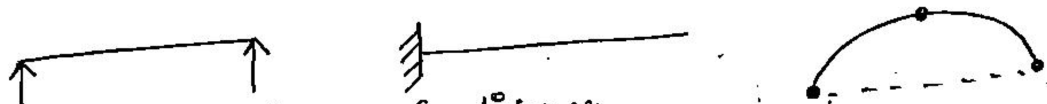
ANALYSIS OF DETERMINATE STRUCTURES 18CV42

MODULE - 1

Q1) a) <u>Statically determinate structure</u> :-

* The structures can be analysed by using the condition of equilibrium ($\Sigma V = 0$, $\Sigma H = 0$, $\Sigma M = 0$), then it is called statically determinate structure
* There is no strain caused due to change in temperature
* There is no stresses caused due to lack of fit

Eg :-

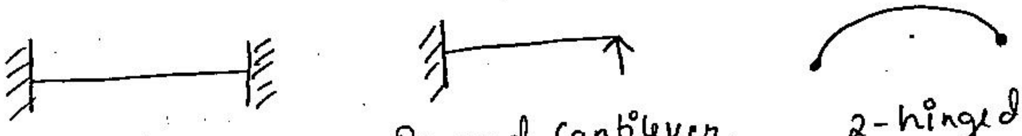


simply supported beam   Cantilever beam   3-hinged arch

<u>Statically indeterminate structure</u> :-

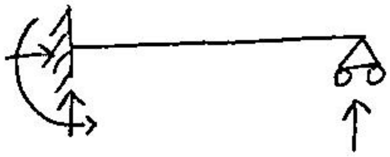
* The structures cannot be analysed just by using the conditions of equilibrium, then it is indeterminate structure.
* Stresses are caused due to change in temperature
* Stresses are developed due to lack of fit

Eg :-



fixed beam   Propped cantilever   2-hinged arch

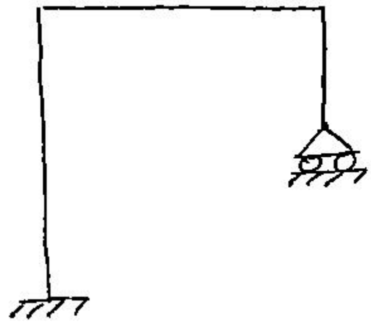
b) [illegible]

(i) 

$D_S = $ [illegible] $= 1$

~~$D_K = 3m + r - 3j = 3(1) + 4 - 3(2) = 1$~~

$D_K = $ [illegible]

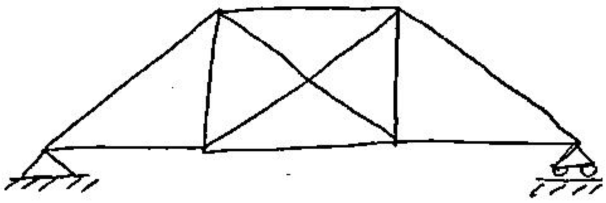
(ii) 

$m = 3, \ r = 4, \ j = 4$

$D_S = $ [illegible] $= 1$

~~$D_K = 3m + r - 3j = 3(3) + 4 - 3($~~

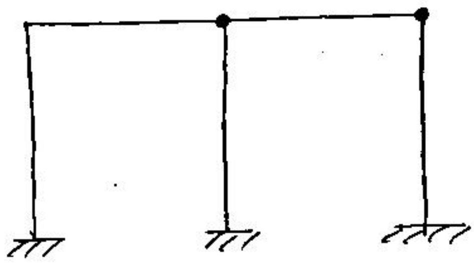
$D_K =$

(iii) 

$m = 10, \ r = 3, \ j = 6$

$D_S = m + r - 2j = 10 + 3 - 2(6)$
$= 1$

$D_K = 2j - R = 2(6) - 3 = 9$

(iv) 

$m = 5, \ r = 9, \ j = 6, \ n = 2$

$D_S = 3m + r - 3j - n$
$= 3(5) + 9 - 3(6) - 2 = 4$

$D_K = 3j - r + n = 3(6) - 9 + 2$
$= 11$

Q2) a) Influence line diagram :- A curve or graph that represents a function like reaction at a support. Shear force at a section the bending moment at a section of a structure etc for a various positions of a unit load in the span of the structure is called influence line diagram (ILD)

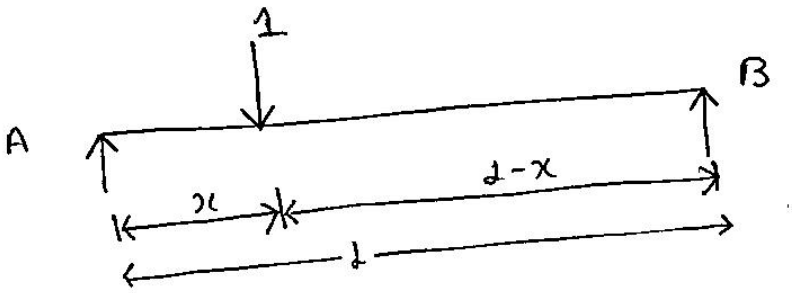
...ations of ILD :-

...he effect of loads that occupy different positions on the structure can be studied by means of influence lines

To identify the positions of loads for maximum shear force & bending moment at specified sections

› ILD is very usefull along with rolling loads

(i)



$\Sigma M_B = 0$ $\quad R_A \times l - 1(l-x) = 0$

$\Rightarrow R_A = \frac{l-x}{l}$

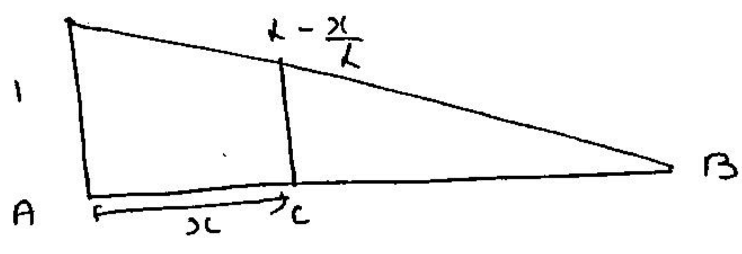
$\Sigma V = 0$ $\quad R_A + R_B = 1$

$R_B = 1 - R_A = 1 - \left(\frac{l-x}{l}\right) \Rightarrow \frac{x}{l}$

Boundry conditions for $(R_A) \Rightarrow (l - x/l)$

$x = 0$ $\quad R_A = \frac{l-0}{l} = 1$

$x = l$ $\quad R_A = \frac{l-l}{l} = 0$

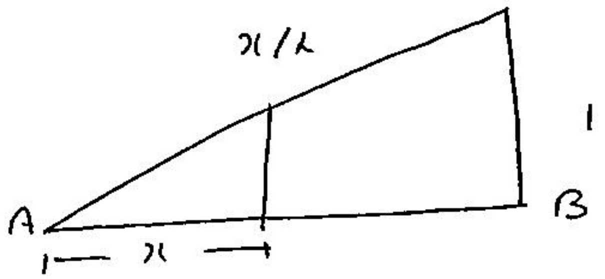


ILD for R_A

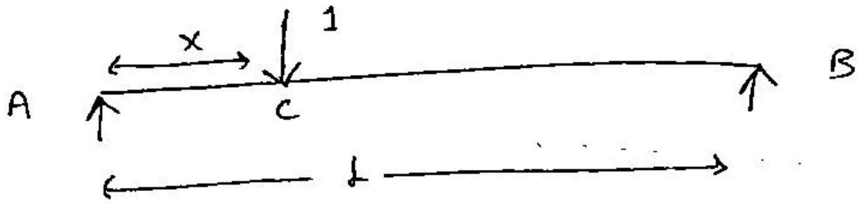
Boundary conditions of ($R_B = x/l$)

@ $x = 0$ $\quad R_B = 0$

$x = l$ $\quad R_B = \frac{l}{l} = 1$



ii)



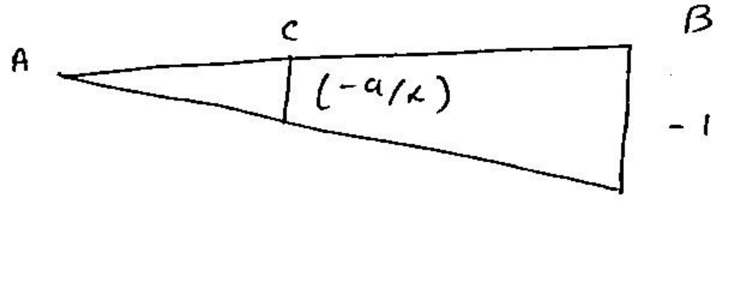
SF @ C => $R_A - 1$ => $\left(\frac{l-x}{l}\right) - 1 = \frac{-x}{l}$

Boundary conditions

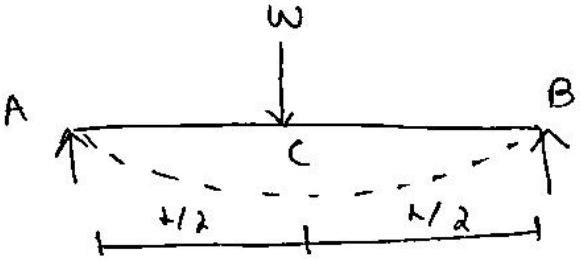
$x = 0$ $\quad$ SF @ A = 0

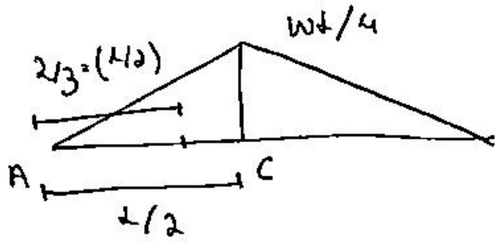
$x = a$ $\quad$ SF @ C = $-a/l$

$x = l$ $\quad$ SF @ B = $-\frac{l}{l} = -1$



MODULE - 3





Moment diagram

At midspan C, due to symmetry the slope is zero

$$\theta_{AC} = \theta_A - \theta_C = \theta_A - 0 = \theta_A$$

$$\theta_A = \text{Area of } \left(\frac{M}{EI}\right)$$

$$\theta_A = \theta_B = \frac{1}{EI}\left(\frac{1}{2} \times \frac{l}{2} \times \frac{WL}{4}\right) \Rightarrow \frac{WL^2}{16EI}$$

$$\boxed{\theta_A = \theta_B = \frac{WL^2}{16EI}}$$

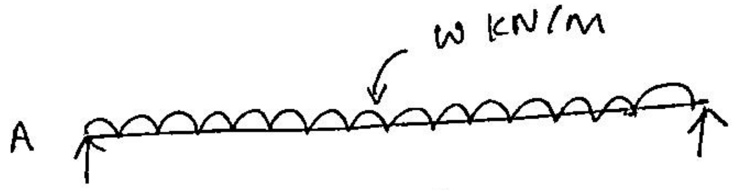
Deflection @ midspan (δ_C)

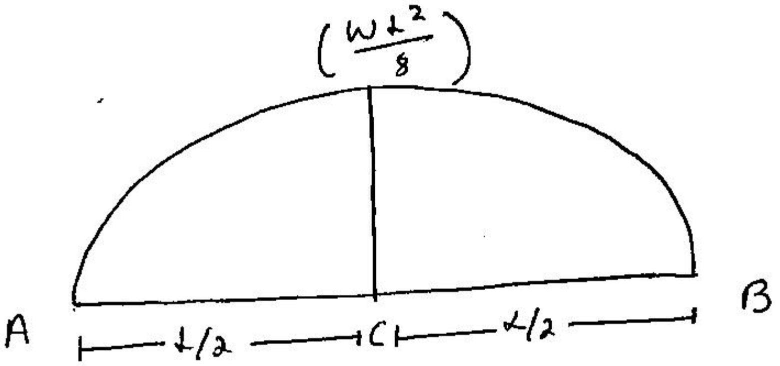
$$\Delta_{AC} \Rightarrow \text{Area of } \left(\frac{M}{EI}\right) \times \bar{x}$$

$$\Delta_A - \Delta_C = \frac{1}{EI}\left[\frac{1}{2} \times \frac{1}{2} \times \frac{WL}{4}\right] \times \left(\frac{2}{3} \times \frac{L}{2}\right)$$

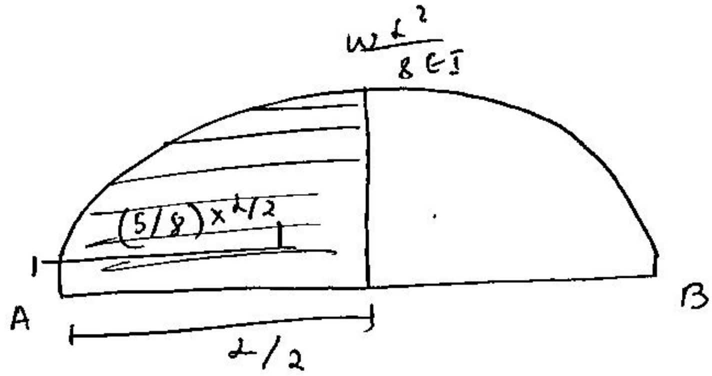
$$\Rightarrow -\Delta_C \Rightarrow \frac{+WL^3}{48EI} \Rightarrow \boxed{\Delta_C = \frac{-WL^3}{48EI}}$$

(b)





BMD



$\left(\frac{M}{EI}\right)$ diagram

Slope θ_{AC} or θ_{BC} $\Rightarrow$ $\theta_A - \theta_C \Rightarrow$ Ar of $\left(\frac{M}{EI}\right)$ b/w A &

$$\Rightarrow \theta_A - 0 \Rightarrow \frac{2}{3} \times \frac{L}{2} \times \left(\frac{wL^2}{8EI}\right) \Rightarrow \frac{wL^3}{24EI}$$

$$\boxed{\theta_A = \frac{wL^3}{24EI}}$$

Deflection @ midspan (C) δ_{AC}

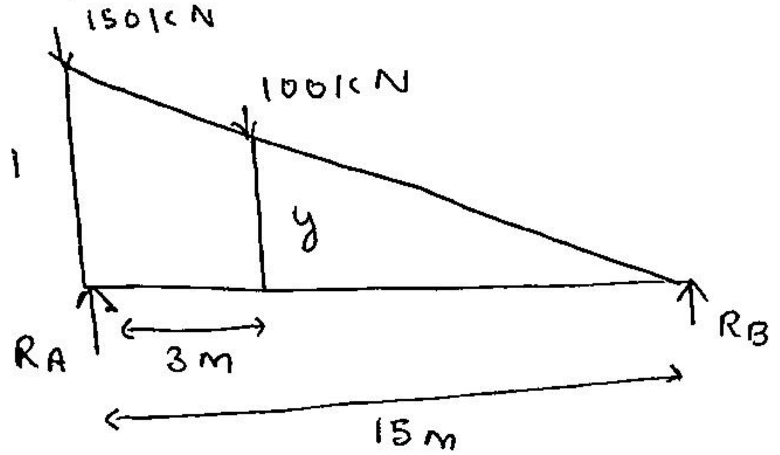
$$\Rightarrow \Delta_A - \Delta_C \Rightarrow 0 - \Delta_C$$

$$\text{Ar of } \left(\frac{M}{EI}\right) \times \bar{x} \Rightarrow \left(\frac{2}{3} \times \frac{1}{2} \times \frac{wL^2}{8EI}\right) \times \frac{5}{8} \times \frac{L}{2}$$

$$\boxed{\Delta_C \Rightarrow \frac{-5wL^4}{384EI}}$$

MODULE - 2

Absolute maximum shear force



$$\frac{1}{15} = \frac{y}{12}$$

$$y = 0.8$$

$R_A = 150 \times 1 + 100 \times 0.8 = 230\,kN$

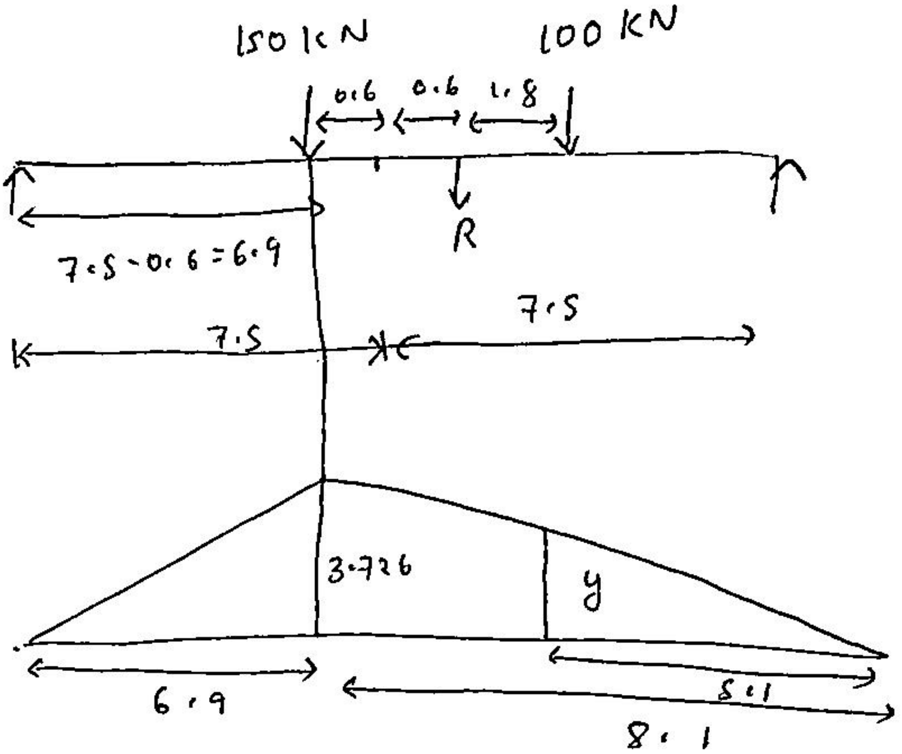
Absolute maximum bending moment

Resultant of external load => $R = 100 + 150 = 250\,kN$

location of R => $\bar{x} = \dfrac{w_1 x_1 + w_2 x_2}{w_1 + w_2}$

$$= \frac{150 \times 0 + 100 \times 3}{150 + 100} = 1.2\,m$$

Selection of loads is it arrangement select havier load [Magnitude are different]. Here magnitude are different, hence take havier loads as 150 kN. The selected load & Resultant load should be equidistance from midpoint



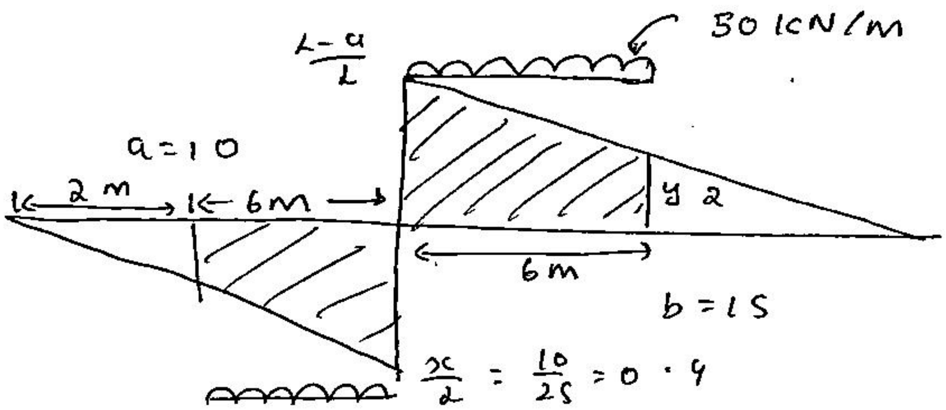
$\frac{y}{5.1} = \frac{3.7}{8}$

$y = 2.$

Absolute BM => $(150 \times 3.726) + (100 \times 2.346)$

$= 793.5$ kN-m

b) Max SF @ section C



$\frac{0.4}{10} = \frac{y_1}{4} \Rightarrow y_1 = 0.16$

$\frac{0.6}{15} = \frac{y_2}{9} \Rightarrow y_2 = 0.36$

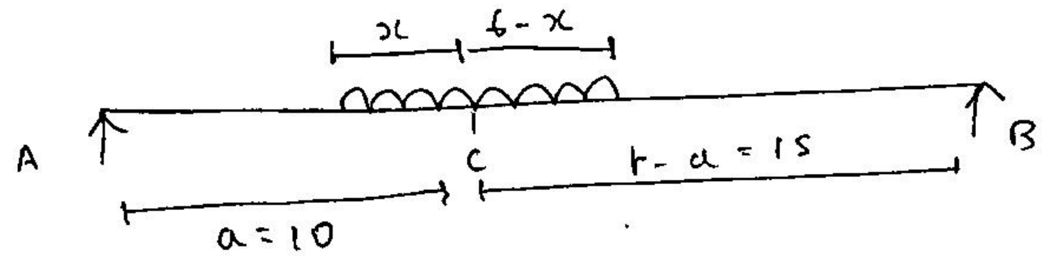
Max +ve SF => load × area of ILD

$= 50 \times \left[\left(\frac{0.6 + 0.36}{2}\right) \times 6\right] = 144$ kN

Max (-ve) SF => load × area of ILD

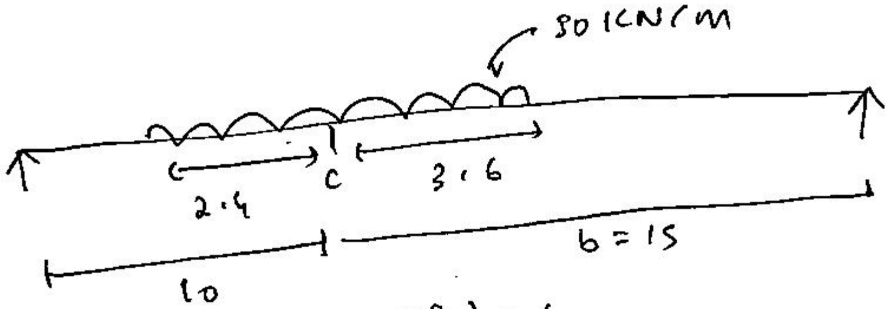
$= 50 \times \left[\left(\frac{0.4 + 0.16}{2}\right) \times 6\right] \Rightarrow 84$ kN

Bending moment at a section

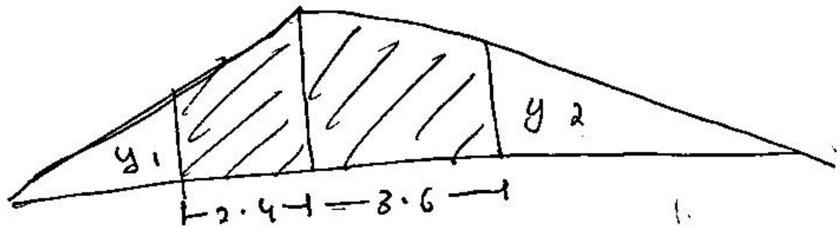


The load positions for max. B.M is "The ratio of span & the ratio of load should be same"

$$\frac{10}{15} = \frac{x}{6-x} \Rightarrow x = 2.4m$$



$$a\left(\frac{l-a}{l}\right) = 6$$



$$\frac{6}{10} = \frac{y_1}{7.6} \Rightarrow y_1 = 4.56$$

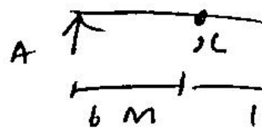
$$\frac{6}{15} = \frac{y_2}{11.4} \Rightarrow y_2 = 4.56$$

Max BM$_c$ => Load × shaded area

$$= 50\left[\left(\frac{6+4.56}{2}\right)\times 2.4 + \left(\frac{6+4.56}{2}\right)\times 3.6\right]$$

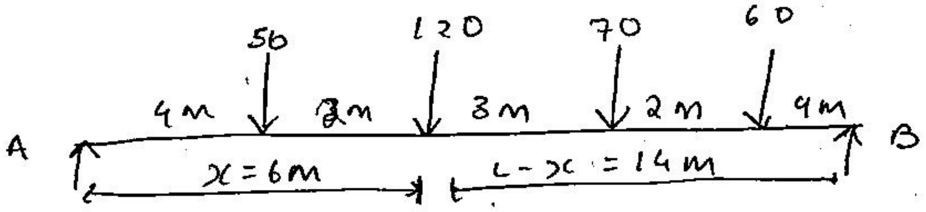
$$= 1584 \text{ kN-m}$$

Q 4) Max. BM at a section ($x = 6m$)

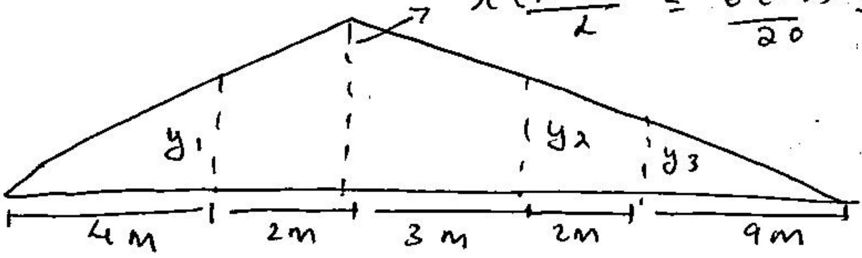


Load	Avg load on AX	Avg load on BX	Remark
60 KN	$\frac{70+120+50}{2} = 40$	$\frac{60}{14} = 4.28$	AX > XB
70 KN	$\frac{120+50}{6} = 28.33$	$\frac{60+70}{14} = 9.28$	AX > XB
120 KN	$\frac{50}{6} = 8.33$	$\frac{60+70+120}{14} = 17.85$	AX < XB

The load which causes change in sign is kept above the section



$\frac{x(l-x)}{l} = \frac{6(14)}{20} = 4.2$



$\frac{4.2}{6} = \frac{y_1}{4} \Rightarrow y_1 = 2.8$

$\frac{4.2}{14} = \frac{y_2}{11} \Rightarrow y_2 = 3.3$

$\frac{4.2}{14} = \frac{y_3}{9} \Rightarrow y_3 = 2.7$

Max BM => Load × ordinate

=> $50 \times 2.8 + 120 \times 4.2 + 70 \times 3.3 + 60 \times 2.7$

Max BM = 1037 KNm //

Absolute Max. BM :-

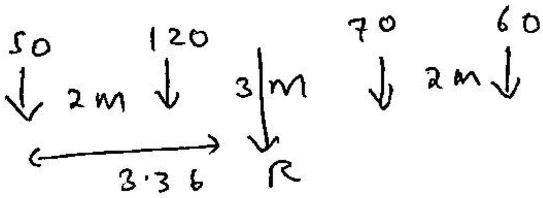
R = 60 + 70 + 120 + 50 = 300 kN

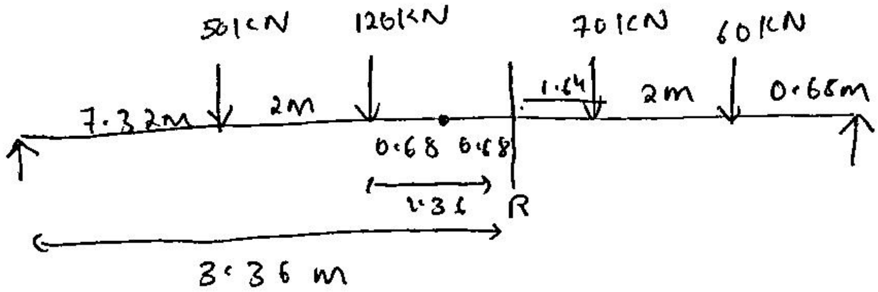
Location of R

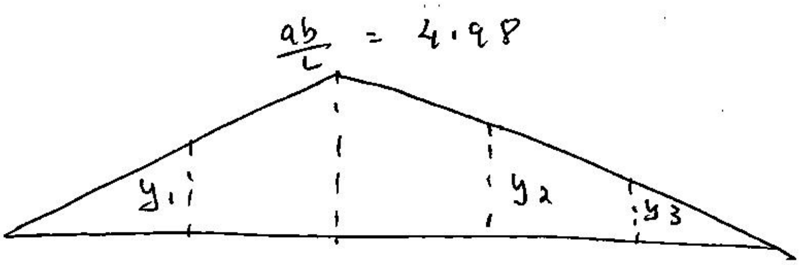
$$\bar{x} = \frac{60 \times 7 + 70 \times 5 + 120 \times 2 + 50 \times 0}{300}$$

$$= 3.36 \text{ m}$$

Selection of load :







$$\frac{4.98}{9.32} = \frac{y_1}{7.32} \Rightarrow y_1 = 3.91$$

$$\frac{4.98}{10.68} = \frac{y_2}{7.68} \Rightarrow y_2 = 3.58$$

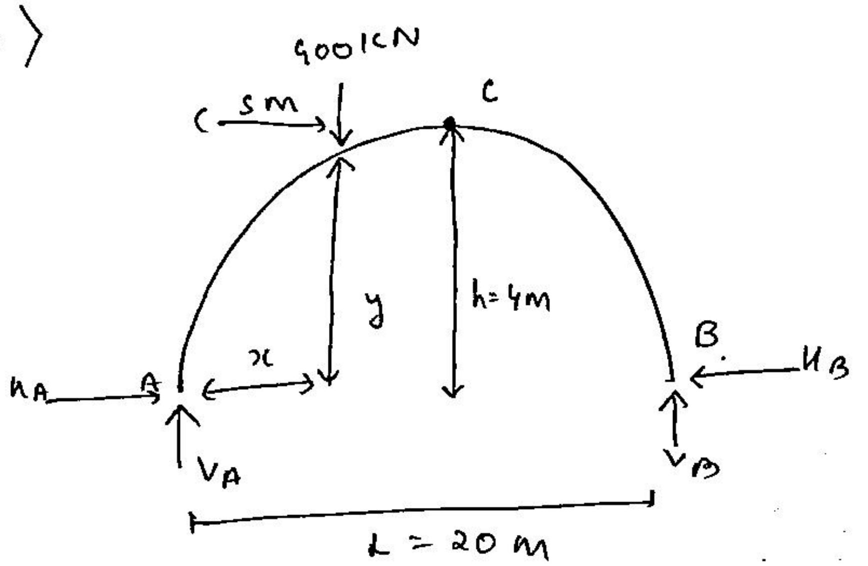
$$\frac{4.98}{10.68} = \frac{y_3}{5.68} \Rightarrow y_3 = 2.648$$

Absolute Max BM : $50 \times 3.91 + 120 \times 4.98 + 70 \times 3.58 + 60 \times 2.64$

$= 1202.11 \text{ kNm}$

MODULE - 5

Q9)



$\Sigma M_A = 0$

$V_B \times 20 - 400 \times 5 = 0$

$V_B = 100 \text{ kN}$

$\Sigma V = 0$

$V_A + V_B = 400$

$V_A = 400 - 100$

$= 300 \text{ kN}$

Finding horizontal reactions

$\Sigma H = 0 \quad H_A - H_B = 0 \quad H_A = H_B$

$\Sigma M_C = 0$ (considering AC)

$V_A \times 10 - H_A \times h - 400 \times 5 = 0$

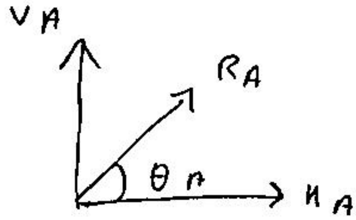
$H_A \times h = V_A \times 10 - 400 \times 5 \Rightarrow 300 \times 10 - 400 \times 5$

$H_A \times 4 \Rightarrow 1000$

$\boxed{H_A = H_B = 250 \text{ kN}}$

resultant reactions at supports

$= \sqrt{H_A^2 + V_A^2}$

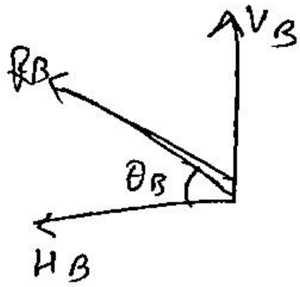


$= \sqrt{250^2 + 300^2}$

$= 390.51$ kN

$= \tan^{-1}\left(\frac{V_A}{H_A}\right) = \tan^{-1}\left[\frac{300}{250}\right] = 50.19^\circ$

$B = \sqrt{H_B^2 + V_B^2}$



$= \sqrt{(250)^2 + (100)^2} = 164.37$ KN

$\theta_B = \tan^{-1}\left[\frac{V_B}{H_B}\right] = \tan^{-1}\left[\frac{100}{250}\right] = 21.8^\circ$

nding BM

$BM_A = BM_B = BM_c = 0$

WKT vertical ordinate y of arch is given by

$y = \frac{4hx[L - x]}{L^2} \Rightarrow \frac{4 \times 4x[20 - x^2]}{20^2}$

$\boxed{y = 0.8x - 0.64x^2}$

BM at 400 kN load : +ve BM below load point

$x = 5$m from left support

$y = 0.8(5) - 0.04(5^2) \Rightarrow 3$m

BM = $V_A \times 5 - H_A \times 3$

BM : $300 \times 5 - 250 \times 3$

$= 750$ kNm

-ve BM occurs @ no load area

Take moment about x distance from B

$BM_x \Rightarrow V_B x - H_B \times y$

$\Rightarrow 100x - 250(0.8x - 0.04x^2)$

$\Rightarrow 100x - 200x + 10x^2$

$\Rightarrow -100x + 10x^2$

for BM to be max $\frac{\delta M_x}{dx} = 0$

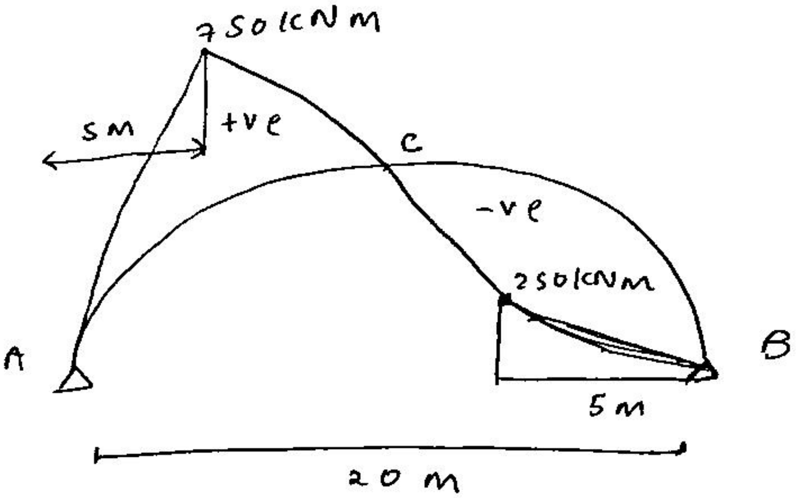
$\frac{d}{dx}(-100x + 10x^2) = 0$

$-100x + 2 \times 10 \times x = 0$

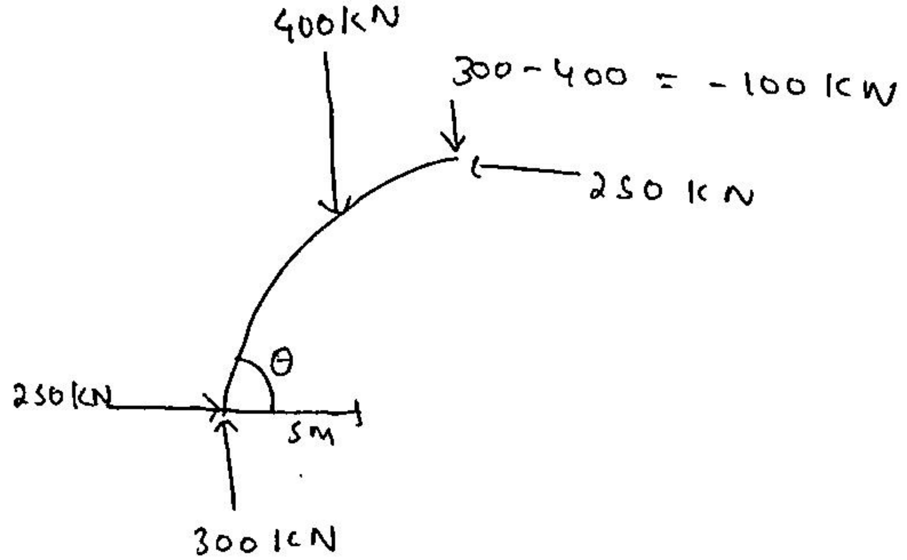
$\boxed{x = 5m}$

$BM_x = -100x + 10x^2$

$\Rightarrow -100(5) + 10(5)^2 \Rightarrow -250 \text{ kNm}$



thrust & radial shear



$$\theta = \frac{dy}{dx} \Rightarrow \frac{d}{dx}\left(0.8x - 0.04x^2\right)$$

$$= 0.8 - 0.04 \times 2 \times x$$

$$= 0.8 - 0.08x$$

when $x = 5$

$$\tan\theta = 0.8 - 0.08 \times 5$$

$$= 0.4$$

$$\theta = 21.8^\circ$$

$$N \cdot T = V\sin\theta + H\cos\theta$$

$$= -100 \sin 21.8 + 250 \times \cos 21.8$$

$$= 92.8 + 232 \Rightarrow 194.9\ kN$$

$$= 194.9\ kN \,//$$

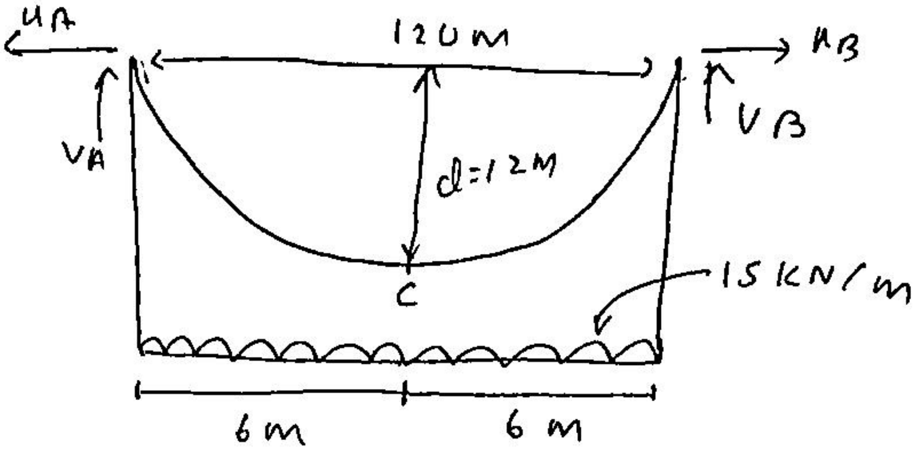
$$\text{Radial shear} = V\cos\theta - H\sin\theta$$

$$= -100 \times \cos 21.8 - 250 \times \sin 21.8$$

$$= -92.8 - 92.75$$

$$= -185.55\ kN \,//$$

Q 10)



$\Sigma H = 0 \Rightarrow H_B - H_A = 0$

$\Sigma V = 0 \Rightarrow V_A + V_B - (15 \times 120) = 0$

$\Rightarrow V_A + V_B = 1800\,kN$

$\Sigma M_A = 0 \Rightarrow -V_B \times 100 + 15 \times 120 \times \frac{120}{2} = 0$

$V_B = 900\,kN$

$V_A = 900\,kN$

Max & Min tension

$T_{max} = \sqrt{V^2 + H^2}$

$M_c = 0$

$\Rightarrow -H_A \times 12 + 900 \times 60 - 15 \times 60 \times \frac{60}{2} = 0$

$H_A = 2250\,kN$

$H_B = 2250\,kN$

$T_{max} = \sqrt{V^2 + H^2}$

$= \sqrt{900^2 + 2250^2} = 2423.32\,kN$

$T_{min} = H = 2250\,kN$

of cable

ss $= \frac{F}{A} \Rightarrow \frac{T_{max}}{\pi d^2/4} \Rightarrow 200 = \frac{2423.32}{\frac{\pi d^2}{4}}$

$d = 124.2mm$ //

gth of cable

$L = l + \frac{8h^2}{3l} \Rightarrow 120 + \frac{8}{3}\frac{(12)^2}{(120)} \Rightarrow 123.2m$ //