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Internal Assesment Test – 3

Sub: Kinematics of Machines				Code: 18ME44
Date:27/08/2022	Duration: 90 mins	Max Marks: 50	Sem: 4	Branch (sections): ME

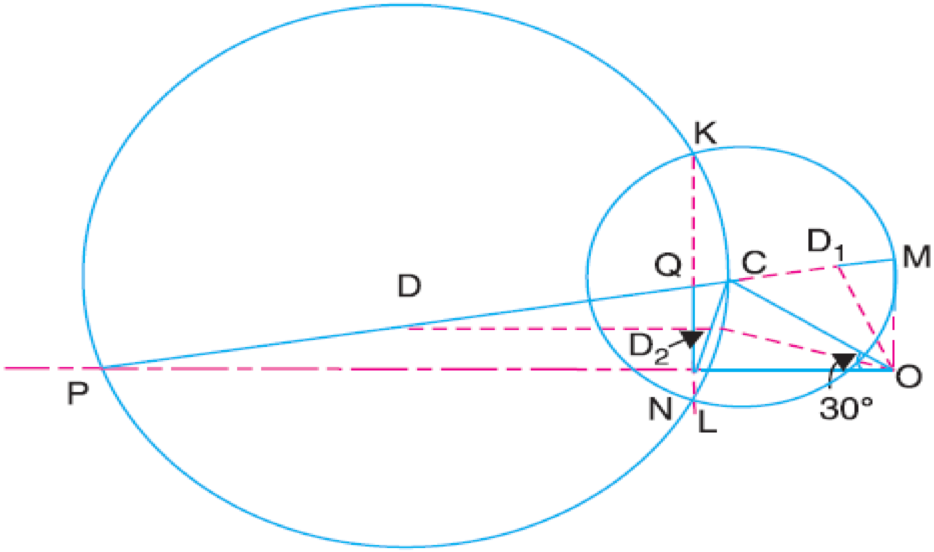
Answer any FOUR questions. Good luck!

		Marks	OBE CO	OBE RBT
1	The crank and connecting rod of a reciprocating engine are 200 mm and 700 mm respectively. The crank is rotating in clockwise direction at 120 rad/s. Find with the help of Klein's construction: **1.** Velocity and acceleration of the piston, **2.** Velocity and acceleration of the midpoint of the connecting rod, and **3.** Angular velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30° to I.D.C.	[12.5]	CO2	L4
2	Explain Klein's construction method to find velocity and acceleration of a slider crank mechanism.	[12.5]	CO2	L3
3	Obtain Freudenstein's equation for four bar mechanism.	[12.5]	CO2	L3
4	Explain function generation for four bar mechanism.	[12.5]	CO2	L3
5	In a slider crank mechanism, the crank OA = 300 mm and connecting rod AB = 1200 mm. The crank OA is turned 30 from inner dead centre. Locate all the instantaneous centres. If the crank rotates at 15 rad/sec clockwise, find i) Velocity of slider B and ii) Angular velocity of connecting rod AB.	[12.5]	CO2	L4
6	An epicyclic gear train consists of a sun wheel (S), a stationary internal gear (E) and three identical planet wheels (P) carried on a star shaped planet carrier (C). The sizes of different toothed wheels are such that the planet carrier C rotates at 1/5 of the speed of the sun wheel. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100Nm. Determine i) Number of teeth on different wheels of train, ii) Torque necessary to keep the internal gear stationary.	[12.5]	CO4	L4

CI **CCI** **HOD**

Q. NO 1:

Solution. Given: OC = 200 mm = 0.2 m ; PC = 700 mm = 0.7 m ; ω = 120 rad/s



The Klein's velocity diagram *OCM* and Klein's acceleration diagram *CQNO* as shown in Fig. is drawn to some suitable scale. By measurement, we find that OM = 127mm = 0.127 m ; CM = 173 mm = 0.173 m ; QN = 93 mm = 0.093 m ; NO = 200 mm= 0.2 m

1. *Velocity and acceleration of the piston*

We know that the velocity of the piston *P*,

$v_P = \omega \times OM = 120 \times 0.127 = 15.24$ m/s **Ans.**

and acceleration of the piston *P*,

$a_P = \omega 2 \times NO = (120)2 \times 0.2 = 2880$ m/s^2 **Ans.**

2. *Velocity and acceleration of the mid-point of the connecting rod*

In order to find the velocity of the mid-point *D* of the connecting rod, divide *CM* at D_1 in the same ratio as *D* divides *CP*. Since *D* is the mid-point of *CP*, therefore D_1 is the mid-point of *CM*, *i.e.*

$CD_1 = D_1M$. Join OD_1. By measurement,

OD_1 = 140 mm = 0.14 m

∴ Velocity of *D*, $v_D = \omega \times OD_1 = 120 \times 0.14 = 16.8$ m/s **Ans.**

In order to find the acceleration of the mid-point of the connecting rod, draw a line DD_2 parallel to the line of stroke *PO* which intersects *CN* at D_2. By measurement,

OD_2 = 193 mm = 0.193 m

∴ Acceleration of *D*,

$a_D = \omega^2 \times OD2 = (120)2 \times 0.193 = 2779.2$ m/s^2 **Ans.**

3. *Angular velocity and angular acceleration of the connecting rod*

We know that the velocity of the connecting rod *PC* (*i.e.* velocity of *P* with respect to *C*),

$v_{PC} = \omega \times CM = 120 \times 0.173 = 20.76$ m/s

∴ Angular acceleration of the connecting rod *PC*,

$$\omega_{pc} = \frac{v_{PC}}{PC} = \frac{20.76}{0.7} = 29.66\ rad/s \quad \textbf{Ans.}$$

We know that the tangential component of the acceleration of *P* with respect to *C*,

$$a_{PC}^{t} = \omega^2 \times QN = (120)^2 \times 0.093 = 1339.2\ m/s^2 \quad \textbf{Ans.}$$

∴ Angular acceleration of the connecting rod *PC*,

$$a_{PC} = \frac{a_{PC}^{t}}{PC} = \frac{1339.2}{0.7} = 1913.14\ rad/s^2 \quad \textbf{Ans.}$$

Q. NO 2:

Klien's Construction

Let *OC* be the crank and *PC* the connecting rod of a reciprocating steam engine, as shown in Fig. 6 (*a*). Let the crank makes an angle θ with the line of stroke *PO* and rotates with uniform angular velocity ω rad/s in a clockwise direction. The Klien's velocity and acceleration diagrams are drawn as discussed below:

Klien's velocity diagram

First of all, draw *OM* perpendicular to *OP*; such that it intersects the line *PC* produced at *M*. The triangle *OCM* is known as ***Klien's velocity diagram***. In this triangle *OCM*,

OM may be regarded as a line perpendicular to *PO*,

CM may be regarded as a line parallel to *PC*, and ...(It is the same line.)

CO may be regarded as a line parallel to *CO*.

We have already discussed that the velocity diagram for given configuration is a triangle *ocp* as shown in Fig. 6 (*b*). If this triangle is revolved through 90°, it will be a triangle $oc_1\,p_1$, in which oc_1 represents v_{CO} (*i.e.* velocity of *C* with respect to *O* or velocity of crank pin *C*) and is paralel to *OC*, op_1 represents v_{PO} (*i.e.* velocity of *P* with respect to *O* or velocity of cross-head or piston *P*) and is perpendicular to *OP*, and c_1p_1 represents v_{PC} (*i.e.* velocity of *P* with respect to *C*) and is parallel to *CP*. A little consideration will show that the triangles oc_1p_1 and *OCM* are similar. Therefore,

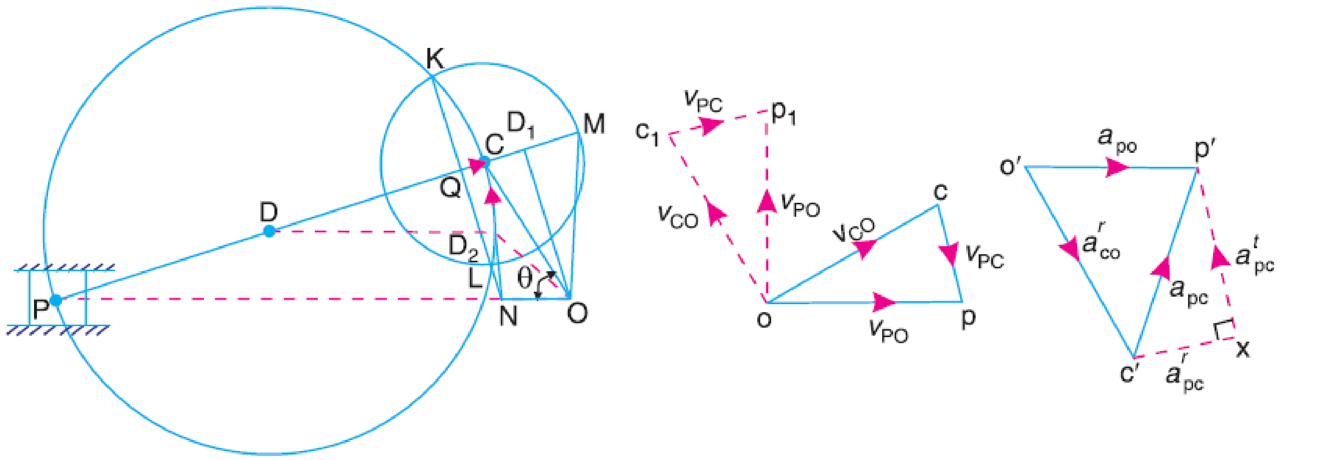
$$\frac{oc_1}{OC} = \frac{op_1}{OM} = \frac{c_1p_1}{CM} = \omega\ (a\ constant)$$

or

$$\frac{v_{CO}}{OC} = \frac{v_{PO}}{OM} = \frac{v_{PC}}{CM} = \omega$$

Therefore, $v_{co} = \omega \times OC$; $v_{PO} = \omega \times OM$ and $v_{PC} = \omega \times CM$

Thus, we see that by drawing the Klien's velocity diagram, the velocities of various points may be obtained without drawing a separate velocity diagram.



(*a*) Klien's acceleration diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig.: Klein's construction

Klien's acceleration diagram

The Klien's acceleration diagram is drawn as discussed below:

1. First of all, draw a circle with *C* as centre and *CM* as radius.
2. Draw another circle with *PC* as diameter. Let this circle intersect the previous circle at *K* and *L*.
3. Join *KL* and produce it to intersect *PO* at *N*. Let *KL* intersect *PC* at *Q*. This forms the quadrilateral *CQNO*, which is known as ***Klien's acceleration diagram*.**

We have already discussed that the acceleration diagram for the given configuration is as shown in Fig. 6 (*c*). We know that

i) *o'c'* represents a^r_{CO} (*i.e.* radial component of the acceleration of crank pin *C* with respect to *O*) and is parallel to *CO*;

ii) *c'x* represents a^r_{PC} (*i.e.* radial component of the acceleration of crosshead or piston *P* with respect to crank pin *C*) and is parallel to *CP* or *CQ*;

iii) *xp'* represents a^t_{PC} (*i.e.* tangential component of the acceleration of *P* with respect to *C*) and is parallel to *QN* (because *QN* is perpendicular to *CQ*); and

iv) *o'p'* represents a_{PO} (*i.e.* acceleration of *P* with respect to *O* or the acceleration of piston *P*) and is parallel to *PO* or *NO*.

A little consideration will show that the quadrilateral *o'c'x p'* [Fig. 6 (*c*)] is similar to quadrilateral *CQNO* [Fig. (*a*)]. Therefore,

$$\frac{o'c'}{OC} = \frac{c'x}{CQ} = \frac{xp'}{QN} = \frac{o'p'}{NO} = \omega^2 \ (a\ constant)$$

$$\frac{a^r_{CO}}{OC} = \frac{a^r_{PC}}{CQ} = \frac{a^t_{PC}}{QN} = \frac{a_{PO}}{NO} = \omega^2$$

Therefore, $a^r_{CO} = \omega^2 \times OC$; $a^r_{PC} = \omega^2 \times CQ$

$a^t_{PC} = \omega^2 \times QN$; and $a_{PO} = \omega^2 \times NO$

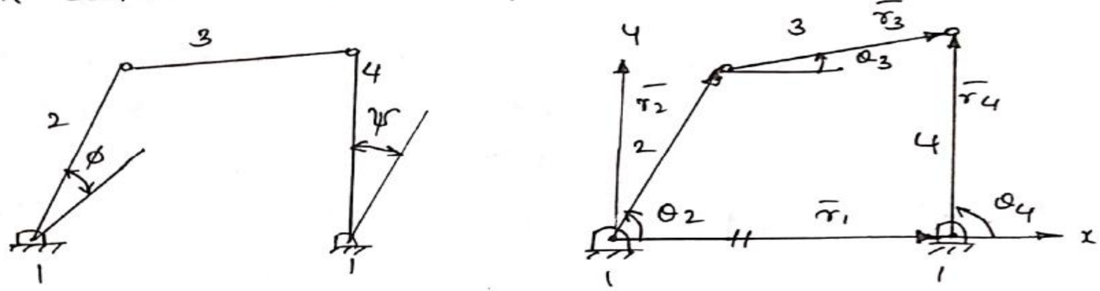
Thus we see that by drawing the Klien's acceleration diagram, the acceleration of various points may be obtained without drawing the separate acceleration diagram.

Q. NO 3:

(1) Freudenstein's Equation for four bar mechanism

A design problem where the link lengths of a four bar mechanism must be determined so that the rotations of the two levers within the mechanism, ϕ and ψ, are functionally related.

The desired relation is represented by $f(\phi, \psi) = 0$.



(a) (b)

Fig.(b) shows the four bar mechanism and the vector loop necessary for the mechanism's analysis. The vector loop equation is,

$$\bar{r}_2 + \bar{r}_3 - \bar{r}_4 - \bar{r}_1 = 0 \qquad -(1)$$

Considering the links to be vectors, displacement along the x-axis is,

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 = 0$$

$$\therefore \; r_3 \cos\theta_3 = -r_2 \cos\theta_2 + r_4 \cos\theta_4 + r_1 \qquad -(2)$$

Squaring equation (2)

$$r_3^2 \cos\theta_3^2 = r_2^2 \cos\theta_2^2 + r_4^2 \cos\theta_4^2 + r_1^2 - 2r_2 r_4 \cos\theta_2 \cos\theta_4 - 2r_2 r_1 \cos\theta_2 + 2r_4 r_1 \cos\theta_4 \qquad -(3)$$

Displacement along y-axis is,

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$

$$\therefore \; r_3 \sin\theta_3 = -r_2 \sin\theta_2 + r_4 \sin\theta_4 \qquad -(4)$$

Squaring equation (4)

$$r_3^2 \sin\theta_3^2 = r_2^2 \sin\theta_2^2 + r_4^2 \sin\theta_4^2 - 2r_2 r_4 \sin\theta_2 \sin\theta_4 \qquad -(5)$$

Equation (3) and (5) can be reduced to a single equation relating θ_2, θ_4 and the four link lengths by eliminating θ_3.

To eliminate θ_3, add both sides of the eqⁿ ③ and ⑤

$$\therefore\ r_3^2\cos^2\theta_3 + r_3^2\sin^2\theta_3 = r_2^2\cos^2\theta_2 + r_4^2\cos^2\theta_4 + r_1^2 - 2r_2r_4\cos\theta_2\cdot\cos\theta_4 - 2r_2r_1\cos\theta_2 + 2r_4r_1\cos\theta_4 + r_2^2\sin\theta_2^2 + r_4^2\sin\theta_4^2 - 2r_2r_4\sin\theta_2\sin\theta_4$$

$$\text{i.e., } r_3^2(\cos^2\theta_3+\sin^2\theta_3) = r_2^2(\cos^2\theta_2+\sin^2\theta_2) + r_4^2(\cos^2\theta_4+\sin^2\theta_4) + r_1^2 - 2r_2r_4\cos\theta_2\cos\theta_4 - 2r_2r_1\cos\theta_2 + 2r_4r_1\cos\theta_4 - 2r_2r_4\sin\theta_2\sin\theta_4$$

$$\text{i.e., } r_2^2 - r_3^2 + r_4^2 + r_1^2 + 2r_4r_1\cos\theta_4 - 2r_2r_1\cos\theta_2 = 2r_2r_4\cos\theta_2\cos\theta_4 + 2r_2r_4\sin\theta_2\sin\theta_4$$

Dividing both sides by $2r_2r_4$ we get,

$$\frac{r_2^2-r_3^2+r_4^2+r_1^2}{2r_2r_4} + \frac{r_1}{r_2}\cos\theta_4 - \frac{r_1}{r_4}\cos\theta_2 = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4 \qquad -(6)$$

Let $\frac{r_1}{r_4} = R_1$; $\frac{r_1}{r_2} = R_2$ and $\frac{r_2^2-r_3^2+r_4^2+r_1^2}{2r_2r_4} = R_3$

Substituting these values in equation (6) we get,

$$R_3 + R_2\cos\theta_4 - R_1\cos\theta_2 = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4$$

$$\therefore\ \boxed{R_3 + R_2\cos\theta_4 - R_1\cos\theta_2 = \cos(\theta_2-\theta_4)} \qquad -(7)$$

Eqⁿ (7) is called <u>Freudenstein's equation</u>.

* It is the relationship between input rotation θ_2 and output rotation θ_4 as determined by the link lengths r_1 through r_4.

In function generation via Freudenstein's equation, the idea is to use equation (7) to determine a set of link lengths that will result in a $(\theta_2 - \theta_4)$ relationship that matches a desired function.

Q. NO. 4:

Function generation – Function generation is similar to curve fitting. There are two basic methods:

(i) Point matching method

(ii) Derivative matching method.

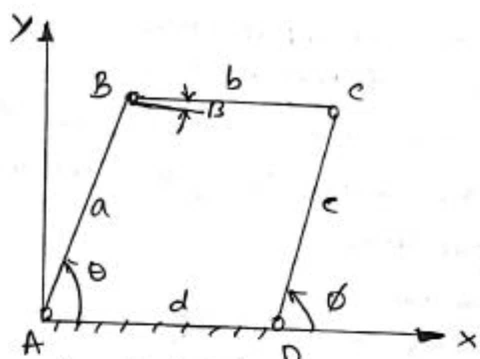
Function generation for four bar mechanism

A four bar mechanism shown in fig. is in equilibrium.

Let a, b, c and d be the magnitudes of the links AB, BC, CD and DA respectively. θ, β and ϕ are the angles of AB, BC and DC respectively with the x-axis.

AD is the fixed link.

AB and DC are the input and output links respectively.



Considering the links to be vectors, displacement along the x-axis is, $a\cos\theta + b\cos\beta - c\cos\phi - d = 0$

$$\therefore b\cos\beta = -a\cos\theta + c\cos\phi + d$$

Squaring on both sides

$$(b\cos\beta)^2 = (-a\cos\theta + c\cos\phi + d)^2$$

$$b^2\cos^2\beta = a^2\cos^2\theta + c^2\cos^2\phi + d^2 - 2ac\cos\theta\cos\phi - 2ad\cos\theta + 2cd\cos\phi \quad \text{— (1)}$$

Displacement along y-axis

$$a\sin\theta + b\sin\beta - c\sin\phi = 0$$

$$\therefore b\sin\beta = -a\sin\theta + c\sin\phi$$

$$b^2\sin^2\beta = a^2\sin^2\theta + c^2\sin\phi - 2ac\sin\phi\sin\theta \quad \text{— (2)}$$

Adding equations ① and ②

$$b^2 = c^2 + a^2 + d^2 - 2ac\cos\theta\cos\phi - 2ad\cos\theta + 2cd\cos\phi - 2ac\sin\theta\sin\phi$$

$$a^2 - b^2 + c^2 + d^2 + 2cd\cos\phi - 2ad\cos\theta = 2ac\,(\cos\theta\cos\phi - \sin\theta\sin\phi)$$

Dividing both sides by $2ac$

$$\frac{a^2 - b^2 + c^2 + d^2}{2ac} + \frac{d}{a}\cos\phi - \frac{d}{c}\cos\theta = \cos(\theta - \phi) \text{ (or) } \cos(\phi - \theta) \quad -(3)$$

Equation ③ is known as Freudenstein's equation.

It can be written as

$$K_3 + K_1\cos\phi + K_2\cos\theta = \cos(\theta - \phi) \quad -(4)$$

Where $K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$; $K_1 = \frac{d}{a}$; $K_2 = -\frac{d}{c}$

Let the input and the output are related by some function such as $y = f(x)$. For the given positions.

$\theta_1, \theta_2, \theta_3$ = three positions of input link.

ϕ_1, θ_2, ϕ_3 : Three positions of output link.

It is required to find the values of a, b, c and d to form a four-link mechanism giving the prescribed motions of the input and output links.

Eqn ④ can be written as

$$K_1\cos\phi_1 + K_2\cos\theta_1 + K_3 = \cos(\theta_1 - \phi_1)$$

$$K_1\cos\phi_2 + K_2\cos\theta_2 + K_3 = \cos(\theta_2 - \phi_2)$$

$$K_1\cos\phi_3 + K_2\cos\theta_3 + K_3 = \cos(\theta_3 - \phi_3)$$

K_1, K_2 and K_3 can be evaluated by Gaussian elimination method or by cramer's rule.

$$\Delta = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & 1 \\ \cos\phi_2 & \cos\theta_2 & 1 \\ \cos\phi_3 & \cos\theta_3 & 1 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} \cos(\theta_1 - \phi_1) & \cos\theta_1 & 1 \\ \cos(\theta_2 - \phi_2) & \cos\theta_2 & 1 \\ \cos(\theta_3 - \phi_3) & \cos\theta_3 & 1 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} \cos\phi_1 & \cos(\theta_1-\phi_1) & 1 \\ \cos\phi_2 & \cos(\theta_2-\phi_2) & 1 \\ \cos\phi_3 & \cos(\theta_3-\phi_3) & 1 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & \cos(\theta_1-\phi_1) \\ \cos\phi_2 & \cos\theta_2 & \cos(\theta_2-\phi_2) \\ \cos\phi_3 & \cos\theta_3 & \cos(\theta_3-\phi_3) \end{vmatrix}$$

K_1, K_2 and K_3 are given by

$$K_1 = \frac{\Delta_1}{\Delta}; \quad K_2 = \frac{\Delta_2}{\Delta}; \quad K_3 = \frac{\Delta_3}{\Delta}$$

Knowing K_1, K_2, and K_3, the values of a, b, c and d can be computed from the relations,

$$K_1 = \frac{d}{a}; \quad K_2 = -\frac{d}{c}; \quad K_3 = \frac{a^2-b^2+c^2+d^2}{2ac}$$

Value of either a or d can be assumed to be unity to get the proportionate values of other parameters.

Q. NO 6:

Solution :

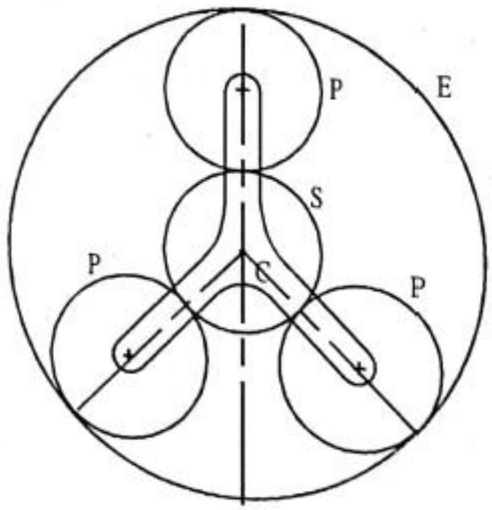


Fig : 7.31

(i) Number of teeth on different wheels

The given arrangement is shown in Fig. 7.31

As the minimum number of teeth on any wheel is 16, take the number of teeth on sun wheel $Z_s = 16$.

Since the pitch circle radius is proportional to number of teeth and the gears have same pitch

$$r_E = r_s + 2r_p$$

$$\text{i.e, } Z_E = Z_S + 2Z_P$$

$$\therefore Z_p = \frac{Z_E - Z_S}{2} \qquad \text{---(i}$$

Tabular Column

Condition of motion	Planet carrier C	Sunwheel S	Planet wheel P	Internal gear E
Fix the planet carrier 'C' and give + 1 rev to sunwheel S	0	+1	$-\frac{Z_S}{Z_P}$	$-\frac{Z_S}{Z_P}.\frac{Z_P}{Z_E}=-\frac{Z_S}{Z_E}$
Multiply by x	0	x	$-\frac{Z_S}{Z_P}.x$	$-\frac{Z_S}{Z_E}.x$
Add y	y	$y+x$	$y-\frac{Z_S}{Z_P}.x$	$y-\frac{Z_S}{Z_E}.x$

Planet carrier C rotates at 1/5 of the speed of the Sunwheel S. i.e., For every 5 revolutions of the Sunwheel S, planet carrier C will make 1 revolution.

$$\therefore \quad y = 1 \text{ and } y + x = 5$$

$$\text{i.e.,} \quad 1 + x = 5, \ \therefore x = 4$$

Internal gear E is stationary

$$\text{i.e.,} \ y - \frac{Z_S}{Z_E}.x = 0$$

$$\text{i.e.,} \ 1 - \frac{Z_S}{Z_E}.4 = 0$$

$$\therefore Z_E = 4Z_S = 4 \times 16 = 64$$

i.e., Number of teeth on internal gear E, $Z_E = 64$

From equation (i)

$$Z_P = \frac{Z_E - Z_S}{2} = \frac{64-16}{2} = 24$$

i.e., Number of teeth on planet wheel P, $Z_P = 24$

(ii) Torque necessary to keep the internal gear stationary.

From energy equation

$$T_S n_S + T_C n_C + T_E n_E = 0$$

$$\text{i.e.,} \ T_S n_S + T_C n_C = 0 \quad (\therefore n_E = 0)$$

$$100 \times 5 + T_C \times 1 = 0$$