effectiveness

$$\varepsilon_{conv} = \sqrt{\frac{kP}{hA_c}}\,\frac{\tanh(ml) + \frac{h}{mk}}{1 + \frac{h}{mk}\tanh(ml)}$$

$$= \sqrt{\frac{380 \times 4}{20 \times 0.005}}\,\frac{\tanh(6.488 \times 0.6) + \frac{20}{6.488 \times 380}}{1 + \frac{20}{6.488 \times 380}\tanh(6.488 \times 0.6)}$$

$$\boxed{\varepsilon_{conv} = 123.187}$$

5)

Given,

Insulated tip

$l = 0.08 m$

$d = 12 mm = 0.012 m$

$k = 15 W/mK$

$T_b = 280^\circ C$

$T_\infty = 30^\circ C$

$Q_{fin} = 7 W$

$\frac{P}{A_c} = \frac{4\pi d}{\pi d^2} = \frac{4}{d}$

$$Q_{fin} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(ml)$$

$$7 = \sqrt{h \times \pi \times 0.012 \times 15 \times \pi \times \frac{0.012^2}{4}}\;(280-30)\;\tanh\left(\sqrt{\frac{h \times 4}{15 \times 0.012}} \times 0.08\right)$$

$$\boxed{h = 15.159\ W/m^2K}$$

Temperature at the tip $x = l$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh(0)}{\cosh\left(\sqrt{\frac{15.159 \times 4}{15 \times 0.012}} \times 0.08\right)} = \frac{T - 30}{280 - 30}$$

$$\boxed{T = 139.35^\circ C}$$