USN											CMRIT

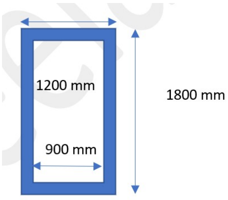
Internal Assessment Test III – June.2022

Sub	Design of Prestressed concrete	Sub Code:	18CV81/ 17CV82	Branch:	Civil Engg
Date:	17.06.2022	Duration: 90 min's	Max Marks: 50	Sem/Sec : 8th sem /All sections	OBE

		MARKS	CO	RBT
1.	A post tensioned bridge girder with unbonded tendons is of box section of overall dimensions 1200 mm wide × 1800 mm deep with wall thickness 150 mm . The high tensile steel has an area of 4000 mm^2 and is located at an effective depth of 1600 mm. The effective prestress in steel after all losses is 1000N/mm^2 and effective span of girder is 24 m. If f_{ck} = 40 N/mm^2 and f_p = 1600 N/mm^2,Estimate the ultimate flexural strength of section.	[10]	CO3	L3

Given data

f_{ck} = 40 N/mm^2 , b_f = 1200 mm, f_{pu} = 1600 N/mm^2 ,d = 1600 mm, A_{ps} = 4000 mm^2 D_f = 150 mm b_w = 300 mm (= 150 + 150)



- STEP 1

  Assume $x_u > D_f$

$$A_{ps} = (A_{pw} + A_{pf})$$
$$A_{pf} = 0.45\, f_{ck}\,(b - b_w)\,(D_f/f_{pu})$$
$$= (0.45 \times 40)\,(1200 - 300)\,(150/1600)$$
$$= 1518 \text{ mm}^2$$
$$A_{pw} = (4000 - 1518) = 2482 \text{ mm}^2$$
$$\text{Ratio}\left(\frac{A_{pw} f_{pu}}{b_w d f_{ck}}\right) = \left(\frac{2482 \times 1600}{300 \times 1600 \times 40}\right) = 0.206$$

- STEP 2
- Use table 11 , find f_{pb} and x_u

interpolate the following values

$$\left(\frac{f_{pb}}{0.87 f_{pu}}\right) = 0.95 \text{ and } \left(\frac{x_u}{d}\right) = 0.414$$

$$f_{pb} = (0.95 \times 0.87 \times 1600) \text{ and } x_u = (0.414 \times 1600) = 662.4 \text{ mm}$$
$$= 1322 \text{ N/mm}^2$$

Assumption of $x_u > D_f$ is correct

- STEP 3

  Find flexural strength of the box girder

$$M_u = f_{pb} A_{pw}(d - 0.42 x_u) + 0.45 f_{ck}(b - b_w) D_f (d - 0.5 D_f)$$
$$= (1322 \times 2482)\,(1600 - 0.42 \times 662.4)$$
$$+ (0.45 \times 40)(1200 - 300)150(1600 - 0.5 \times 150)$$
$$= (4338 + 3705) \times 10^6 \text{ N mm}$$
$$= 8043 \text{ kN m}$$

		MARKS	CO	RBT
2.	A precast pretensioned T-beam has a flange width of 1200 mm and thickness of 150 mm. The width and depth of the rib are 300 and 1500 mm, respectively. The high tensile steel tendons of cross-sectional area 4700 mm^2 are located at an effective depth of 1600 mm. If the characteristic strength of concrete and steel are 40 and 1600 N/mm^2, respectively, calculate the flexural strength of the T-section using Indian Standard Code provisions.	[10]	CO2	L3

Given data: f_{ck} = 40 N/mm^2 b_f = 1200 mm f_{pu} = 1600 N/mm^2 d = 1600 mm A_{ps} = 4700 mm^2 D_f = 150 mm b_w = 300 mm

- STEP 1

	Assume x_u < D_f , then put b = b_f find $\frac{A_{ps} \times f_{pu}}{b_f \times d \times f_{ck}} = \frac{4700 \times 1600}{1200 \times 1600 \times 40} = 0.097$ Find $\frac{x_u}{d}$ through interpolation using Table 11, Page 51 corresponding to $\frac{A_{ps} \times f_{pu}}{b_f \times d \times f_{ck}} = 0.1$, $\frac{x_u}{d} \approx 0.217$ $x_u \approx 0.217 \times 1600 \approx 347.2$ mm But x_u > D_f , so assumption is wrong. • STEP 2 Recalculate effective reinforcement ratio , by putting the values of b_w and A_{pw} in $\frac{A_{pw} \times f_{pu}}{b_w \times d \times f_{ck}}$ $A_p = (A_{pw} + A_{pf})$ and $A_{pf} = 0.45 f_{ck} (b - b_w)(D_f / f_{pu})$ $= (0.45 \times 40)(1200 - 300)\left(\frac{150}{1600}\right)$ $= 1518$ mm^2 ∴ $A_{pw} = (4700 - 1518) = 3182$ mm^2 Hence, the ratio $\left(\frac{A_{pw} . f_{pu}}{b_w . d . f_{ck}}\right) = \left(\frac{3182 \times 1600}{300 \times 1600 \times 40}\right) = 0.265$ From Table 11, the corresponding values are interpolated as, $\left(\frac{f_{pb}}{0.87 f_{pu}}\right) = 1.0$ and $\left(\frac{x_u}{d}\right) = 0.56$ $f_{pb} = (0.87 \times 1600) = 1392$ N/mm^2 and $x_u = (0.56 \times 1600) = 896$ mm • STEP 3 Calculate Moment of resistance or flexural strength of the T-section $\left(\frac{f_{pb}}{0.87 f_{pu}}\right) = 1.0$ and $\left(\frac{x_u}{d}\right) = 0.56$ $f_{pb} = (0.87 \times 1600) = 1392$ N/mm^2 and $x_u = (0.56 \times 1600) = 896$ mm $M_u = f_{pb} A_{pw} (d - 0.42 x_u) + 0.45 f_{ck} (b - b_w) D_f (d - 0.5 D_f)$ $= (1392 \times 3182)(1600 - 0.42 \times 896)$ $+ (0.45 \times 40 (1200 - 300) 150 (1600 - 0.5 \times 150)$ $= [(5420 \times 10^6) + (3705 \times 10^6)]$ $= (9125 \times 10^6)$ N mm $= 9125$ kN m			
3.	A prestressed girder has to be designed to cover a span of 12 m, to support an uniformly distributed live load of 15 kN/m. M-45 Grade concrete is used for casting the girder. The permissible stress in compression under transfer may be assumed as 14 N/mm2 and 1.4 N/mm2 in tension under transfer and service load conditions. Assume 15 per cent losses in prestress during service load conditions.	[10]	CO2	L3

	The preliminary section proposed for the girder consists of a symmetrical I-section with flanges 300 mm wide and 150 mm thick. The web is 120 mm wide by 450 mm deep. (a) Check the adequacy of the section provided to resist the service loads. (b) Design the minimum prestressing force and the corresponding eccentricity for the section. **Solution.** $L = 12$ m Loss ratio = $\eta = 0.85$ $f_{ct} = 14$ N/mm^2 $f_{tt} = f_{tw} = -1.4$ N/mm^2 $q = 15$ kN/m $y = y_t = y_b = 375$ mm Area of section = $A = [(2 \times 200 \times 150) + (120 \times 450)] = 144000$ mm^2 Section moment of area = $I = \left[\frac{300 \times 750^3}{12} - \frac{180 \times 450^3}{12}\right] = (918 \times 10^7)$ mm^4 Section modulus = $Z = Z_t = Z_b = \left[\frac{I}{y}\right] = \left[\frac{(918 \times 10^7)}{375}\right] = (24.48 \times 10^6)$ mm^3			

Self-weight of girder $= g = \left[\frac{144000}{10^6}\right] 24 = 3.456$ kN/m

Dead-load moment $= M_g = \left[\frac{gL^2}{8}\right] = \left[\frac{3.456 \times 12^2}{8}\right] = 62.208$ kN m

Live-load moment $= M_q = \left[\frac{qL^2}{8}\right] = \left[\frac{15 \times 12^2}{8}\right] = 270$ kN m

$$f_{br} = (\eta f_{ct} - f_{tw}) = [(0.85 \times 14) - (-1.4)] = 13.3 \text{ N/mm}^2$$

(a) Check for adequacy of section

$$Z_b \geq \left[\frac{M_q + (1-\eta)M_g}{f_{br}}\right] = \left[\frac{(270 \times 10^6) + (1 - 0.85)(62.208 \times 10^6)}{13.3}\right]$$

$$= (21 \times 10^6) \text{ mm}^3 > Z_b \text{ provided}$$

Hence, the section provided is adequate to resist the loads safely.

(b) Minimum prestressing force and corresponding eccentricity

$$f_t = \left[f_{tt} - \frac{M_g}{Z_t}\right] = \left[-1.4 - \frac{(62.208 \times 10^6)}{(24.48 \times 10^6)}\right] = -3.94 \text{ N/mm}^2$$

$$f_b = \left[\frac{f_{tw}}{\eta} + \frac{M_g + M_q}{\eta Z_b}\right] = \left[\frac{-1.4}{0.85} + \frac{(62.208 + 270) \times 10^6}{0.85(24.48 \times 10^6)}\right] = 14.36 \text{ N/mm}^2$$

Prestressing force is computed using the relation

$$P = \left[\frac{A(f_t Z_t + f_b Z_b)}{(Z_t + Z_b)}\right] = \left[\frac{144000[(-3.94 + 14.36)24.48 \times 10^6]}{(2 \times 24.48 \times 10^6)}\right]$$

$$= (747.28 \times 10^6)\text{N} = 747.28 \text{ kN}$$

Eccentricity is obtained by the relation

$$e = \left[\frac{Z_t Z_b (f_b - f_t)}{A(f_t Z_t + f_b Z_b)}\right] = \left[\frac{(24.48^2)(10^{12})[14.36 - (-3.94)]}{144000[(-3.94 + 14.36)24.48 \times 10^6]}\right] = 298.5 \text{ mm}$$

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4 | [10] | CO3 | L3

A prestressed concrete beam (span = 10 m) of rectangular section 120 mm wide and 300 mm deep, is axially prestressed by a cable carrying an effective force of 180 kN. The beam supports a total uniformly distributed load of 5 kN/m which includes the self-weight of the member. If the prestressing is done using a parabolic cable of eccentricity 100 mm at the centre of the beam . Calculate the magnitude of principal tension developed in the beam for both straight and parabolic profile of cable and compare the results.

A = 120 x 300 = 36 x 10^3 mm^2

P = 180 kN

Total load , w = 5 kN/m

L = 10m

- Max shear stress developed in a rectangular beam $\tau_{vmax} = \frac{3 \times V}{2\, b\, h}$

Max S F @ supports , V $= \frac{wL}{2} = \frac{50 \times 10}{2} = 25$ kN

Max Shear stress $\tau_{vmax} = \frac{3 \times 25 \times 10^3}{2 \times 120 \times 300} = 1.042$ N/mm^2

i) with Prestress

$$f_{\substack{max \\ min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$$

where f_x and f_y are the direct stresses and τ_v, is the shear stress acting at the point.

Axial prestress $f_x = \frac{180 \times 10^3}{120 \times 300} = 5$ N/mm^2

$f_{max,min} = \frac{5}{2} \pm \frac{1}{2}\sqrt{(5^2 + 4 \times 1.042^2)}$

$= 2.5 \pm 2.71$

$f_{max} =$ 5.21 N/mm^2 (compression)

$f_{min} =$ - 0.21 N/mm^2

Principal tension = $f_{min} =$ - 0.21 N/mm^2

ii) without Prestress

	$f_{\substack{max\\min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$ where f_x and f_y are the direct stresses and τ_v, is the shear stress acting at the point. Axial prestress $f_x = f_y = 0$ $f_{max,min} = 0 \pm \frac{1}{2}\sqrt{((0^2) + 4 \times 1.042^2)}$ $= \pm 1.042$ N/mm^2 Principal tension = f_{min} = - 1.042 N/mm^2 Since with the actual prestress, principal tension is reduced to 1.042 N/mm^2 to 0.21 N/mm^2 ie $\frac{1.042-0.21}{1.042} \times 100 = 79.85$ % Calculation of slope of the cable Calculation of slope at support y = $\frac{4 \times e}{L^2}(Lx - x^2)$ $\frac{dy}{dx} = \frac{4 \times e}{L^2}(L - 2x)$ At support x =0, θ = $\frac{dy}{dx} = \frac{4 \times e}{L} = \frac{4 \times 100}{10000} = \frac{1}{25}$ radians θ in degrees = $\frac{1}{25} \times \frac{180}{\pi} = 2.292^{o}$ Vertical component of prestressing force = P x sin 2.29 = 180 × sin 2.29 = 7.2 kN Horizontal component of prestressing force = P x cos 2.29 = 180 × cos 2.29 = 179.8 kN Net vertical shear force acting on the section will be V = 25 – 7.2 = 17.8kN Max Shear stress $\tau_{vmax} = \frac{3 \times 17.8 \times 10^3}{2 \times 120 \times 300}$ = 0.742 N/mm^2 Axial prestress $f_x = \frac{180 \times 10^3}{120 \times 300}$ = 5 N/mm^2 $f_{\substack{max\\min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$ $f_{max,min} = \frac{5}{2} \pm \frac{1}{2}\sqrt{(5^2 + 4 \times 0.742^2)}$ $= 2.5 \pm 2.61$ f_{max} = 5.11 N/mm^2 (compression) f_{min} = - 0.11 N/mm^2 Principal tension = f_{min} = - 0.11 N/mm^2 i) Comparing with axial prestressing $\frac{0.211 - 0.11}{0.211} \times 100 = 47.62$ % ii) Comparing with without axial prestressing $\frac{1.042 - 0.11}{1.042} \times 100 = 89.44\%$			
5.	A beam of symmetrical I section with flanges 450 x 150 mm and the web thickness 150 mm , the overall depth of the beam 1000 mm span of the beam is 20 m. The cable is parabolic with zero eccentricity at supports and max eccentricity of 300 mm at mid-span. The effective prestressing force 1250 kN. Live load = 20 kN. Determine the principal tension at the support section and at the junction of web and flange. Solution:- Ac = 150 x 450 + 150 x 700 = 240 x 10^3 mm^2 $I = \frac{450 \times 1000^3}{12} - \frac{300 \times 700^3}{12} = 2.8975 \times 10^{10}\ mm^4$	[10]	CO2	L3

Calculation of slope at support $y = \frac{4 \times e}{L^2}(Lx - x^2)$

$\frac{dy}{dx} = \frac{4 \times e}{L^2}(L - 2x)$

At support x =0, $\theta = \frac{dy}{dx} = \frac{4 \times e}{L} = \frac{4 \times 300}{20000}$ = 0.06 radians

θ in degrees = 0.06 $\times \frac{180}{\pi}$ = 3.42°

self weight = 25 x .024 = 5.76 kN/m

Vertical component of prestressing force = P x sin 3.42 = 1250 × sin 3.42 =1250 kN

Horizontal component of prestressing force = P x cos 3.42 = 1250 × cos 3.42 = 74.96 kN

Total load = 25.76 kN/m

Shear force at support due to applied load = 25.76 x 20 /2 = 257.6 kN

Net shear force at support section = 257.6 – 74.96 = 182.64 kN

Shear stress

- At centroid

  $\tau = \frac{F \times A \times \bar{y}}{I \times b} = \frac{182640 \times (450 \times 150 \times 425 + 150 \times 350 \times \frac{350}{2})}{2.8975 \times 10^{10} \times 150}$, (taking b = bw)

  = 1.594 N/mm^2

- At junction of the web

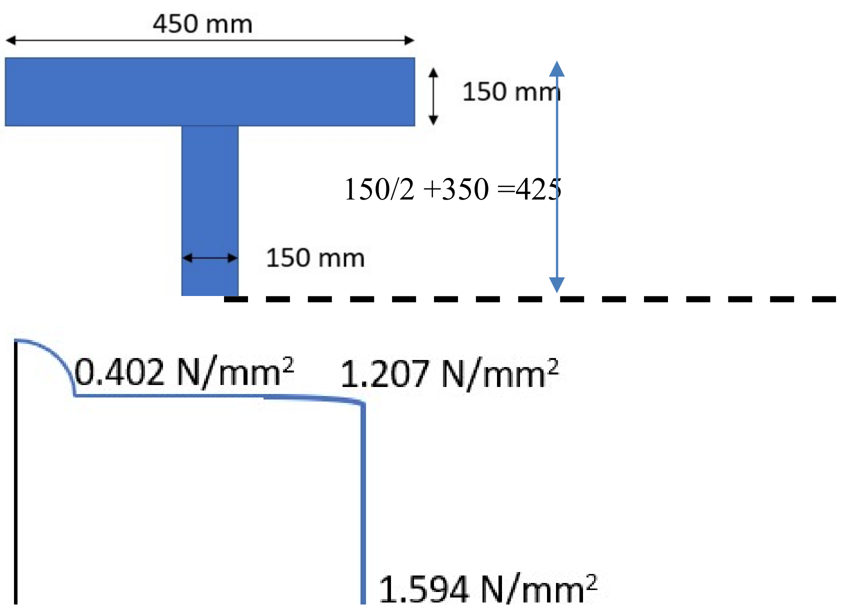
  $\tau = \frac{F \times A \times \bar{y}}{I \times b} = \frac{182640 \times (450 \times 150 \times (500 - \frac{150}{2})}{2.8975 \times 10^{10} \times 150}$, (taking b = bw)

  = 1.207 N/mm^2

- At junction of the flange

  $\tau = \frac{F \times A \times \bar{y}}{I \times b} = \frac{182640 \times (450 \times 150 \times (500 - \frac{150}{2})}{2.8975 \times 10^{10} \times 450}$, (taking b = bf)

  = 0.402 N/mm^2



1. Principal tension along centrodial axis

$$f_{\substack{max \\ min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$$

$f_x = \frac{1250000}{240 \times 1000}$ = 5.21 N/mm^2

	$f_{max,min} = \frac{5.21}{2} \pm \frac{1}{2}\sqrt{(5.21^2 + 4 \times 1.549^2)}$ $= 2.605 \pm 3.504$ $f_{max} = 2.605 + 3.504 = +\ 5.6$ N/mm^2 (compression) $f_{min} = 2.605 - 3.504 = -\ 0.449$ N/mm^2 (tension) 2. Principal tension at the junction of flange $f_{\substack{max \\ min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$ $f_x = \frac{1250000}{240 \text{ x } 1000} = 5.21$ N/mm^2 $f_{max,min} = \frac{5.21}{2} \pm \frac{1}{2}\sqrt{(5.21^2 + 4 \times 0.402^2)}$ $= 2.605 \pm 2.63$ $f_{max} = 2.605 + 2.63 = +\ 5.24$ N/mm^2 (compression) $f_{min} = 2.605 - 2.63 = -\ 0.031$ N/mm^2 (tension) 3. Principal tension at the junction of web $f_{\substack{max \\ min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$ $f_x = \frac{1250000}{240 \text{ x } 1000} = 5.21$ N/mm^2 $f_{max,min} = \frac{5.21}{2} \pm \frac{1}{2}\sqrt{(5.21^2 + 4 \times 1.207^2)}$ $= 2.605 \pm 2.87$ $f_{max} = 2.605 + 2.87 = +\ 5.47$ N/mm^2 (compression) $f_{min} = 2.605 - 2.87 = -\ 0.266$ N/mm^2 (tension)				