Sub:	Fluid Mechanics					Sub Code:	18ME43	Branch:
Date:	27/08/2022	Duration:	90 min's	Max Marks:	50	Sem / Sec:	IV	
Note: Answer any **5** questions.								MARKS

Scheme

1.	Neat diagram and Derivation for discharge in venture meter.	4+6
2.	Step by step method involved in Rayleigh method of the dimensional analysis.	4+4+2
3.	Sketch and derive the relation for actual discharge through an orifice meter.	4+6
4.	A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take C=0.98.	4+6
5.	The frictional torque T of a disc diameter D rotating at a speed N in a fluid of viscosity μ and density ρ in a turbulent flow is given by $T=D^5N^2\rho\Phi[\mu/D^2N\rho]$. Prove this by the method of dimensions.	5+5
6.	Check whether the following equations (with their usual notations) are dimensionally homogenous or not? $V=\sqrt{2gH}$ 2. $h_f=\frac{4fLV^2}{2gd}$	5+5
7.	Determine the dimensions of the quantities given below. a. Force b. Discharge c. dynamic viscosity d. Density	2.5*4

Solution:

1.

6.7.1 Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for rate of flow through venturimeter

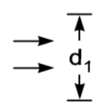
Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) = $\frac{\pi}{4}d_1^2$



and d_2, p_2, v_2, a_2 are corresponding values at section (2).

Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

As pipe is horizontal, hence $z_1=z_2$

$$\therefore \quad \frac{p_1}{\rho g}+\frac{v_1^2}{2g}=\frac{p_2}{\rho g}+\frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1-p_2}{\rho g}=\frac{v_2^2}{2g}-\frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$\therefore$

$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge,

$$Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$\therefore$

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

2.

▶ 12.4 METHODS OF DIMENSIONAL ANALYSIS

If the number of variable involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods :

1. Rayleigh's method, and
2. Buckingham's π-theorem.

12.4.1 Rayleigh's Method. This method is used for determining the expression for a variable which depends upon maximum three or four variables only. If the number of independent variables becomes more than four, then it is very difficult to find the expression for the dependent variable.

Let X is a variable, which depends on X_1, X_2 and X_3 variables. Then according to Rayleigh's method, X is function of X_1, X_2 and X_3 and mathematically it is written as $X = f[X_1, X_2, X_3]$.

This can also be written as $X = KX_1^a \cdot X_2^b \cdot X_3^c$

where K is constant and a, b and c are arbitrary powers.

The values of a, b and c are obtained by comparing the powers of the fundamental dimension on both sides. Thus the expression is obtained for dependent variable.

6.7.2 Orifice Meter or Orifice Plate. It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section (2), which is at a distance of about half the diameter of the orifice on the downstream side from the orifice plate.

Let p_1 = pressure at section (1),
v_1 = velocity at section (1),
a_1 = area of pipe at section (1), and

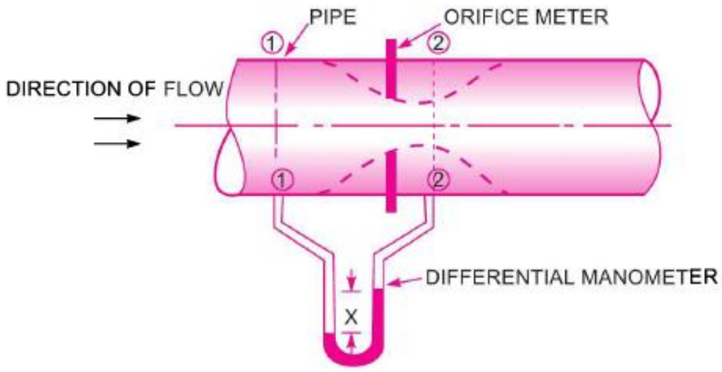


Fig. 6.12. *Orifice meter.*

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$\therefore$
$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = {v_2}^2 - {v_1}^2$$

or
$$v_2 = \sqrt{2gh + v_1^2} \qquad \text{...}(i)$$

Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$\therefore$
$$a_2 = a_0 \times C_c \qquad \text{...}(ii)$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2}{a_1} v_2 = \frac{a_0 C_c}{a_1} v_2 \qquad \text{...}(iii)$$

Substituting the value of v_1 in equation (*i*), we get

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

or $$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \text{ or } v_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

∴ $$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

∴ The discharge $Q = v_2 \times a_2 = v_2 \times a_0 C_c$ $\{\because a_2 = a_0 C_c$ from $(ii)\}$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad ...(iv)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

∴ $$C_c = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in equation (iv), we get

$$Q = a_0 \times C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}. \quad ...(6.13)$$

where C_d = Co-efficient of discharge for orifice meter.

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

4.

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

∴ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

∴ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer = x = 20 cm of mercury.

∴ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_o = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3\text{/s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

5.

Problem 12.13 *The frictional torque T of a disc of diameter D rotating at a speed N in a fluid of viscosity μ and density ρ in a turbulent flow is given by* $T = D^5N^2 \rho\phi\left[\frac{\mu}{D^2N\rho}\right]$.

Prove this by the method of dimensions.

Solution. Given : $T = f(D, N, \mu, \rho)$ or $f_1(T, D, N, \mu, \rho) = 0$ ...(*i*)

∴ Total number of variables, $n = 5$

Dimensions of each variable are expressed as

$$T = ML^2T^{-2},\ D = L,\ N = T^{-1},\ \mu = ML^{-1}T^{-1},\ \rho = ML^{-3}$$

∴ Number of fundamental dimensions, $m = 3$

Number of π-terms $= n - m = 5 - 3 = 2$

Hence equation (*i*) can be written as $f_1(\pi_1, \pi_2) = 0$ ...(*ii*)

Each π-term contains $m + 1$ variable, *i.e.*, 3 + 1 = 4 variables. Three variables are repeating variables.

Choosing D, N, ρ as repeating variables, the π-terms are

$$\pi_1 = D^{a_1}.\ N^{b_1}.\ \rho^{c_1}.\ T$$
$$\pi_2 = D^{a_2}.\ N^{b_2}.\ \rho^{c_2}.\ \mu$$

Dimensional Analysis of π_1

$$\pi_1 = D^{a_1}\ .\ N^{b_1}\ .\ \rho^{c_1}\ .\ T$$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_1}\ .\ (T^{-1})^{b_1}\ (ML^{-3})^{c_1}\ .\ ML^2T^{-2}.$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1 + 1$, ∴ $c_1 = -1$

Power of L, $0 = a_1 - 3c_1 + 2$, ∴ $a_1 = 3c_1 - 2 = -3 - 2 = -5$

Power of T, $0 = -b_1 - 2$, ∴ $b_1 = -2$

Substituting the values of a_1, b_1, c_1 in π,

$$\pi_1 = D^{-5}\ .\ N^{-2}\ .\ \rho^{-1}\ .\ T = \frac{T}{D^5N^2\rho}.$$

Dimensional Analysis of π_2

$$\pi_2 = D^{a_2}\ .\ N^{b_2}\ .\ \rho^{c_2}\ .\ \mu$$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1}T^{-1}.$$

Equating the powers of M, L, T on both sides,

Power of M, $\quad 0 = c_2 + 1, \quad \therefore \quad c_2 = -1$

Power of L, $\quad 0 = a_2 - 3c_2 - 1, \quad \therefore \quad a_2 = 3c_2 + 1 = -3 + 1 = -2$

Power of T, $\quad 0 = -b_2 - 1, \quad \therefore \quad b_2 = -1$

Substituting the values of a_2, b_2 and c_2 in π_2,

$$\pi_2 = D^{-2}N^{-1}\rho^{-1} \cdot \mu = \frac{\mu}{D^2 N\rho}.$$

Substituting the values of π_1 and π_2 in equation (*ii*),

$$f_1\left(\frac{T}{D^5N^2\rho}, \frac{\mu}{D^2N\rho}\right) = 0 \quad \text{or} \quad \frac{T}{D^5N^2\rho} = \phi\left(\frac{\mu}{D^2N\rho}\right)$$

or

$$T = \mathbf{D^5N^2}\,\rho\,\phi\left[\frac{\mu}{\mathbf{D^2N}\rho}\right]. \textbf{ Ans.}$$

6.

▶ 12.3 DIMENSIONAL HOMOGENEITY

Dimensional homogeneity means the dimensions of each terms in an equation on both sides are equal. Thus if the dimensions of each term on both sides of an equation are the same the equation is known as dimensionally homogeneous equation. The powers of fundamental dimensions (*i.e.*, L, M, T) on both sides of the equation will be identical for a dimensionally homogeneous equation. Such equations are independent of the system of units.

Let us consider the equation, $V = \sqrt{2gH}$

Dimension of L.H.S. $\quad = V = \dfrac{L}{T} = LT^{-1}$

Dimension of R.H.S. $\quad = \sqrt{2gH} = \sqrt{\dfrac{L}{T^2} \times L} = \sqrt{\dfrac{L^2}{T^2}} = \dfrac{L}{T} = LT^{-1}$

Dimension of L.H.S. $\quad$ = Dimension of R.H.S. $= LT^{-1}$

$\therefore$ Equation $V = \sqrt{2gH}$ is dimensionally homogeneous. So it can be used in any system of units.

7.

(*iii*) Discharge $\quad$ = Area × Velocity $= L^2 \times \dfrac{L}{T} = \dfrac{L^3}{T} = L^3T^{-1}$.

Volume Volume L

(*vii*) Dynamic viscosity, μ is derived in (*iv*) as $\mu = ML^{-1}T^{-1}$.

(*v*) Force $\quad$ = Mass × Acceleration $= M \times \dfrac{\text{Length}}{(\text{Time})^2} = \dfrac{ML}{T^2} = MLT^{-2}$.

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} = ML^{-3}$$