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Internal Assessment Test 2 – Aug 2022

Sub:	Applied Hydraulics				Sub Code:	18CV43	Branch:	CV
Date:	04.08.2022	Duration:	90 mins	Max Marks:	50	Sem/Sec:	IV	OBE

Answer all questions. Provide neat sketches wherever necessary. Assume data wherever required.

		MARKS	CO	RBT
1	Explain three types of similarities in model analysis.	[10]	CO1	L2
2	Using Buckingham's Π - theorem, show that the velocity through a circular orifice is given by, $v=\sqrt{2gH}\,\phi\left[\frac{D}{H},\frac{\mu}{\rho vH}\right]$ where H is head causing flow, μ is coefficient viscosity, ρ is mass density and g is gravitational acceleration.	[10]	CO1	L4
3	Derive the conditions for most economical trapezoidal section	[10]	CO2	L3
4	An open channel is to be constructed of trapezoidal section and with side slope 1V:1.5H. Find relationship between bottom width and depth of flow for min excavation. If flow is to be 2.7cumec, calculate the bottom width and depth of flow assuming C=44.5 and bed slope =1/4000.	[10]	CO2	L4
5	Derive an equation for the force exerted and work done by a jet of water on a fixed curved plate in the direction of the jet when the jet strikes at the centre of the plate.	[10]	CO2	L4

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SIMILITUDE

- It is defined as the similarity between the model and its prototype. It means it has similar properties. There are 3 types of similarities which must exist between model and prototype.

1. Geometric Similarity
2. Kinematic Similarity
3. Dynamic Similarity

Geometric similarity

- When the ratios of the linear dimensions in the model and prototype are equal, it is said to be geometrically similar.

Let

L_m = Length of model, L_p = Length of prototype

B_m = Breadth of model, B_p = Breadth of prototype

H_m = Height of model, H_p = Height of prototype

A_m = Area of model, A_p = Area of prototype

V_m = Volume of model, V_p = Volume of prototype

For geometric similarity,

$$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{H_p}{H_m} = L_r$$

L_r = scale ratio

For area

$$\frac{A_p}{A_m} = \frac{L_p}{L_m} * \frac{B_p}{B_m} = L_r^{\ 2}$$

For volume

$$\frac{V_p}{V_m} = \frac{L_p}{L_m} * \frac{B_p}{B_m} * \frac{H_p}{H_m} = L_r^{\ 3}$$

Kinematic similarity

- When the ratios of the velocity and acceleration at the corresponding points in the model and corresponding points in the prototype are same, it is said have kinematic similarity.
- The direction of the vector quantities (velocity and acceleration) should be same.

In the fluid, let

v_{m1} = velocity at pt 1 in model, v_{p_1} = velocity at pt 1 in prototype

v_{m2} = velocity at pt 2 in model, v_{p_2} = velocity at pt 2 in prototype

a_{m1} = acc at pt 1 in model, a_{p_1} = acc at pt 1 in prototype

a_{m2} = acc at pt 2 in model, a_{p_2} = acc at pt 2 in prototype

For kinematic similarity

$$\frac{v_{p_1}}{v_{m1}} = \frac{v_{p_2}}{v_{m2}} = v_r$$

v_r is velocity ratio

$$\frac{a_{p_1}}{a_{m1}} = \frac{a_{p_2}}{a_{m2}} = a_r$$

a_r is acceleration ratio

DYNAMIC SIMILARITY

- When the ratios of the forces acting at the corresponding points in the model and in the prototype are same, it is said have dynamic similarity.
- The direction of the forces should be same.

At a point, let

$(F_i)_m$ = Inertial force in model, $(F_i)_p$ = Inertial force in prototype

$(F_v)_m$ = Viscous force in model, $(F_v)_p$ = Viscous force in prototype

$(F_g)_m$ = Gravity force in model, $(F_g)_p$ = Gravity force in prototype

For dynamic similarity

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

F_r is force ratio

Types of forces in moving fluid

1. Inertial Force, F_i

It is equal to the product of mass, m and acceleration, a and acts in the opposite direction to the direction of acceleration.

$$F_i = m * a$$

$$= \rho * Volume * \frac{Velocity}{Time} = \rho * \frac{Volume}{Time} * Velocity$$

$$= \rho * Area * \frac{Length}{Time} * Velocity$$

$$= \rho * Area * Velocity * Velocity$$

$$= \rho A v^2$$

2. Viscous Force, F_v

It is equal to the product of shear stress, τ due to viscosity and surface area of flow, A. Considered in problems where viscosity is significant.

$$F_v = \tau * A$$

Newton's law of viscosity, $\tau = \mu \frac{du}{dy}$

$$= \mu \frac{du}{dy} * A$$

$$= \mu \frac{v}{L} A$$

3. Gravity Force, F_g

It is equal to the product of mass, m and acceleration due to gravity, g of the flowing fluid. This force is considered in open channel flow.

$$F_g = m * g$$

$$= \rho * Volume * g$$

$$= \rho * Area * Length * g$$

$$= \rho A L g$$

4. Pressure Force, F_p

It is equal to the product of pressure intensity, p and crosssection area, A of the flowing fluid. It is considered in pipe flow.

$$F_p = p * A$$

$$= pA$$

5. Surface tension force, F_s

It is equal to the product of surface tension, σ and length of surface of flow, L.

$$F_s = \sigma * L$$

$$= \sigma L$$

6. Elastic force, F_e

It is equal to the product of elastic stress, K and area of flowing fluid, A.

$$F_e = K * A$$

$$= KA$$

Problem 2 – Buckingham's Π theorem

- Using Buckingham's Π – theorem, show that the velocity through a circular orifice is given by

$$V = \sqrt{2gH}\,\phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$$

$$V = \sqrt{2gH} * \phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$$

$1.\ g = LT^{-2}$

$2.\ H = L$

$3.\ D = L$

$4.\ \rho = (kg/m^3) = ML^{-3}$

$5.\ \mu = ML^{-1}T^{-1}$

$6.\ V = LT^{-1}$

$$V = f(g, H, D, \mu, \rho)$$

$$f_1(V, g, H, D, \mu, \rho) = 0$$

There 6 variables and

3 fundamental dimensions

$\therefore 6 - 3 = 3\ \pi$ - terms

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

Repeating variables

H, g, ρ

$$\pi_1 = H^{a_1} g^{b_1} \rho^{c_1} . V$$

$$\pi_2 = H^{a_2} g^{b_2} \rho^{c_2} . D$$

$$\pi_3 = H^{a_3} g^{b_3} \rho^{c_3} . \mu$$

$$\pi_1 = H^{a_1} g^{b_1} \rho^{c_1} . V$$

$$M^0 L^0 T^0 = L^{a_1} (LT^{-2})^{b_1} (ML^{-3})^{c_1} . LT^{-1}$$

$$a_1 = -1/2 \quad b_1 = -1/2 \quad c_1 = 0$$

$$\pi_1 = H^{-1/2} g^{-1/2} \rho^0 . V \rightarrow \pi_1 = \frac{V}{\sqrt{gH}}$$

$$\pi_2 = H^{a_2} g^{b_2} \rho^{c_2} . D$$

$$a_2 = -1 \quad b_2 = 0 \quad c_2 = 0$$

$$\pi_2 = H^{-1} g^0 \rho 0 . D \rightarrow \pi_2 = \frac{D}{H}$$

$$\pi_3 = H^{a_3} g^{b_3} \rho^{c_3} . \mu$$

$$a_3 = -3/2 \quad b_3 = -1/2 \quad c_3 = -1$$

$$\pi_3 = H^{-3/2} g^{-1/2} \rho^{-1} . \mu \rightarrow \pi_3 = \frac{\mu}{\rho H^{3/2} \sqrt{g}}$$

Solving for a_i, b_i and c_i in respective eqns

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

$$\pi_1 = \frac{V}{\sqrt{gH}} = \frac{V}{\sqrt{2gH}} \text{ (applying principles of } \pi - \text{term)}$$

$$\pi_2 = \frac{D}{H}$$

$$\pi_3 = \frac{\mu}{\rho H^{3/2}\sqrt{g}} = \frac{\mu}{\rho H\sqrt{gH}} * \frac{V}{V} = \frac{\mu}{\rho HV} * \frac{V}{\sqrt{gH}} = \frac{\mu}{\rho HV} * \pi_1$$

$$\pi_3 = \frac{\mu}{\rho HV} * \pi_1 \rightarrow \pi_3 / \pi_1 = \frac{\mu}{\rho HV} \text{ (applying principles of } \pi - \text{term)}$$

$$f_1\left(\frac{V}{\sqrt{2gH}}, \frac{D}{H}, \frac{\mu}{\rho HV}\right) = 0 \quad \rightarrow \quad \frac{V}{\sqrt{2gH}} = \phi\left(\frac{D}{H}, \frac{\mu}{\rho VH}\right)$$

$$V = \sqrt{2gH} * \phi\left(\frac{D}{H}, \frac{\mu}{\rho VH}\right) \qquad \textit{Hence}\text{ proved.}$$

MOST ECONOMICAL SECTION – TRAPEZOIDAL

Let

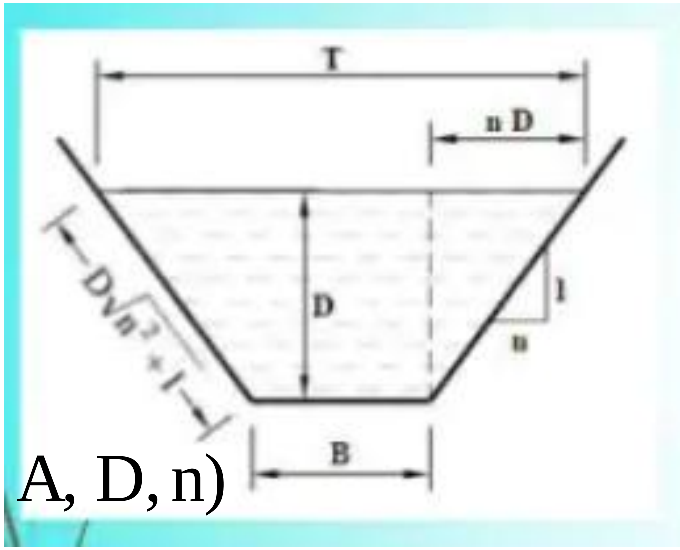
Depth of flow - D

Bed width - B

Side slope - 1/n

Wetted area, $A = D * (B + nD)$

Re writing the above eqn(B in terms of A, D, n)



$$\frac{A}{D} = B + nD \rightarrow B = \frac{A}{D} - nD.....(1)$$

Wetted Perimeter- $P = B + 2D\sqrt{1+n^2}$.....(2)

Sub (1) in (2)

$$P = \frac{A}{D} - nD + 2D\sqrt{1+n^2}$$

For most economical section, P is min

$$\frac{dP}{dD}=0$$

$$\frac{dP}{dD}=-\frac{A}{D^2}-n+2\sqrt{1+n^2}=0$$

$$-\frac{A}{D^2}-n+2\sqrt{1+n^2}=0$$

$$\frac{A}{D^2}+n=2\sqrt{1+n^2}$$

Sub the value of A

$$\frac{D*(B+nD)}{D^2}+n=2\sqrt{1+n^2}$$

$$\frac{(B+nD)}{D}+n=2\sqrt{1+n^2}$$

$$(B+nD)+nD=2D\sqrt{1+n^2}$$

$$(B+2nD)=2D\sqrt{1+n^2}$$

$$\frac{(B+2nD)}{2}=D\sqrt{1+n^2}$$

1. Half of the top width is equal to one of the sloping sides

Calculation of R

$$R = \frac{A}{P} = \frac{D*(B+nD)}{B+2D\sqrt{1+n^2}} \;....(3)$$

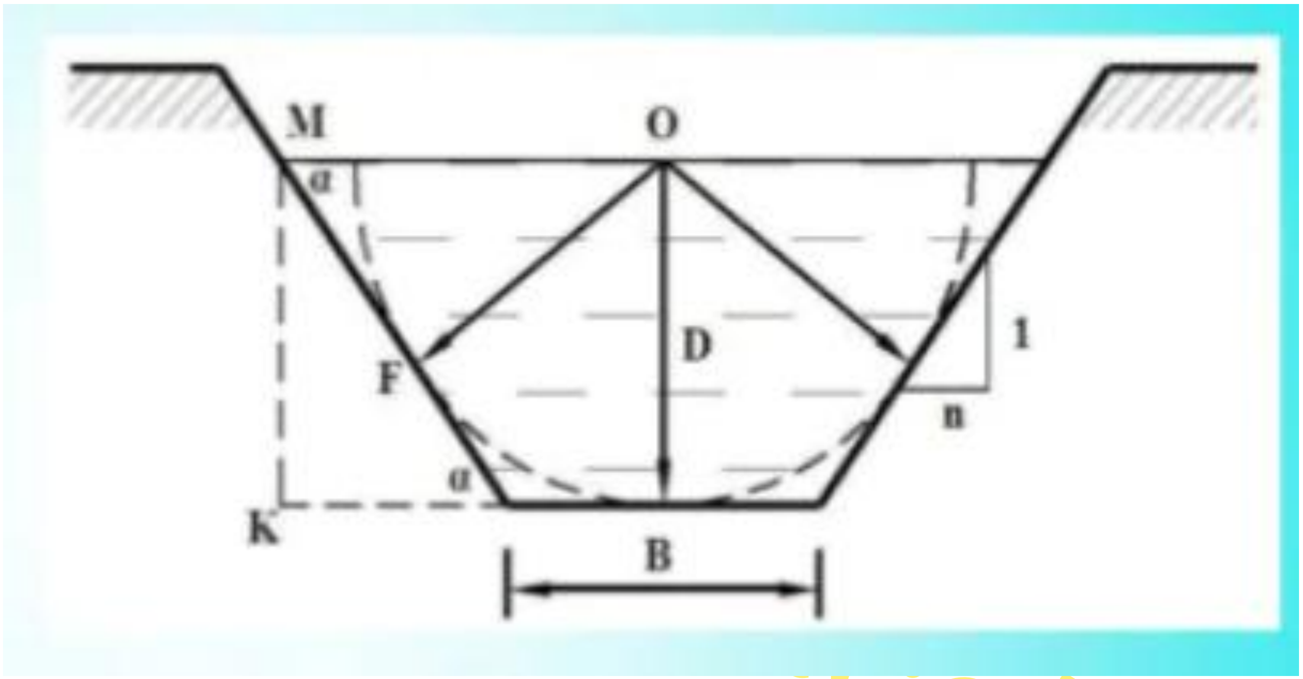
$$D\sqrt{1+n^2} = \frac{B+2nD}{2} \;....(4)$$

$Sub\,(4)$ in (3)

$$R = \frac{A}{P} = \frac{D*(B+nD)}{B+2(\frac{B+2nD}{2})} = \frac{D*(B+nD)}{B+B+2nD} = \frac{D*(B+nD)}{2(B+nD)}$$

$$R = \frac{D}{2}$$

2. For most economical trapezoidal section, the hydraulic radius is equal to half of the depth of flow.


M
O
α
D
1
F
n
α
K
B

Let

α - angle made by the sloping side wrt to x - axis(horizontal axis)

O - centre of the top width

OF - perpendicular line to the sloping side MN

Taking ΔOFM(right angled triangle)

$$\sin\alpha = \frac{OF}{OM} \rightarrow OF = OM * \sin\alpha .....(i)$$

Taking ΔMKN (right angled triangle)

$$\sin\alpha = \frac{MK}{MN} = \frac{D}{D*\sqrt{1+n^2}} .....(ii)$$

Sub (ii) in (i)

$$OF = OM * \sin\alpha$$

$$OF = OM * \frac{D}{D*\sqrt{1+n^2}}$$

$$Sub\,OM = half \text{ of Top width, T} = D * \sqrt{1+n^2} \; (first \text{ condition})$$

$$OF = D * \sqrt{1+n^2} * \frac{D}{D * \sqrt{1+n^2}} = D$$

3. Thus for a most economical trapezoidal section, a semi-circle with centre O (centre of top width) and radius equal to the depth of flow, D will be tangential to the three sides of the most economical trapezoidal section.

PROBLEM

- An open channel is to be constructed of trapezoidal section and with side slope 1V:1.5H. Find relationship between bottom width and depth of flow for min excavation. If flow is to be 2.7cumec, calculate the bottom width and depth of flow assuming C=44.5 and bed slope =1/4000.

Let B be the bottom width and y be the depth of flow

Most eonomical trapezoidal section

1. $\frac{B+2ny}{2} = y*\sqrt{1+n^2}$ ....(1)

2. R = y/2

n = 1.5

$$\frac{B+2*1.5y}{2} = y*\sqrt{1+1.5^2}$$

$$B+3y = y*2*\sqrt{1+1.5^2}$$

$$B = 0.605y$$

$$Q = 2.7m^3/s$$

$$C = 44.5$$

$$S = 1/4000$$

$$Q = A*C*\sqrt{RS}$$

$$A = y(B+ny)$$

$$2.7 = y(0.605y+1.5y)*44.5*\sqrt{\frac{y}{2}*\frac{1}{4000}}$$

$$\frac{2.7*20*\sqrt{2}}{44.5} = 2.105y^2*\sqrt{y}$$

$$\frac{2.7*20*\sqrt{20}}{44.5*2.105} = y^2*\sqrt{y} = y^{(2+1/2)} = y^{5/2}$$

$$y = \left(\frac{2.7*20*\sqrt{20}}{44.5*2.105}\right)^{2/5} = 1.46m$$

$$B = 0.605y = 0.605*1.46 = 0.885m$$

Force exerted by the jet on Stationary curved plate at the centre

- Force in the horizontal direction(in the direction of jet,

$$F_x = (m/\Delta t)*(v_{1x} - v_{2x})$$

$$F_x = \rho a v*(v - (-v\cos\theta))$$

$$F_x = \rho a v^2 (1 + \cos\theta)$$

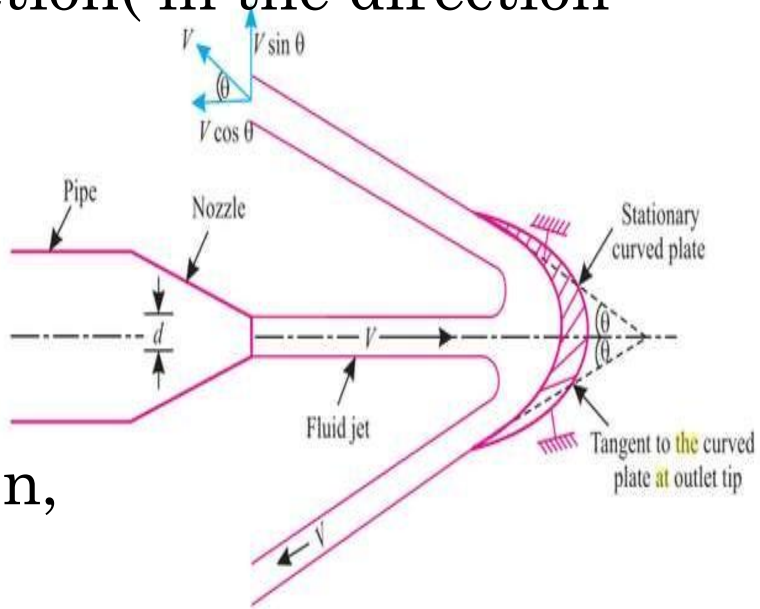


Fig. 1.3. Fluid jet striking a stationary curved plate.

- Force in the vertical direction,

$$F_y = (m/\Delta t)*(v_{1y} - v_{2y})$$

$$F_y = \rho a v*(0 - v\sin\theta)$$

$$F_y = -\rho a v^2 \sin\theta$$

- Angle of deflection = (180° – θ)