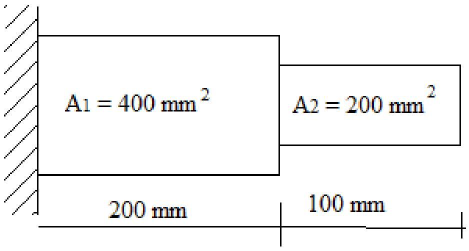
Internal Assessment Test III – July 2022

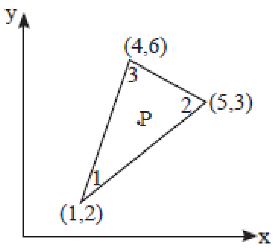
Sub:	Finite Element Methods	**Code:**	18ME61
Date: 08/ 07 / 2022	**Duration:** 90 mins	**Max Marks:** 50	**Sem:** VI
		Branch:	MECH

Note: Answer all questions.

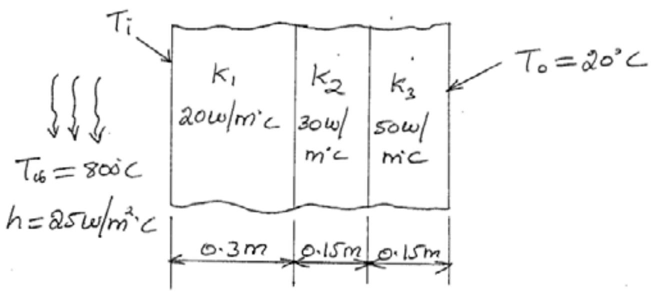
		Marks	OBE CO	OBE RBT
1	Evaluate Eigen values & Eigen vector for stepped bar shown in figure 1. Take E = 200 GPa, Density = 7850 kg/m^3. Draw mode shapes	**20**	CO4	L3



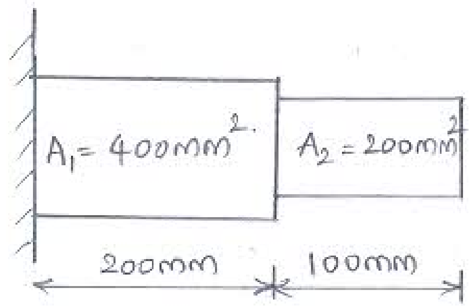
		Marks	CO	RBT
2	Explain the various boundary conditions in steady state heat transfer problems with simple sketches.	**6**	CO5	L2
3	The nodal co-ordinates of a triangular element are shown in fig. The x - coordinates of an interior point P is 3.3 and shape function. $N_1 = 0.3$. Determine N_2, N_3 and y - coordinate point P.	**10**	CO4	L2



		Marks	CO	RBT
4	A composite wall is as shown in figure consists of three materials. The outer temperature T_0 is 20^0C. Convective heat transfer takes place on inner surface of the wall with $T_\infty = 800^0$C. The convective heat transfer coefficient is 25 W/m^{2}C. Determine the temperature distribution in the wall.	**14**	CO5	L3

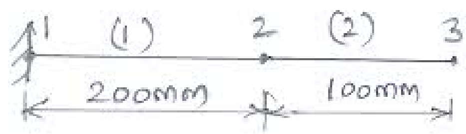


(P) Evaluate eigen vectors & eigen values for the stepped bar shown. Take $E = 200\,GPa$; density $= 7850\,kg/m^3$, Draw mode shapes.



Sol:

F.E. Model



Elemental Stiffness matrix

$$K = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_1 = \frac{200 \times 10^3 \times 400}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^5 \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

$$k_2 = \frac{200 \times 10^3 \times 200}{100}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^5 \begin{array}{cc} \begin{array}{cc} 2 & 3 \end{array} & \\ \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array}$$

Global Stiffness matrix

$$K = 10^5 \begin{array}{cc} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{bmatrix} 4 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

Elemental mass matrix

$$m^e = \frac{\rho A l_e}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$m^{e_1} = \frac{7850\times10^{-9}\times 400\times 200}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{array}{cc} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} 0.2093 & 0.1047 \\ 0.1047 & 0.2093 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

$$m^{e_2} = \frac{7850\times10^{-9}\times 200\times 100}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{array}{cc} \begin{array}{cc} 2 & 3 \end{array} & \\ \begin{bmatrix} 0.0523 & 0.0261 \\ 0.0261 & 0.0523 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array}$$

Global mass matrix

$$m^e = \begin{array}{cc} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{bmatrix} 0.2093 & 0.1047 & 0 \\ 0.1047 & 0.157 & 0.0261 \\ 0 & 0.0261 & 0.0523 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$KU - \lambda M U = 0$$

$$\left| 10^5 \begin{bmatrix} 4 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 0.2093 & 0.1047 & 0 \\ 0.1047 & 0.157 & 0.0261 \\ 0 & 0.0261 & 0.0523 \end{bmatrix} \right| = 0$$

$$\left| 10^5 \begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 0.157 & 0.0261 \\ 0.0261 & 0.0523 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 8\times10^5 & -4\times10^5 \\ -4\times10^5 & 4\times10^5 \end{bmatrix} - \begin{bmatrix} 0.157\lambda & 0.0261\lambda \\ 0.0261\lambda & 0.0523\lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 8\times10^5 - 0.157\lambda & -(4\times10^5 + 0.0261\lambda) \\ -(4\times10^5 + 0.0261\lambda) & 4\times10^5 - 0.0523\lambda \end{vmatrix} = 0$$

$$\Rightarrow (8\times10^5 - 0.157\lambda)(4\times10^5 - 0.0523\lambda) - (4\times10^5 + 0.0261\lambda)^2 = 0$$

$$\Rightarrow (3.2\times10^{11} - 41840\lambda - 62800\lambda + 8.211\times10^{-3}\lambda^2)$$

$$- (1.6\times10^{11} + 6.8121\times10^{-4}\lambda^2 + 20880\lambda) = 0$$

$$\Rightarrow \quad 1.6\times10^{11} - 125520\lambda + 7.53\times10^{-3}\lambda^2 = 0$$

$$\lambda_1 = 1.39\times10^6 \qquad \lambda_2 = 15.3\times10^6$$

$$\lambda_1 = \omega_1^2 = 1.39\times10^6 \qquad \omega_2^2 = 15.3\times10^6$$

$$\omega_1 = 1178.98 \text{ r/s} \qquad \omega_2 = 3911.52 \text{ r/s}$$

<u>For Mode Shapes</u>

<u>1st mode shape</u>

$$[K - \lambda M]x = 0$$

$$\left\{10^5\begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix} - \lambda_1\begin{bmatrix} 0.157 & 0.0261 \\ 0.0261 & 0.0523 \end{bmatrix}\right\}\begin{bmatrix} x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left\{10^5\begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix} - 1.39\times10^6\begin{bmatrix} 0.157 & 0.0261 \\ 0.0261 & 0.0523 \end{bmatrix}\right\}\begin{bmatrix} x_2^{(1)} \\ x_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

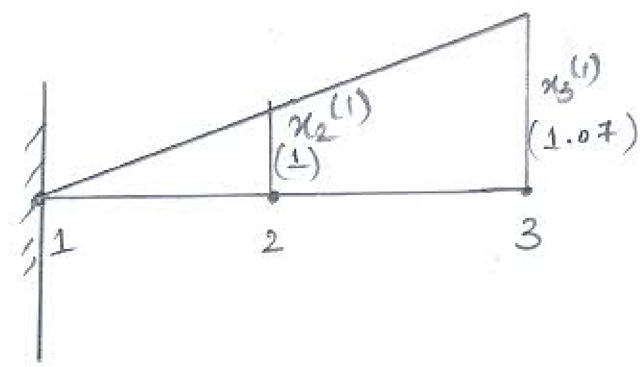
$$\left\{10^5\begin{bmatrix}8 & -4\\ -4 & 4\end{bmatrix} - \begin{bmatrix}2.18\times10^5 & 0.036\times10^5\\ 0.36\times10^5 & 0.727\times10^5\end{bmatrix}\right\}\begin{bmatrix}x_2^{(1)}\\ x_3^{(1)}\end{bmatrix} = 0$$

$$8\times10^5\, x_2^{(1)} - 2.18\times10^5\, x_2^{(1)} \quad -4\times10^5\, x_3^{(1)} - 0.36\times10^5\, x_3^{(1)} = 0$$

$$\left(5.82\times10^5\right) x_2^{(1)} - 486000\, x_3^{(1)} = 0$$

$$x_3^{(1)} = 1.07\, x_2^{(1)}$$

$$x^{(1)} = \begin{bmatrix}x_2^{(1)}\\ x_3^{(1)}\end{bmatrix} = \begin{bmatrix}x_2^{(1)}\\ 1.07\,x_2^{(1)}\end{bmatrix} = x_2^{(1)}\begin{bmatrix}1\\ 1.07\end{bmatrix}$$



<u>Second mode Shape</u>

$$[K - \lambda_2 M] = 0$$

$$\left\{10^5\begin{bmatrix}8 & -4\\ -4 & 4\end{bmatrix} - \lambda_2\begin{bmatrix}0.157 & 0.0261\\ 0.0261 & 0.0523\end{bmatrix}\right\}\begin{bmatrix}x_2^{(2)}\\ x_3^{(2)}\end{bmatrix} = 0$$

$$\left\{ 10^5 \begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix} - 15.3\times10^6 \begin{bmatrix} 0.157 & 0.0261 \\ 0.0261 & 0.0523 \end{bmatrix} \right\} \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = 0$$

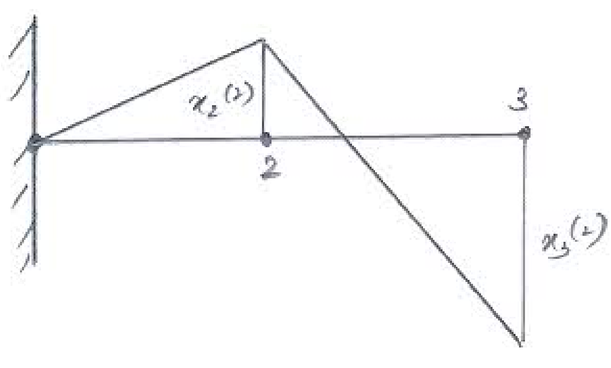
$$\left\{ 10^5 \begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix} - \begin{bmatrix} 2.4\times10^6 & 0.399\times10^6 \\ 0.399\times10^6 & 0.8\times10^6 \end{bmatrix} \right\} \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = 0$$

$$\left(8\times10^5 - 2.4\times10^6\right) x_2^{(2)} \cdot \left(-4\times10^5 - 0.399\times10^6\right) x_3^{(2)} = 0$$

$$-1.6\times10^6\, x_2^{(2)} - 0.799\times10^6\, x_3^{(2)} = 0$$

$$x_3^{(2)} = -2.00\, x_2^{(2)}$$

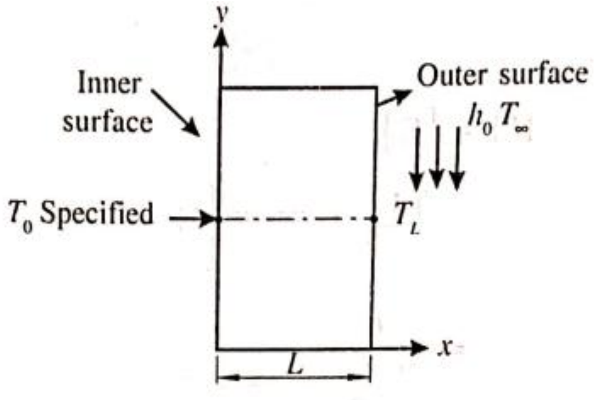
$$x^{(2)} = \begin{bmatrix} x_2^{(2)} \\ x_3^{(2)} \end{bmatrix} = \begin{bmatrix} x_2^{(2)} \\ -2\, x_2^{(2)} \end{bmatrix} = x_2^{(2)} \begin{bmatrix} 1 \\ -2.00 \end{bmatrix}$$



BOUNDARY CONDITIONS

- Specified Temperature Boundary Condition
- Specified Heat Flux Boundary Condition
- Convection Boundary Condition

Specified Temperature Boundary Condition

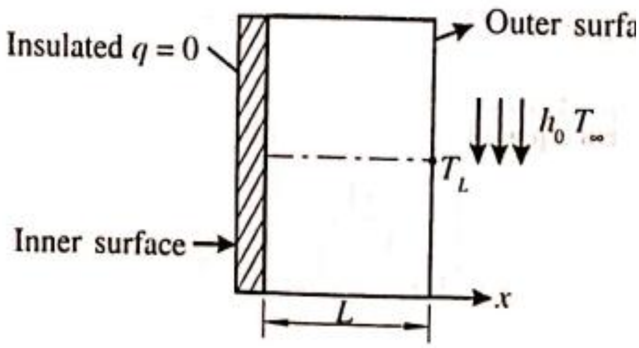


The boundary condition for this problem are

At $x = 0$, $T = T_0$ (Specified temperature)

At $x = L$ $q = h_0 (T_L - T_\infty)$

Specified Heat Flux Boundary Condition



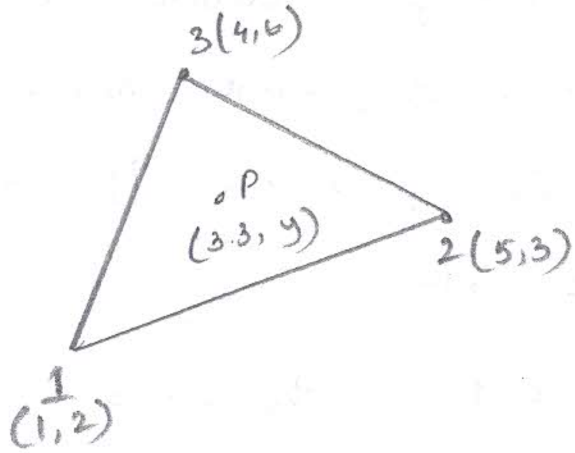
The boundary condition for this problem are

At $x = 0$, $q = q_0 = 0$

At $x = L$, $q = h_0 (T_L - T_\infty)$

Where, q_0 is the specified heat flux on the boundary.

4



$$x = N_1 x_1 + N_2 x_2 + N_3 x_3$$

$$3.3 = 0.3(1) + \eta(5) + (1-\xi-\eta)4$$

$$3.3 = 0.3 + 5\eta + 4 - 4\xi - 4\eta$$

$$-4\xi + \eta = -1.$$

$$-4(0.3) + \eta = -1$$

$$\eta = 0.2$$

$$\therefore N_2 = \eta = 0.2 //$$

$$N_3 = 1 - \xi - \eta = 1 - 0.3 - 0.2 = 0.5$$

$$N_3 = 0.5 //$$

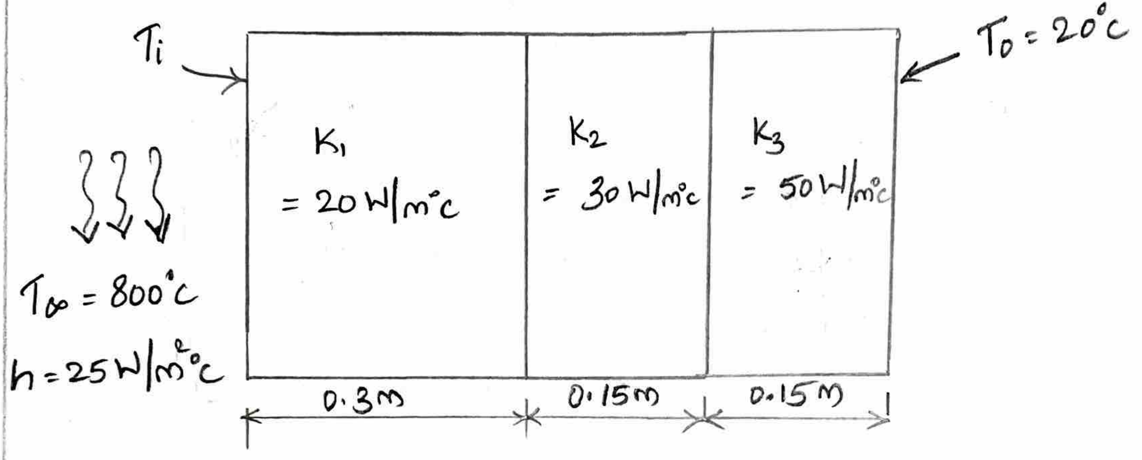
<u>y Coordinate</u>

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3$$

$$= 0.3(2) + 0.2(3) + 0.5(6)$$

$$y = 4.2 //$$

5 A Composite wall is as shown in figure Consists of three materials. The outer temperature T_o is 20°c. Convective heat transfer takes place on inner Surface of the wall with T_∞ = 800°c. The Convective heat transfer Coefficient is 25 $W/m^2 °c$. Determine the temperature distribution in the wall.

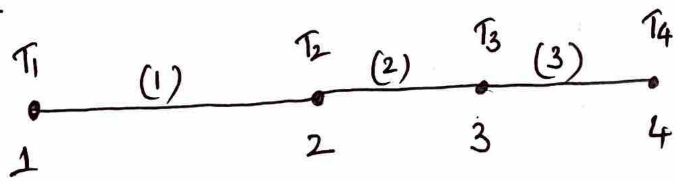


Sol

$K_1 = 20\ W/m°c$; $K_2 = 30\ W/m°c$; $K_3 = 50\ W/m°c$

$h = 25\ W/m^2°c$; $T_\infty = 800°c$

F.E. Model



Element Stiffness matrix

$$[K] = \frac{AK}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

For element 1

$$[K_1] = \frac{1 \times 20}{0.3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{c|cc|c} & 1 & 2 & \\ & \left[\begin{matrix} 66.67 \\ -66.67 \end{matrix}\right. & \left.\begin{matrix} -66.67 \\ 66.67 \end{matrix}\right] & \begin{matrix} 1 \\ 2 \end{matrix} \end{array}$$

For element 2

$$K_2 = \frac{1(30)}{0.15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} 2 & 3 \end{array}$$

$$= \begin{array}{cc} & \begin{array}{cc} 2 & 3 \end{array} \\ & \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{array}{c} 2 \\ 3 \end{array} \end{array}$$

For element 3

$$K_3 = \frac{1(50)}{0.15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{array}{cc} & \begin{array}{cc} 3 & 4 \end{array} \\ & \begin{bmatrix} 333.33 & -333.33 \\ -333.33 & 333.33 \end{bmatrix} \begin{array}{c} 3 \\ 4 \end{array} \end{array}$$

Global stiffness

$$[K] = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ & \begin{bmatrix} 66.67 & -66.67 & 0 & 0 \\ -66.67 & 266.67 & -200 & 0 \\ 0 & -200 & 533.33 & -333.33 \\ 0 & 0 & -333.33 & 333.33 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

At node 1 ; Convection is taking place

$$[K_h] = hA\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 25(1)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ & \begin{bmatrix} 25 & 0 \\ 0 & 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

Global stiffness

$$[K] = [K] + [K_h]$$

$$[K] = \begin{bmatrix} 91.67 & -66.67 & 0 & 0 \\ -66.67 & 266.67 & -200 & 0 \\ 0 & -200 & 533.33 & -333.33 \\ 0 & 0 & -333.33 & 333.33 \end{bmatrix}$$

Force Vector

$$[F_h] = A h T_\infty \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= 1 (25)(800) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 20,000 \\ 0 \end{bmatrix}$$

Equilibrium eqn $[K][T] = [F]$

$$\begin{bmatrix} 91.67 & -66.67 & 0 & 0 \\ -66.67 & 266.67 & -200 & 0 \\ 0 & -200 & 533.33 & -333.33 \\ 0 & 0 & -333.33 & 333.33 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 20000 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Since Temp at node 4 ; $T_4 = 20^\circ C$

$$-200 T_2 + 533.33 T_3 - 333.33 T_4 = 0$$

$$-200 (T_2) + 533.33 T_3 - 333.33 (20) = 0$$

$$-200 T_2 + 533.33 T_3 = 6666.67$$

$$\therefore \begin{bmatrix} 91.67 & -66.67 & 0 \\ -66.67 & 266.67 & -200 \\ 0 & -200 & 533.33 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 20000 \\ 0 \\ 6666.67 \end{bmatrix}$$

$T_1 = 304.76°c$; $T_2 = 119.05°c$; $T_3 = 57.14°c$

Nodal temp.

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{bmatrix} 304.76 \\ 119.05 \\ 57.14 \end{bmatrix} °c$$