CBCS SCHEME

USN | | | | | | | | | |

18MATDIP41

Fourth Semester B.E. Degree Examination, July/August 2022
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix
$$A = \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$
(06 Marks)

b. Solve the system of equations: $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$ by Gauss elimination method. (07 Marks)

c. Find all the eigen values and corresponding eigen vectors of $\begin{pmatrix} -5 & 9 \\ -6 & 10 \end{pmatrix}$ (07 Marks)

OR

2 a. Find the rank of the matrix
$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{pmatrix}$$
(06 Marks)

b. Using Gauss elimination method solve the system of equations
$x + 2y + 3z = 6$; $2x + 4y + z = 7$; $3x + 2y + 9z = 14$. (07 Marks)

c. Find the eigen values of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{pmatrix}$ (07 Marks)

Module-2

3 a. Use an appropriate Interpolation formula to compute f(6).

x	1	2	3	4	5
y	1	-1	1	-1	1

(07 Marks)

b. Evaluate $\int_0^6 3x^2 dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking $n = 6$. (07 Marks)

c. Find a real root of the equation $x^3 - 2x - 5 = 0$ by Newton Raphson method. (06 Marks)

OR

4 a. Find solution using Newton's Interpolation formula, at $x = -1$.

x	0	1	2	3
f(x)	1	0	1	10

(07 Marks)

b. Find the real root of the equation $\cos x = 3x - 1$ using Regula Falsi method. (07 Marks)

c. Evaluate $\int_4^{5.2} \log_e x$ taking n = 6 by Weddle's rule. (06 Marks)

Module-3

5 a. Solve : $(D^3 - 2D^2 + 4D - 8)y = 0$ (06 Marks)

b. Solve : $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{2x}$ (07 Marks)

c. Solve : $\frac{d^2y}{dx^2} + 4y = \cos 4x$ (07 Marks)

OR

6 a. Solve : $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$ (06 Marks)

b. Solve : $(D^2 - 6D + 9)y = 7e^{-2x} - \log 2$ (07 Marks)

c. Solve : $\frac{d^2y}{dx^2} - 16y = \sin 16x$ (07 Marks)

Module-4

7 a. Form the partial differential equation by eliminating the arbitrary constants from $z = (x-a)^2 + (y-b)^2$ (06 Marks)

b. Solve : $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ (07 Marks)

c. Solve : $\frac{\partial^2 z}{\partial y^2} - z = 0$; given that $z = \cos x$ and $\frac{\partial z}{\partial y} = \sin x$, when $y = 0$. (07 Marks)

OR

8 a. Form the partial differential equation by eliminating the arbitrary function 'f' from $f(x^2 + y^2, z - xy) = 0$ (06 Marks)

b. Solve the equation $\frac{\partial^2 z}{\partial y^2} = \sin xy$ (07 Marks)

c. Form the partial differential equation by eliminating the arbitrary constants $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (07 Marks)

Module-5

9 a. Define : (i) Mathematical definition of probability
(ii) Mutually exclusive events
(iii) Independent events (06 Marks)

b. If A and B are two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$.

Find (i) P(A/B) (ii) P(B/A) (iii) $P(\overline{A}/\overline{B})$ (iv) $P(\overline{B}/\overline{A})$ (07 Marks)

c. In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25%, 40% of the total production. Out of these 5%, 4%, 3%, 2% are defective. If a bolt drawn at random was found defective, what is the probability that it was manufactured by A? (07 Marks)

OR

10 a. State and prove Baye's theorem. **(06 Marks)**

b. A card is drawn at random from a pack of cards. (i) What is the probability that it is a heart? (ii) If it is known that the card drawn is red, what is the probability that it is a heart? **(07 Marks)**

c. An Urn 'A' contains 2 white and 4 black balls. Another Urn 'B' contains 5 white and 7 black balls. A ball is transferred from the Urn A to the Urn B. Then a ball is drawn from the Urn B. Find the probability that it is white. **(07 Marks)**

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1) a) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

Soln

Given $A = \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$$\begin{array}{cccc} 3 & 1 & 0 & 2 \\ 3 & 0 & -9 & -3 \\ - & - & + & + \\ \hline 0 & 1 & 9 & 5 \end{array}$$

$$\begin{array}{cccc} 1 & 1 & -2 & 0 \\ 1 & 0 & -3 & -1 \\ - & - & + & + \\ \hline 0 & 1 & 1 & 1 \end{array}$$

$R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 9 & 5 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 9 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$R_4 \to R_2 - R_4$

$$\sim \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 9 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 8 & 4 \end{bmatrix}$$

CMR

$R_4 \to R_4 - 8R_3$

$$\sim \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 9 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$$\begin{array}{cccc} 0 & 0 & 8 & 4 \\ 0 & 0 & 8 & 8 \\ - & - & - & - \\ \hline 0 & - & - & - \end{array}$$

No. of non-zero rows is equal to four

$$\boxed{\therefore\ \rho(A) = 4}$$

b) Solve the System of equations $x+y+z=9$; $x-2y+3z=8$; $2x+y-z=3$ by Gauss elimination method.

Soln:- The augmented matrix of the System is

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 1 & -2 & 3 & : & 8 \\ 2 & 1 & -1 & : & 3 \end{bmatrix}$$

$R_2 \to -R_1 + R_2$, $R_3 \to -2R_1 + R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & -1 & -3 & : & -15 \end{bmatrix}$$

$R_3 \to R_2 + (-3)R_3$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 11 & : & 44 \end{bmatrix}$$

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CMR

Hence we have, $x+y+z=9$

$-3y+2z=-1$

$11z=44$

$\therefore z=4$, $-3y+8=-1$

$\therefore y=3$, $x+y+z=9$

$\Rightarrow x+3+4=9$

$\therefore x=9-7 \Rightarrow x=2$

$\boxed{\therefore x=2, y=3, z=4}$

c) Find all the eigen Values and corresponding eigen Vectors of $\begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$

Sol^n: The characteristic equation of Given matrix

$|A-\lambda I|=0$

W.K.T. $\lambda^2-(\Sigma d)\lambda+|A|=0$

$\Sigma d = -5+10 = 5$ $\quad |A| = -50+54 = 4$

$\lambda^2-5\lambda+4=0$

$\lambda^2-4\lambda-\lambda+4=0$

$\lambda(\lambda-4)-1(\lambda-4)$

$(\lambda-1)(\lambda-4)$ $\therefore$ $\boxed{\lambda=1,4}$ $\rightarrow$ eigen Values

Now consider $[A-\lambda I][X]=0$

$$\Rightarrow \begin{bmatrix} (-5-\lambda) & 9 \\ -6 & 10-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$(-5-\lambda)x + 9y = 0$

$-6x + (10-\lambda)y = 0$

Case (i): let $\lambda = 1$

$\Rightarrow (-5-1)x + 9y = 0 \Rightarrow -6x + 9y = 0$

$-6x + (10-1)y = 0 \quad -6x + 9y = 0$

$2x - 3y = 0$

$\frac{x}{y} = \frac{3}{2}$

$\Rightarrow \cancel{2x = 0, y = 0}$

$\therefore \cancel{X = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$ ~~is~~ $\therefore X_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is the eigen vector corresponding to $\lambda = 1$

Case (ii): let $\lambda = 4$

$\therefore (-5-4)x + 9y = 0 \Rightarrow -9x + 9y = 0$

$-6x + (10-4)y = 0 \quad -6x + 6y = 0$

$\Rightarrow x - y = 0 \Rightarrow \frac{x}{y} = \frac{1}{1}$

$\therefore X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda = 4$

2)
a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

Sol: Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\begin{array}{ccc} 2 & 6 & 5 \\ 2 & 4 & 6 \\ \hline 0 & 2 & -1 \end{array}$$

$R_3 \to R_2 - R_3$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\therefore \rho(A) = 2}$$

CMR

b) Using Gauss elimination method Solve the System of equations

$x+2y+3z=6,\ 2x+4y+z=7,\ 3x+2y+9z=14$

Sol^n

Augmented matrix is

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 2 & 3 & 6 \\ 2 & 4 & 1 & 7 \\ 3 & 2 & 9 & 14 \end{array}\right]$$

~~$R_2 \to 3R_2 - 2R_3$~~

$$\sim \cancel{\left[\begin{array}{ccc:c} 1 & 2 & 3 & 6 \\ 0 & 8 & -15 & -7 \end{array}\right]}$$

$\cancel{6\ 12\ 3\ 21}$

$\cancel{6\ 4\ 18\ 28}$

$\cancel{0\ 8\ -15\ -7}$

$R_2 \to R_2 - 2R_1,$

$R_3 \to R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 6 \\ 0 & 0 & -5 & -5 \\ 0 & -4 & 0 & -4 \end{array}\right]$$

$2\ 4\ 1\ 7$

$2\ 4\ 6\ 12$

$0\ 0\ -5\ -5$

$3\ 2\ 9 : 14$

$3\ 6\ 9 : 18$

$0\ -4\ 0 : -4$

$R_2 \leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 3 & 6 \\ 0 & -4 & 0 & -4 \\ 0 & 0 & -5 & -5 \end{array}\right]$$

$-5z = -5 \Rightarrow z = 1$

$-4y = -4 \Rightarrow y = 1$

$x + 2y + 3z = 6$

$x + 2 + 3 = 6$

$x = 1$

$$\boxed{\therefore\ x = 1,\ y = 1,\ z = 1}$$

c) Find the eigen values of matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$

Soln Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$

characteristic equation = $|A - \lambda I|$

w.k.t $\lambda^3 - (\Sigma d)\lambda^2 + (\text{Sum of minors of } a_1, b_2 + c_3) - |A| = 0$

$\therefore \Sigma d = (1-2-3) = -4$

Sum of minors of diagonal elements

$= (6-0) + (-3-0) + (-2-0) = 6-3-2$

$= 1$

$\therefore$ ~~λ^3~~. $|A| = 1(6-0) - 2(0) + 3(0) = 6$

$\therefore$ characteristic equation is $\lambda^3 + 4\lambda^2 + \lambda - 6 = 0$

~~on solving we get~~ This can be written as

$(\lambda+3)(\lambda+2)(\lambda-1) = 0$

$\therefore \lambda = -3, -2, 1$

$\boxed{\therefore \text{ eigen values are } -3, -2, 1}$

Module-2

3) a) use an appropriate Interpolation formula to compute $f(6)$

x	1	2	3	4	5
y	1	-1	1	-1	1

Sol^n:

Here we have to find y at $x=6$

Since 6 is near to end value 5, Newton's backward interpolation formula is appropriate.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	1				
		-2			
2	-1		4		
		2		-8	
3	1		-4		16
		-2		8	
4	-1		4		
		2			
5	1				

We have Newton's backward interpolation formula.

$$y_x = y_n + x\nabla y_n + \frac{x(x+1)}{2!}\nabla^2 y_n + \frac{x(x+1)(x+2)}{3!}\nabla^3 y_n$$

$$+ \frac{x(x+1)(x+2)(x+3)}{4!}\nabla^4 y_n + \cdots$$

where $x = \frac{x-x_n}{h} = \frac{6-5}{1} = 1$

from table $\nabla y_n = 2$, $\nabla^2 y_n = 4$, $\nabla^3 y_n = 8$, $\nabla^4 y_n = 16$

$1\times2(3)$ $\frac{1\times2\times3\times4}{24}$

$$\cancel{\text{Sr}}\ f(6) = 1 + 1(2) + \frac{1(1+1)}{2}4 + \frac{1(1+1)(1+2)}{6}8 + \frac{1(1+1)(1+2)(1+3)}{24}16$$

$$f(6) = 1+2+4+8+16$$

$$\boxed{f(6) = 31}$$

b) Evaluate $\int_0^6 3x^2 dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking $n=6$

Sol^n Given $\int_0^6 3x^2 dx$ when $n=6$

x	0	1	2	3	4	5	6
y	0	3	12	27	48	75	108

Simpson's $1/3^{rd}$ rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5+\dots+y_{n-1}) + 2(y_2+y_4+\dots+y_{n-2})\right]$$

Here $n=6$, ~~test~~ $h = \frac{6-0}{6} = 1$, $a=0$, $b=6$

$$\int_0^6 3x^2 dx = \frac{1}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(0+108) + 4(3+27+75) + 2(12+48)\right]$$

$$\boxed{\therefore \int_0^6 3x^2 dx = 216}$$

c) Find a real root of the equation $x^3-2x-5=0$ by Newton Raphson method.

Sol^n

We shall find an interval (a,b) where a real root of the equation lies and then locate the approximate root.

Let $f(x) = x^3-2x-5$

$f(0) = -5<0, \quad f(1) = -6<0 \quad f(2) = -1<0, \quad f(3) = 16>0$

A real root lies in $(2,3)$. It will be in the neighbourhood of 2 and let the approximate root $x_0 = 2$

The first approximation is given by,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)}$$

We have, $f(x) = x^3-2x-5$; $f'(x) = 3x^2-2$

$$\therefore x_1 = 2 - \frac{(-1)}{3(4)-2} = 2+\frac{1}{10} = 2.1$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.1 - \frac{f(2.1)}{f'(2.1)}$$

$$x_2 = 2.1 - \frac{[(2.1)^3-2(2.1)-5]}{[3(2.1)^2-2]} = 2.0946$$

$|||^{ly}$ $$x_3 = 2.0946 - \frac{[(2.0946)^3-2(2.0946)-5]}{[3(2.0946)^2-2]} = 2.0946$$

$\boxed{\therefore \text{ real root of Given equation is } 2.0946}$

4)
9) Find solution using Newton's Interpolation formula at $x=-1$

x	0	1	2	3
$f(x)$	1	0	1	10

Sol^n

Here we have to find $f(x)$ at $x=-1$

we use Newton's forward interpolation

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		−1		
1	0		2	
		1		6
2	1		8	
		9		
3	10			

$$y_r = y_0 + r\Delta y_0 + \frac{r(r+1)}{2!}\Delta^2 y_0 + \frac{r(r+1)(r+2)}{3!}\Delta^3 y_0$$

Here $r = \frac{x-x_0}{h} : \frac{-1-0}{1} = -1$

$$1 + (-1)(-1) + \frac{(-1)(0)2}{2!} + \frac{(-1)(0)(1)6}{3!}$$

$1+1=2$

$$\boxed{\therefore f(-1) = 2}$$

CMR

b) Find the real root of the equation $\cos x = 3x - 1$ using Regula falsi method.

Sol^n

Let $f(x) = \cos x + 1 - 3x$

In radians $f(0) = 2 > 0$, $f(1) = -1.46 < 0$

A real root lies in $(0, 1)$ and we can expect the root in the neighbourhood of 1.

Consider, $f(0.6) = -0.0253 > 0$, $f(0.7) = -0.3352 < 0$

$\therefore$ root lies in $(0.6, 0.7)$

Iteration:-1 : $a = 0.6$, $f(a) = -0.0253$

$b = 0.7$, $f(b) = -0.3352$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0.6(-0.3352) - 0.7(-0.0253)}{-0.3352 + 0.0253}$$

$\therefore x_1 = 0.607$

Iteration:-2 : $f(x_1) = f(0.607) = 0.00036 > 0$

root lies in $(0.607, 0.7)$

Now $a = 0.607$, $f(a) = 0.00036$

$b = 0.7$, $f(b) = -0.3352$

$\therefore \cancel{x_2 = 0.607}$. $x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)}$

on simplifying we get

$x_2 = 0.607$

$\boxed{\text{Hence the real root is } 0.607}$

CMR

c) Evaluate $\int_4^{5.2} \log_e x$ taking $n=6$ by weddle's rule

Sol^n The length of each strip $(h) = \frac{5.2-4}{6} = 0.2$; $n=6$

The values of x and $y = \log_e x$ are tabulated

x	4	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

weddle's rule for $n=6$ is given by

$$\int_a^b y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

$$\therefore \int_4^{5.2} \log_e x\,dx = \frac{3(0.2)}{10}\left[1.3863 + 5(1.4351) + 1.4816 + 6(1.5261) + 1.5686 + 5(1.6094) + 1.6487\right]$$

Thus $\boxed{\int_4^{5.2} \log_e x\,dx = 1.8279}$

Module-3

5) a) Solve $(D^3-2D^2+4D-8)y=0$

Soln: we have $(D^3-2D^2+4D-8)y=0$

~~when~~

Auxiliary equation is $m^3-2m^2+4m-8=0$

$m^2(m-2)+4(m-2)=0$

$(m-2)(m^2+4)=0$

$m=2,\quad m^2=-4$

$m=\sqrt{-4} \Rightarrow m=\pm 2i$

$\therefore$ Roots of AE are $2, -2i, 2i$

~~Thus $y=c_1e^{2x}+c_2\cos 2x+$~~

Thus $\boxed{y=c_1e^{2x}+c_2\cos 2x+c_3\sin 2x}$

b) Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=e^{2x}$

Soln: Given $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=e^{2x}$

$\Rightarrow (D^2+4D+3)y=e^{2x}$

AE:- $m^2+4m+3=0$

$m^2+3m+m+3=0$

$m(m+3)+1(m+3)$ $\Rightarrow m=-1,-3$

$(m+1)(m+3)=0$

$\therefore$ $C.F = c_1 e^{-x} + c_2 e^{-3x}$

$$P.I = \frac{e^{2x}}{D^2+4D+3} = \frac{e^{2x}}{4+8+3} = \frac{e^{2x}}{15}$$

complete solution $y = C.F + P.I$

$$\boxed{y = c_1 e^{-x} + c_2 e^{-3x} + \frac{e^{2x}}{15}}$$

c) Solve $\frac{d^2y}{dx^2} + 4y = \cos 4x$

Soln Given $\frac{d^2y}{dx^2} + 4y = \cos 4x$

~~A.E~~: $(D^2+4)y = \cos 4x$

A.E $m^2 + 4 = 0$: $m = \pm 2i$

$C.F = c_1 \cos 2x + c_2 \sin 2x$

$$P.I = \frac{\cos 4x}{D^2+4}$$

Here $a = 4$ and hence replace D^2 by $-a^2 = -16$

$$= \frac{\cos 4x}{-16+4} = \frac{\cos 4x}{-12}$$

$\therefore$ $y = C.F + P.I$

$$\boxed{y = c_1 \cos 2x + c_2 \sin 2x - \frac{\cos 4x}{12}}$$

CMR

6)
a) Solve $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$

Soln Given $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 0$

$\Rightarrow (D^3 - 3D + 2)y = 0$

A.E = $m^3 - 3m + 2 = 0$

put $m=1 \Rightarrow 1 - 3 + 2 = 0$

$\therefore$ 1 is a root by inspection.

other two roots can be found using Synthetic division

1	1	0	-3	2
	0	1	1	-2
	1	1	-2	0

$\Rightarrow m^2 + m - 2 = 0$

$(m+2)(m-1) = 0$

$m = -2, 1$

Hence $m = 1, 1, -2$

$$\boxed{\therefore\ y = (c_1 + c_2 x)e^{x} + c_3 e^{-2x}}$$

b) Solve $(D^2 - 6D + 9)y = 7e^{-2x} - \log 2$

Soln Given $(D^2 - 6D + 9)y = 7e^{-2x} - \log_2$

A.E: $m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) - 3(m-3) = 0$

$(m-3)(m-3) = 0$

$m = 3, 3$

$C.F = (c_1 + c_2 x)e^{3x}$

$P.I = \frac{1}{D^2 - 6D + 9}(7e^{-2x} - \log 2)$

$= \frac{7e^{-2x}}{D^2 - 6D + 9} - \frac{\log 2}{9}$

$= \frac{7e^{-2x}}{4 + 12 + 9} - \frac{\log 2}{9}$

$\therefore P.I = \frac{7e^{-2x}}{25} - \frac{\log 2}{9}$

$\therefore$ ~~$y = (c_1 + c_2)e$~~ $\boxed{y = (c_1 + c_2 x)e^{3x} + \frac{7e^{-2x}}{25} - \frac{\log 2}{9}}$ //.

c) $\frac{d^2y}{dx^2} - 16y = \sin 16x$

Soln

Given $\frac{d^2y}{dx^2} - 16y = \sin 16x$

$\Rightarrow (D^2 - 16)y = \sin 16x$

A.E: $m^2 - 16 = 0$

$m^2 = 16$

$m = \pm 4$

$\therefore C.F = c_1 e^{4x} + c_2 e^{-4x}$

CMR

$$P.I = \frac{\sin 16x}{D^2 - 16}$$

Here $a = 16$ replacing D^2 by $-a^2 = -256$

$$P.I = \frac{\sin 16x}{-256-16} = \frac{\sin 16x}{-272}$$

$$\boxed{\therefore y = c_1 e^{4x} + c_2 e^{-4x} - \frac{\sin 16x}{272}}$$

Module - 4

7) a) Form the partial differential equation by eliminating the arbitrary constants from $z = (x-a)^2 + (y-b)^2$

Soln Given $z = (x-a)^2 + (y-b)^2$

$$\frac{\partial z}{\partial x} = p = 2(x-a) \Rightarrow (x-a) = \frac{p}{2} \rightarrow ①$$

$$\frac{\partial z}{\partial y} = q = 2(y-b) \Rightarrow (y-b) = \frac{q}{2} \rightarrow ②$$

eq ① & ② in $z = (x-a)^2 + (y-b)^2$

$$z = \left(\frac{p}{2}\right)^2 + \left(\frac{q}{2}\right)^2$$

$$z = \frac{p^2}{4} + \frac{q^2}{4} \Rightarrow \boxed{4z = p^2 + q^2}$$

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b) Solve: $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$

The given PDE can be written as $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = x^2 y$

Integrating w.r.t. x treating y as constant

$$\frac{\partial z}{\partial y} = y\int x^2\,dx + f(y)$$

$$\frac{\partial z}{\partial y} = y\frac{x^3}{3} + f(y)$$

Integrating now w.r.t y

$$z = \frac{x^3}{3}\int y + \int f(y)\,dy + g(x)$$

$$z = \frac{x^3}{3}\frac{y^2}{2} + F(y) + g(x)$$

$$\boxed{z = \frac{x^3y^2}{6} + F(y) + g(x)}$$ where $F(y) = \int f(y)\,dy$

c) Solve: $\frac{\partial^2 z}{\partial y^2} - z = 0$, given that $z = \cos x$ and $\frac{\partial z}{\partial y} = \sin x$, when $y = 0$

Sol^n Let us suppose that z is a function of y only.

The given PDE assumes the form of ODE

$$\frac{d^2 z}{dy^2} - z = 0 \quad \text{or} \quad (D^2-1)z = 0$$

AE is $m^2 - 1 = 0 \Rightarrow m = \pm 1$

$$z = c_1 e^{y} + c_2 e^{-y}$$

Solution of PDE is got by replacing c_1 and c_2 by functions of x

$\therefore z = f(x)e^{y} + g(x)e^{-y}$

Now we shall apply given conditions to find $f(x)$ & $g(x)$

Given $z = \cos x$ at $y=0$

$\cos x = f(x)e^{0} + g(x)e^{0}$

$\cos x = f(x) + g(x) \longrightarrow (1)$

Also Given $\frac{\partial z}{\partial y} = \sin x$ at $y=0$

$\frac{\partial z}{\partial y} = f(x)e^{y} - g(x)e^{-y}$

$\Rightarrow \sin x = f(x)e^{y} - g(x)e^{-y}$

$\Rightarrow \sin x = f(x)e^{0} - g(x)e^{0}$

$\therefore \sin x = f(x) - g(x) \rightarrow (2)$

Adding (1) & (2) we get

$\cos x + \sin x = 2f(x) \Rightarrow f(x) = \frac{\cos x + \sin x}{2}$

on subtracting (1) & (2) we get

$\cos x - \sin x = 2g(x) \Rightarrow g(x) = \frac{\cos x - \sin x}{2}$

$$\boxed{\therefore z = \frac{1}{2}\left[(\cos x + \sin x)e^{y} + (\cos x - \sin x)e^{-y}\right]}$$

8) a) Form the partial differential equation by eliminating the arbitrary function 'f' from

$$f(x^2+y^2,\ z-xy)=0$$

Soln We have by data $f(u,v)=0 \rightarrow (1)$

where $u=x^2+y^2$, $v=z-xy \rightarrow (2)$

Diff partially (1) w.r.t. x & y by applying chain rule

$$\frac{\partial f}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial x}=0 \quad (or) \quad \frac{\partial f}{\partial u}\frac{\partial u}{\partial x}=-\frac{\partial f}{\partial v}\frac{\partial v}{\partial x} \rightarrow (3)$$

and $$\frac{\partial f}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial y}=0 \quad (or) \quad \frac{\partial f}{\partial u}\frac{\partial u}{\partial y}=-\frac{\partial f}{\partial v}\frac{\partial v}{\partial y} \rightarrow (4)$$

Dividing (3) by (4) we get

$$\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}}=\frac{\frac{\partial v}{\partial x}}{\frac{\partial v}{\partial y}} \Rightarrow \frac{\cancel{2}x}{\cancel{2}y}=\frac{\cancel{-y}}{\cancel{-x}}\ \frac{\partial x}{\partial y}=\frac{p-y}{q-x}$$

$$\frac{x}{y}=\frac{p-y}{q-x} \Rightarrow x(q-x)=y(p-y)$$

$$qx-x^2=py-y^2$$

$$\therefore \boxed{py-qx=y^2-x^2}$$

Sin 2θ   2 sin θ cos θ
2 cos 2θ   2 cos θ ..

b) Solve the equation $\frac{\partial^2 z}{\partial y^2} = \sin xy$

Sol: Given $\frac{\partial^2 z}{\partial y^2} = \sin xy$

$$\Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \sin xy$$

I w.r.t y treating x as constant

$$\frac{\partial z}{\partial y} = -x \cos xy + f(x)$$

Again integrating partially w.r.t y treating x as constant

$$\boxed{z = -x^2 \sin xy + F(x) + g(x)}$$ where $F(x) = \int f(x)dx$

c) Form the partial differential equation by eliminating the arbitrary constants, $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Sol: Given $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \rightarrow (1)$

D. (1) partially w.r.t x and y we get

$$\frac{2x}{a^2} + \frac{2z}{c^2}p = 0 \quad \text{and} \quad \frac{2y}{b^2} + \frac{2z}{c^2}q = 0$$

i.e; $\frac{x}{a^2} + \frac{zp}{c^2} = 0 \rightarrow (2)$

$\frac{y}{b^2} + \frac{zp}{c^2} = 0 \rightarrow (3)$

Since there are the arbitrary constants, we differentiate further

Differentiating (2) w.r.t. x partially again we get

$$\frac{1}{a^2} + \frac{1}{c^2}(zr + p^2) = 0 \longrightarrow (5)$$

$$\because \frac{\partial p}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x^2} = r$$

Now from (2), $\frac{x}{a^2} = \frac{-zp}{c^2}$ or $\frac{1}{a^2} = \frac{-zp}{c^2 x}$

Substituting this in (5) we get

$$\frac{-zp}{c^2 x} = \frac{-1}{c^2}(zr + p^2) \text{ or } zp = x(zr + p^2)$$

Thus $\boxed{z\frac{\partial z}{\partial x} = xz\frac{\partial^2 z}{\partial x^2} + x\left(\frac{\partial z}{\partial x}\right)^2}$

Module -5

9) a) Define : (i) Mathematical definition of probability
(ii) Mutually exclusive events
(iii) Independent events

Sol:

Mathematical definition of probability :

If the outcome of a trial consists of n exhaustive, mutually exclusive, equally possible cases, of which m of them are favourable cases to an event E, then the probability of the happening of the event E, usually denoted by P(E) or simply p is defined to be equal to m/n.

That is $P(E) = p = \dfrac{\text{number of favourable cases}}{\text{number of possible cases}}$,

Mutually exclusive events

Two or more events are said to be mutually exclusive if the happening of one event prevent the simultaneous happening of the others.

Ex:- In tossing a coin, getting head and tail are mutually exclusive inview of the fact that if head is the turn out, getting tail is not possible.

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Independent events: Two @ more events are said to be independent if the happening @ nonhappening of one event does not prevent the happening @ non happening of the others.

Ex:- When two coins are tossed the event of getting head is an independent event as both the coins can turn out heads.

b) If A and B are two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Find (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(\bar{A}/\bar{B})$ (iv) $P(\bar{B}/\bar{A})$

Soln Given $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$

(i) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$

$$\boxed{P(A/B) = \frac{3}{4}}$$

(ii) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{2}{4} = \frac{1}{2}$

$$\boxed{P(B/A) = \frac{1}{2}}$$

(iii) $P(\bar{A}/\bar{B}) = \dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$

But $P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

we have De-morgans Law $(\overline{A\cup B}) = \bar{A}\cap\bar{B}$

$P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B)$

$= 1 - [P(A) + P(B) - P(A\cap B)]$

$P(\bar{A}\cap\bar{B}) = 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right] = \frac{5}{12}$

$\rightarrow P(\bar{A}/\bar{B}) = \dfrac{\frac{5}{12}}{\frac{2}{3}} = \frac{5}{\cancel{12}_4} \times \frac{\cancel{3}}{2} = \frac{5}{8}$

$\boxed{P(\bar{A}/\bar{B}) = \frac{5}{8}}$

Also $P(\bar{B}/\bar{A}) = \dfrac{P(\bar{B}\cap\bar{A})}{P(\bar{A})} = \dfrac{5/12}{1/2} = \frac{5}{6}$ $\left(\because P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}\right)$

$\boxed{\therefore P(\bar{B}/\bar{A}) = \frac{5}{6}}$ //..

c) In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25%, 40% of the total production. out of these 5%, 4%, 3%, 2% are defective. If a bolt drawn at random was found defective, what is the probability that it was manufactured by A

Sol^n: By data A, B, C, D manufacture 20%, 15%, 25% and 40% of the total production.

Hence we have

$$P(A) = 0.2,\quad P(B) = 0.15,\quad P(C) = 0.25,\quad P(D) = 0.4$$

let X be the event of selection of a defective bolt. Then

$$P(X/A) = 0.05,\quad P(X/B) = 0.04,\quad P(X/C) = 0.03,\quad P(X/D) = 0.02$$

we need to compute ~~P(A∪B/X)~~ $P(A/X)$

we have Baye's theorem

$$P(A/X) = \frac{P(A)\cdot P(X/A)}{P(A)\cdot P(X/A) + P(B)\cdot P(X/B) + P(C)\cdot P(X/C) + P(D)\cdot P(X/D)}$$

$$\therefore P(A/X) = \frac{(0.2)(0.05)}{(0.2)(0.05) + (0.15)(0.04) + (0.25)(0.03) + (0.4)(0.02)}$$

$$\boxed{P(A/X) = \frac{0.01}{0.0315} \approx 0.317}$$

//.

10) a) State and prove Baye's theorem

Soln:

Statement: Let $A_1, A_2, A_3, \ldots A_n$ be a set of exhaustive and mutually exclusive events of the sample space S with $P(A_i) \neq 0$ for each i, if A is any other event associated with A_i,

$(A \subset \bigcup_{i=1}^{n} A_i)$ with $P(A) \neq 0$ then

$$P(A_i/A) = \frac{P(A_i)\,P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A/A_i)}$$

Proof: We have $S = A_1 \cup A_2 \cup \cdots \cup A_n$ and $A \subset S$

$\therefore A = S \cap A = (A_1 \cup A_2 \cup \cdots \cup A_n) \cap A$

using distributive law in the RHS we have

$A = (A_1 \cap A) \cup (A_2 \cap A) \cup \cdots (A_n \cap A)$

Since $A_i \cap A$ for $i=1$ to n are mutually exclusive, we have by applying the additional rule of probability

$P(A) = P(A_1 \cap A) + P(A_2 \cap A) + \cdots + P(A_n \cap A)$

Now applying multiplication rule onto each term in RHS we have,

$P(A) = P(A_1)\,P(A/A_1) + P(A_2)\,P(A/A_2) + \cdots + P(A_n)\,P(A/A_n)$

That is $$\boxed{P(A) = \sum_{i=1}^{n} P(A_i)\, P(A/A_i)} \longrightarrow (1)$$

The conditional probability of A_i for any i given A is defined by

$$P(A_i/A) = \frac{P(A_i \cap A)}{P(A)} = \frac{P(A_i)\, P(A/A_i)}{P(A)}$$

using (1) in the denominator of RHS we get

$$\boxed{P(A_i/A) = \frac{P(A_i)\, P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\, P(A/A_i)}}$$

c) An urn 'A' contains 2 white and 4 black balls. Another urn 'B' contains 5 white and 7 black balls. A ball is transferred from the urn A to B and then a ball is drawn from urn B. Find the probability that it is white.

Soln

Total no of balls in urn A = 2W + 4B = 6

" " " " " " B = 5W + 7B = 12

Since a ball is transferred from A to B, two cases arise.

Case (i) :- Suppose the transferred ball is white

probability of the transfer of a white ball is $2/6 = 1/3$

Then urn B will have 6W and 7B = 13 balls

Hence probability of getting a white ball from B after the transfer is $6/13$.

$\therefore$ probability of transferring a white ball and getting white from B is $\frac{1}{3} \times \frac{6}{13} = \frac{2}{13}$

Case (ii): Suppose the transferred ball is black. probability of transferred is $4/6 = 2/3$

Then urn B will have 5W and 8B = 13 balls

Hence the probability of getting a white ball after the transfer is $5/13$.

$\therefore$ probability of transferring a black ball and getting white from B is $\frac{2}{3} \times \frac{5}{13} = \frac{10}{39}$

Either of these two cases are favourable to the event

Thus the required probability by addition theorem is

$$\frac{2}{13} + \frac{10}{39} = \frac{6+10}{39} = \frac{16}{39}$$

b) A card is drawn at random from a pack of cards.

(i) what is the probability that it is heart?

(ii) If it is known the card drawn is red, what is the probability that it is a heart?

Soln

Number of possible (exhaustive) cases = 52 (n)

(i) Number of favorable cases of getting heart = 13 (m)

$\therefore$ probability of getting heart $= \frac{13}{52} = \frac{1}{4}$

~~(ii) probability of getting red card = $\frac{26}{52} = \frac{1}{2}$~~

(ii) No of red cards in a pack = 26

No of favorable cases of getting heart = 13

$\therefore$ probability of getting red heart card $= \frac{13}{26} = \frac{1}{2}$