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CMRIT
CMR INSTITUTE OF TECHNOLOGY, BENGALURU.
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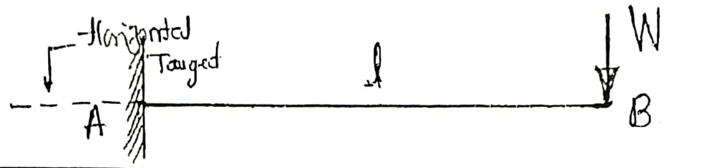
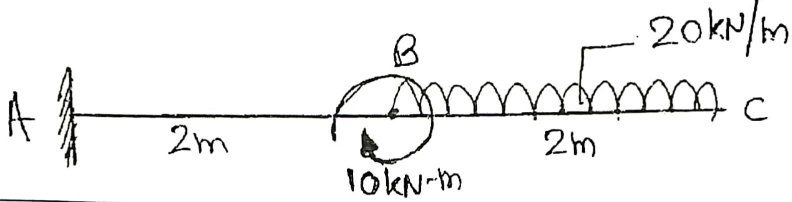
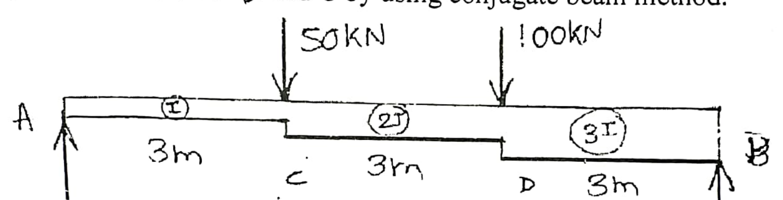
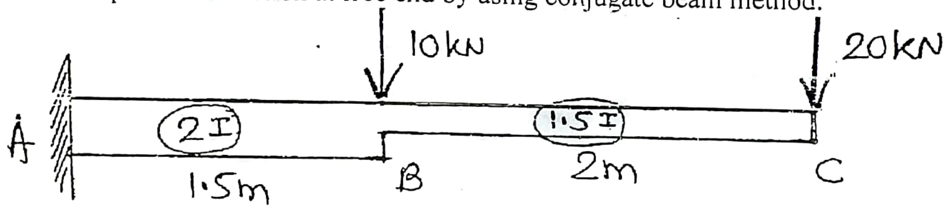
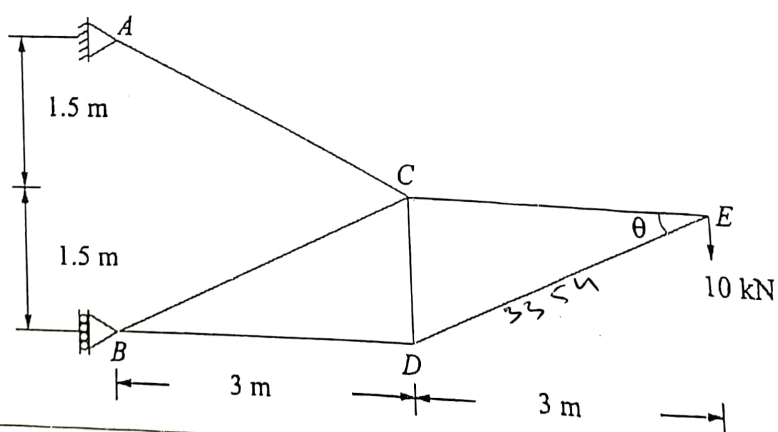
Internal Assessment Test III – Aug. 2022

Sub:	Analysis of Determinate Structures				Sub Code:	18CV42	Branch:	Civil Engg.
Date:	29/08/2022	Duration:	90 min's	Max Marks: 50	Sem / Sec:	4th sem /A section		OBE

Answer any FIVE FULL Questions

		MARKS	CO	RBT
1 (a)	State the Mohr's Theorems with formula.	[04]	CO3	L1
(b)	Determine the slope and deflection at free end by using moment area method. 	[06]	CO3	L3
2 (a)	Calculate the slope and deflection at free end by using moment area method. 	[10]	CO3	L3
3 (a)	Write the difference between real beam and conjugate beam.	[05]	CO3	L1
(b)	What do you mean by principle of virtual displacement and principle of virtual force?	[05]	CO4	L1
4 (a)	Determine the slope and deflection at D and C by using conjugate beam method. 	[10]	CO3	L3
5 (a)	Determine the slope and deflection at free end by using conjugate beam method. 	[10]	CO3	L3
6 (a)	Determine the vertical deflection at E of the loaded frame shown in below figure. Cross-sectional area are 1000 mm^2. E = 200 kN/mm^2. 	[10]	CO4	L3

1

Analysis of determinate structures

IAT – III Solutions -

Sol 1(a) To state Mohr's theorem with formula -

Theorem (I) - It states that, the change in the slope between two points on a straight member under flexure is equal to the area of $\left[\frac{M}{EI}\right]$ diagram between these two points.

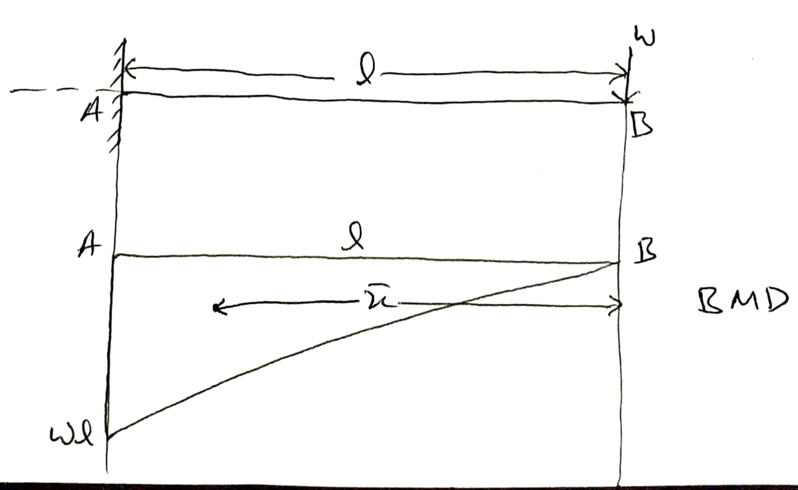
$$\text{Slope } \theta = \int \frac{M}{EI}\,dx = \frac{\text{Area of BMD}}{EI}$$

Theorem (II) - It states that, deflection at a point in a beam in the direction perpendicular to its original straight line position measures from the tangent to the elastic curve at another point is given by moment of $\left(\frac{M}{EI}\right)$ diagram about the point where deflection is required.

$$\text{deflection } \delta = \int \frac{Mx}{EI}\,dx = \frac{(\text{Area of BMD})\,\bar{x}}{EI}$$

where $\bar{x}$ = centroid or centre of gravity.

Sol b,



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(a) Slope at B (Max. slope)

$$\theta_B = \theta_{max} = \frac{\text{Area of BMD}}{EI}$$

$$= -\frac{1}{EI}\left[\frac{1}{2}(l)(wl)\right]$$

$$\boxed{\theta_B = -\frac{wl^2}{2EI}}$$

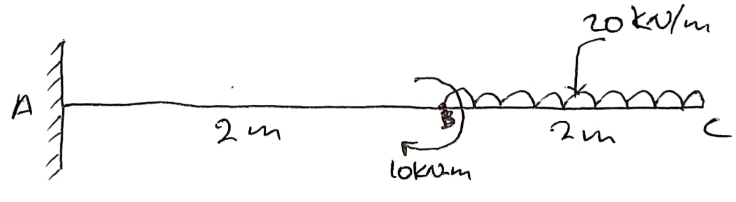
(b) Deflection at B (Max deflection)

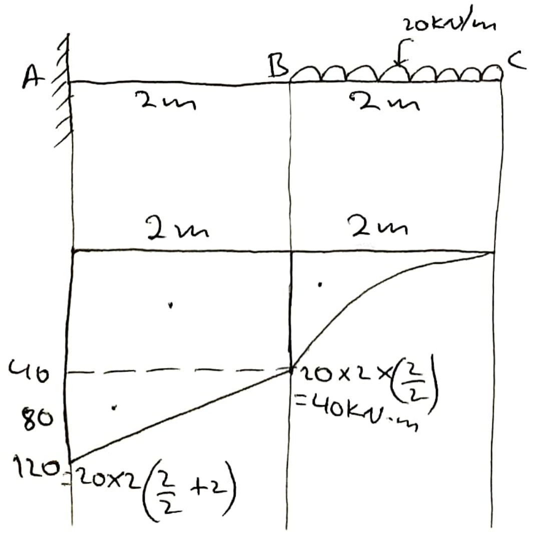
$$\delta_B = \delta_{max} = \frac{(\text{Area})\bar{x}}{EI}$$

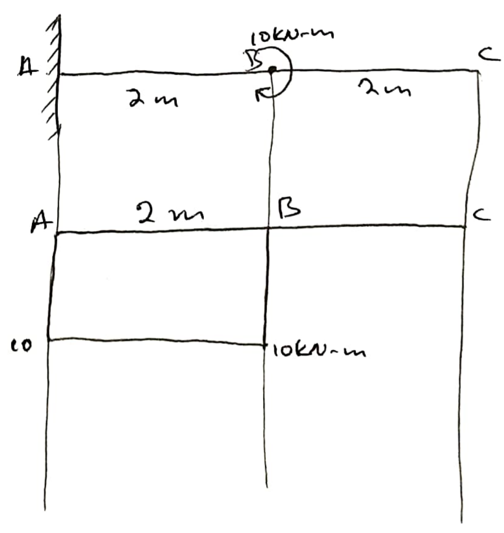
$$= -\frac{1}{EI}\left[\frac{1}{2}(l)(wl)\right]\left[\frac{2}{3}l\right]$$

$$\boxed{\delta_B = -\frac{wl^3}{3EI}}$$

Sol 2







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(a) Slope at 'C' -

$$\theta_c = \frac{\text{Area}}{EI}$$

$$= -\frac{1}{EI}\left[\frac{1}{3}(2)(40) + 2\times 40 + \frac{1}{2}(2)(80)\right] + 2\times 10$$

$$= -\frac{1}{EI}\left[\frac{80}{3} + 80 + 80\right] + 20$$

$$= -\frac{1}{EI}\left[206.66\right]$$

$$\boxed{\theta_c = -\frac{206.66}{EI}}$$

(b) Slope at 'B'

$$\theta_B = \frac{\text{Area}}{EI}$$

$$= -\frac{1}{EI}\left[2\times 40 + \frac{1}{2}(2)(80) + 2\times 10\right]$$

$$\theta_B = -\frac{1}{EI}(180)$$

$$\boxed{\theta_B = -\frac{180}{EI}.}$$

Deflection at point 'c'

$$\delta_c = \left(\frac{\text{Area}}{EI}\right)\bar{x}$$

$$\cancel{\delta_c EI = -\frac{1}{EI}\left[\left(\frac{1}{3}\times 2\times 40\right)\frac{3}{4}\times 2 + (2\times 40)\left(\frac{2}{2}+2\right)\right.}$$

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$$\delta_c = -\frac{1}{EI}\left[\left(\frac{1}{3}\times 2\times 40\right)\frac{3}{4}\times 2 + (2\times 40)\left(\frac{2}{2}+2\right) + \left(\frac{1}{2}\times 2\times 80\right)\left(2+\frac{2}{3}\times 2\right)\right] + (2\times 10)\left(\frac{2}{2}+2\right)$$

$$\delta_c = -\frac{1}{EI}\left[\left(\frac{80}{3}\right)\frac{6}{4} + 80(3) + 80\left(2+\frac{4}{3}\right)\right] + (20)(3)$$

$$\delta_c = -\frac{606.66}{EI}$$

Deflection at point 'B'

$$\delta_B = \frac{\text{Area}}{EI}(\bar{x})$$

$$\delta_B = -\frac{1}{EI}\left[\left(\frac{1}{2}\right)(2)(80)\left(\frac{2}{3}\times 2\right) + (2\times 40)\left(\frac{2}{2}\right) + (2\times 10)\left(\frac{2}{2}\right)\right]$$

$$\delta_B = -\frac{1}{EI}\left[80\left(\frac{4}{3}\right) + 80 + 20\right]$$

$$\delta_B = -\frac{1}{EI}(206.66)$$

$$\delta_B = -\frac{206.66}{EI}$$

Sol 3(a) Difference between real and conjugate beam

Real Beam	Conjugate Beam
(1) 	(1) 
(2) 	(2) 

5.

(3)

(3)

(4)

(4)

(5)

(5)

"Slope" in real beam

"Deflection" in real beam

"Shear force" in Conjugate beam

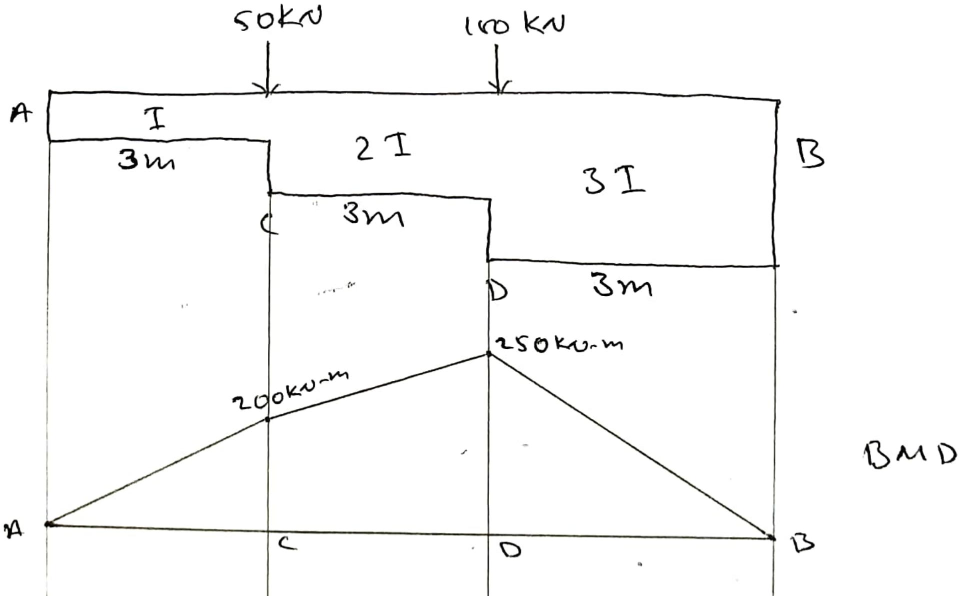
"Bending Moment" in Conjugate beam.

Sol 5)

<u>Virtual displacement</u>: A virtual displacement is any displacement consistent with the contraints of the structure, i.e., that satisfy the boundary conditions at the supports.

<u>Virtual force</u>: A virtual force is a any force sytem in equilibrium.

Sol 4)



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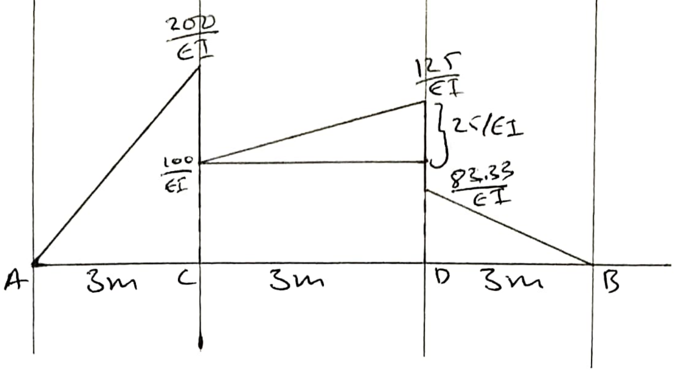

Real beam

$\Sigma V = 0$

$VA + VB - 50 - 150 = 0$

$VA + VB = 150 \quad \text{--- (1)}$

$\Sigma M_A = 0$

$50 \times 3 + 150 \times 6 - VB \times 9 = 0$

$-9VB = -(150 + 600)$

$-9VB = -750$

$\boxed{VB = 83.33 \text{ kN}}$ put in n(1)

$\boxed{VA = 66.67 \text{ kN}}$

Conjugate beam

$\Sigma V = 0,$

$$VA + VB = \left(\frac{1}{2}(3)\left(\frac{200}{EI}\right)\right) + \left(3 \times \frac{100}{EI}\right) + \left(\frac{1}{2}(3)\left(\frac{25}{EI}\right)\right) + \left(\frac{1}{2}(3)\left(\frac{83.33}{EI}\right)\right)$$

$$VA + VB = \frac{762.50}{EI}$$

$\Sigma M_A = 0,$

$$-VB \times 9 + \left(\frac{1}{2} \times 3 \times \frac{200}{EI}\right)\frac{2}{3} \times 3 + \left(3 \times \frac{100}{EI}\right)4.5 + \left(\frac{1}{2} \times 3 \times \frac{25}{EI}\right)\left(\frac{2}{3} \times 3 + 3\right)$$
$$+ \left(\frac{1}{2} \times 3 \times \frac{83.33}{EI}\right)\left(\frac{1}{3} \times 3 + 6\right) = 0$$

$$VB = \frac{334.72}{EI} \qquad \therefore VA = \frac{427.78}{EI}$$

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(1) $\theta_D = -\dfrac{334.72}{EI} + \left(\dfrac{1}{2}\times(3)\times\left(\dfrac{83.33}{EI}\right)\right)$

$\theta_D = -\dfrac{334.72}{EI} + \dfrac{\cancel{8}3}{2}\left(\dfrac{83.33}{EI}\right)$

$\theta_D = -\dfrac{334.72}{E\cancel{E}I} + \cancel{\dfrac{280}{EI}}\ \dfrac{125}{EI}$

$\boxed{\theta_D = -\dfrac{209.72}{EI}}$

$\theta_c = \dfrac{427.78}{EI} - \left(\dfrac{1}{2}\times 3\times\dfrac{200}{EI}\right)$

$\theta_c = \dfrac{427.78}{EI} - \dfrac{300}{EI}$

$\boxed{\theta_c = \dfrac{127.78}{EI}}$

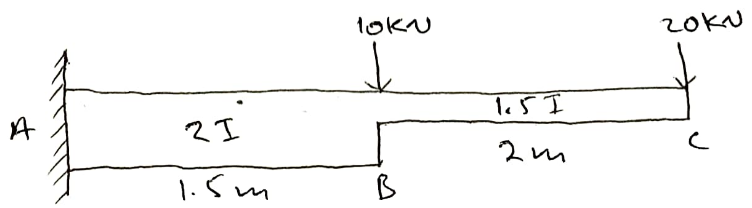
(2) $\delta_c = \dfrac{427.78}{EI}\times 3 - \left(\dfrac{1}{2}\times 3\times\dfrac{200}{EI}\right)\dfrac{1}{3}\times 3$

$\boxed{\delta_c = \dfrac{983.3}{EI}}$

$\delta_D = \dfrac{334.72}{EI}\times 3 - \left(\dfrac{1}{2}\times 3\times\dfrac{83.33}{EI}\right)\dfrac{1}{3}\times 3$

$\boxed{\delta_D = \dfrac{879.16}{EI}}$

Sol 5,



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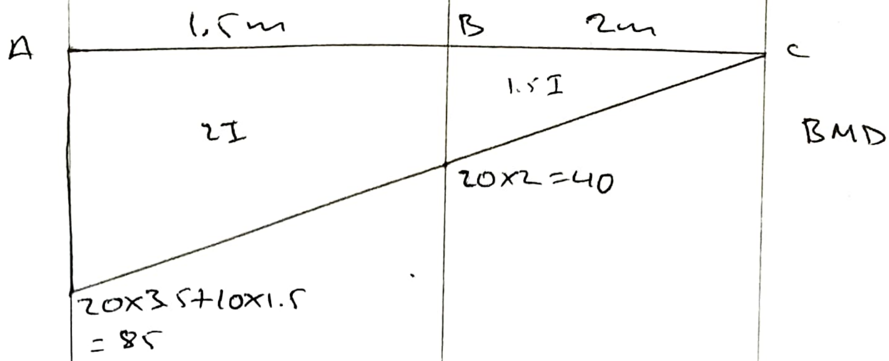
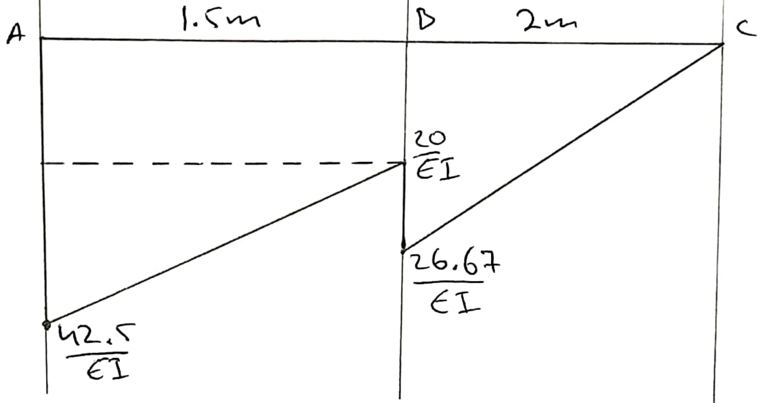




(1) Slope at C, $\theta_c = \frac{1}{2}(2)\left(\frac{26.67}{EI}\right) + \left(1.5\times\frac{20}{EI}\right) + \left(\frac{1}{2}(1.5)\left(\frac{22.5}{EI}\right)\right)$

$$\boxed{\theta_c = \frac{73.55}{EI}}$$

$\theta_B = \left(1.5\times\frac{20}{EI}\right) + \left(\frac{1}{2}(1.5)\left(\frac{22.5}{EI}\right)\right)$

$$\boxed{\theta_B = \frac{46.87}{EI}}$$

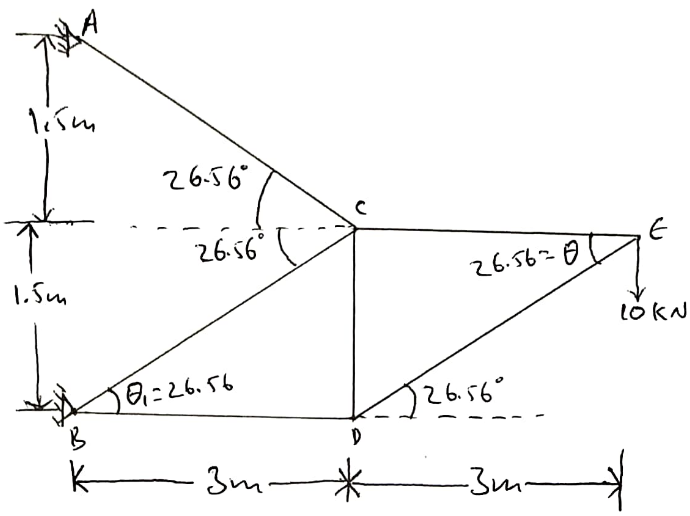
(2) Deflection at C, $\delta_c = \left[\frac{1}{2}(2)\left(\frac{26.67}{EI}\right)\frac{2}{3}\times 2 + \left(1.5\times\frac{20}{EI}\right)\left(2+\frac{1.5}{2}\right) + \left(\frac{1}{2}(1.5)\left(\frac{22.5}{EI}\right)\right)\left(2+\frac{2}{3}\times 1.5\right)\right]$

$$\boxed{\delta_c = \frac{168.68}{EI}}$$

$\delta_B = \left[\left(1.5\times\frac{20}{EI}\right)\left(\frac{1.5}{2}\right) + \left(\frac{1}{2}(1.5)\left(\frac{22.5}{EI}\right)\right)\left(\frac{2}{3}\times 1.5\right)\right]$

$$\boxed{\delta_B = \frac{39.37}{EI}}$$

sol 6,



In Δ CED

$\tan\theta = \frac{DC}{CE}$

$\tan\theta = \frac{1.5}{3}$

$\tan\theta = 0.5 \Rightarrow \theta = \tan^{-1}(0.5)$

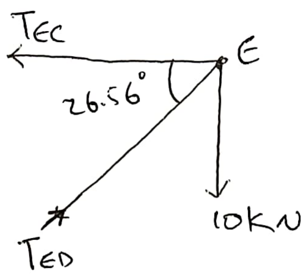
$\boxed{\theta = 26.56^\circ}$

In Δ BDC

$\tan\theta = \frac{1.5}{3}$

$\boxed{\theta_1 = 26.56^\circ}$

Consider joint E,



$\Sigma V = 0$

$T_{ED} \sin\theta - 10 = 0$

$T_{ED} = \frac{10}{\sin\theta} \Rightarrow T_{ED} = \frac{10}{\sin(26.56)}$

$\boxed{T_{ED} = 22.36 \text{ KN}}$

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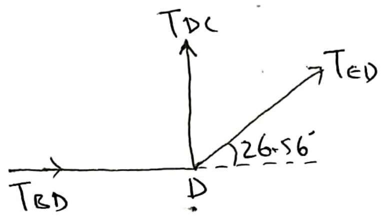
$\Sigma H = 0$

$-F_{EC} + T_{DE}\cos\theta = 0$

$T_{EC} = 22.36\cos(26.56)$

$\boxed{T_{EC} = 20\ kN}$

Consider joint 'D',



$\Sigma V = 0$

$T_{DC} + T_{ED}\sin(26.56°) = 0$

$T_{DC} = -22.36\sin(26.56)$

$\boxed{T_{DC} = -9.997 \approx 10\ kN}$

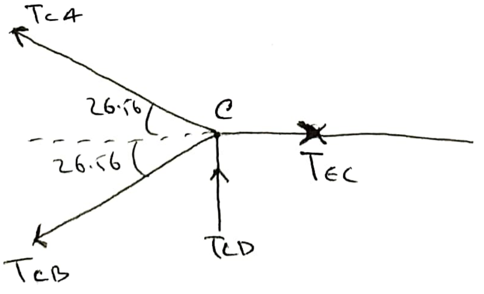
$\Sigma H = 0$

$T_{BD} + T_{ED}\cos 26.56 = 0$

$T_{DB} = -22.36\cos 26.56$

$\boxed{T_{DB} = -20\ kN}$

Consider joint 'c'



$\Sigma H = 0$

$-T_{CA}\cos(26.56) - T_{CB}\cos(26.56) - T_{EC} = 0$

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$-0.894T_{AC} - 0.894T_{CB} = -20$

$0.894T_{AC} + 0.894T_{CB} = 20$ ——— (1)

$\Sigma V = 0$

$T_{AC}\sin(26.56) - T_{CB}\sin(26.56) - \cancel{20}\,10 = 0$

$0.447T_{AC} - 0.447T_{CB} = \cancel{20}\,10$ ——— (ii)

Multiply eq(2) by 2, we get

$0.894F_{AC} - 0.894F_{CB} = 20$ ——— (iii)

Add eq(1) and eq(iii), we get

$1.788T_{AC} = 40$

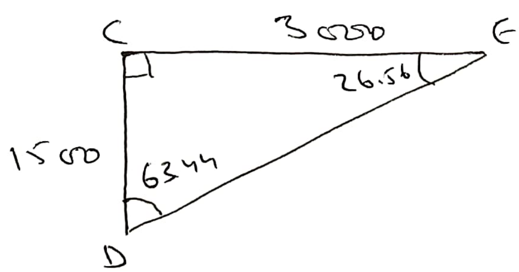
$\boxed{T_{AC} = 22.37\ kN}$ put in eq(iii), we get

$0.894(22.37) - 0.894T_{CB} = 20$

$-0.894T_{CB} = 20 - 20$

$\boxed{T_{CB} = 0\ kN}$

Let take Δ DCE



$DE^2 = DC^2 + CE^2$

$DE^2 = 11250000$

$\boxed{DE = 3354\ mm}$

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Member	Length	Area (mm^2)	P (KN)	$P^2 l/A$
BD	3000	1000	20	1200
CE	3000	1000	20	1200
CD	1500	1000	10	150
DE	3354	1000	22.36	1676.8
BC	3354	1000	0	0
AC	3354	1000	22.36	1676.8
				5913.6

$$\Delta = \frac{1}{wE} \sum \frac{P^2 L}{A}$$

$$\Delta = \frac{1}{10 \times 200} (5913.6)$$

$$\boxed{\Delta = 2.95 \text{ mm}}$$ Ans