Q.1 Defination of periodic signal. 1M
Example. — 1M.
Defination of aperiodic signal 1M
Example – 1M.
Differentiation of Energy & power – 5M.
Examples of each – 1M.

Q.2 a) o/p due to linear combination of I/P
$y(n) = \{a x_1(n) + b x_2(n)\} g(n)$
o/p due to $x_1(n)$
$y_1(n) = x_1(n) g(n)$
o/p due to $x_2(n)$
$y_2(n) = x_2(n) g(n)$.
$y(n) = a y_1(n) + b y_2(n) = a x_1(n) g(n) + b x_2(n) g(n)$
∴ Linear.

5M o/p depends on present state of I/P only so.
Memory less & causal.
For stability let $|x(n)| = M_x$
$|y(n)| < |x(n)| |g(n)|$
$< M_x |g(n)|$ stable if $|g(n)| < \infty$
else astable (Non stable)

o/p due to shifted I/P
$y(n,k) = x(n-k) g(n)$
shifted o/p is $y(n-k) = x(n-k) \cdot g(n-k)$
∴ $y(n,k) \neq y(n-k)$ sys. is time variant

CMR

Q.2 b. O/P due to linear combination of I/P

$$y(t) = \frac{d}{dt}\left[e^{-t}\{a x_1(t) + b x_2(t)\}\right] = a\frac{d}{dt}\left[e^{-t}x_1(t)\right] + b\frac{d}{dt}\left[e^{-t}x_2(t)\right]$$

O/P due to $x_1(t)$

$$y_1(t) = \frac{d}{dt}\left[e^{-t}x_1(t)\right]$$

O/P due to $x_2(t)$

$$y_2(t) = \frac{d}{dt}\left[e^{-t}x_2(t)\right]$$

Linear combination of O/P.

$$a y_1(t) + b y_2(t) = a\frac{d}{dt}\left[e^{-t}x_1(t)\right] + b\frac{d}{dt}\left[e^{-t}x_2(t)\right]$$

$\therefore y(t) = a y_1(t) + b y_2(t)$ sys. is Linear.

O/P depends on only present state of I/P. Therefore sys. is Memoryless (static) and Causal.

5M

Let $|x(t)| = M_x$.

$$|y(t)| = \frac{d}{dt}|e^{-t}||x(t)| = \frac{d}{dt}|e^{-t}|M_x$$

$|y(t)| < \infty$ if $|e^{-t}| < \infty$

but $|e^{-t}|$ is not bounded for $t < 0$

$\therefore$ sys. is not stable.

O/P due to shifted I/P

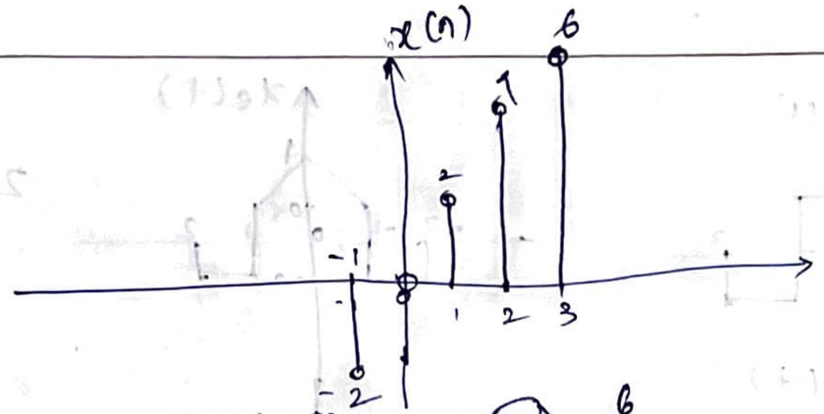
$$y(t, t_0) = \frac{d}{dt}\left[e^{-t}x(t - t_0)\right]$$

O/P shifted by an amount of t_0

$$y(t - t_0) = \frac{d}{dt}\left[e^{-(t - t_0)}x(t - t_0)\right]$$

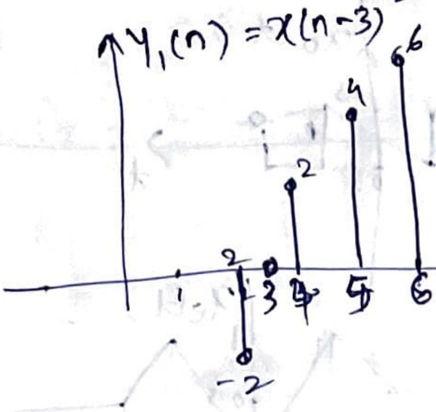
Now $y(t, t_0) \neq y(t - t_0)$ $\therefore$ Sys. is time variant.

Q.3



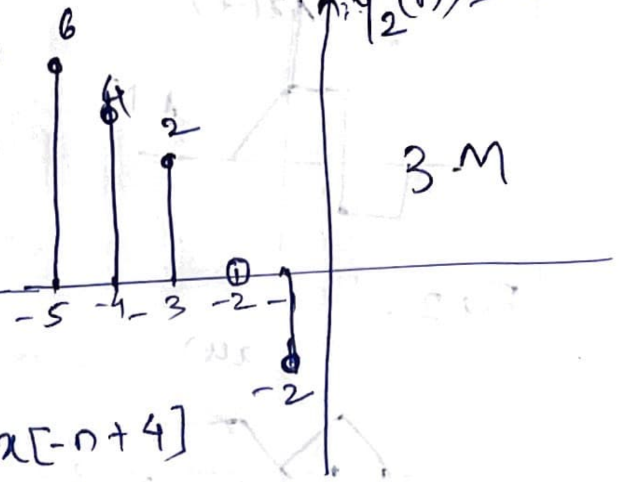
1M

a) $y_1(n) = x(n-3)$



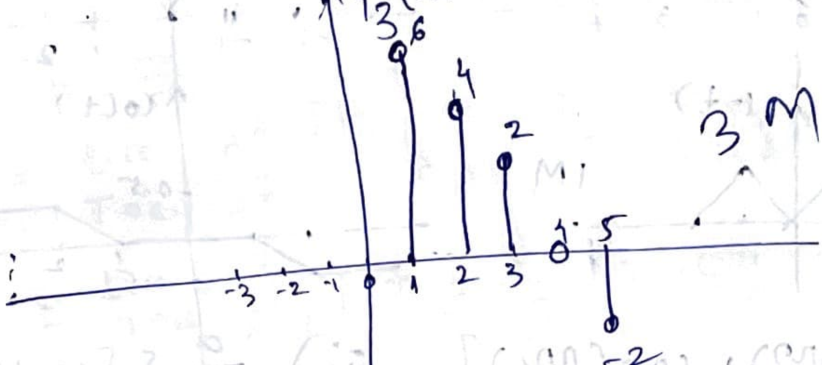
3M

b) $y_2(n) = x(-n-2)$



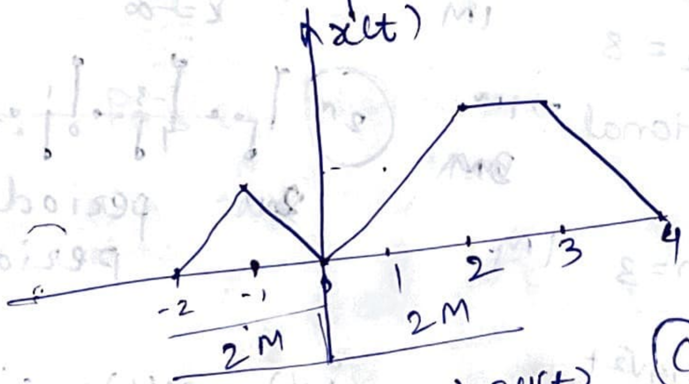
3M

c) $y_3(n) = x[-n+4]$



3M

Q.4. a)

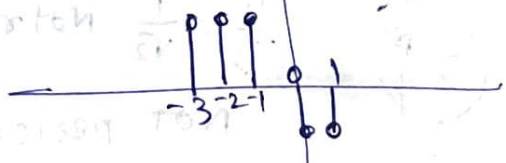


4M

b) $x(t) = u(t+2) - 2u(t) + u(t-2)$

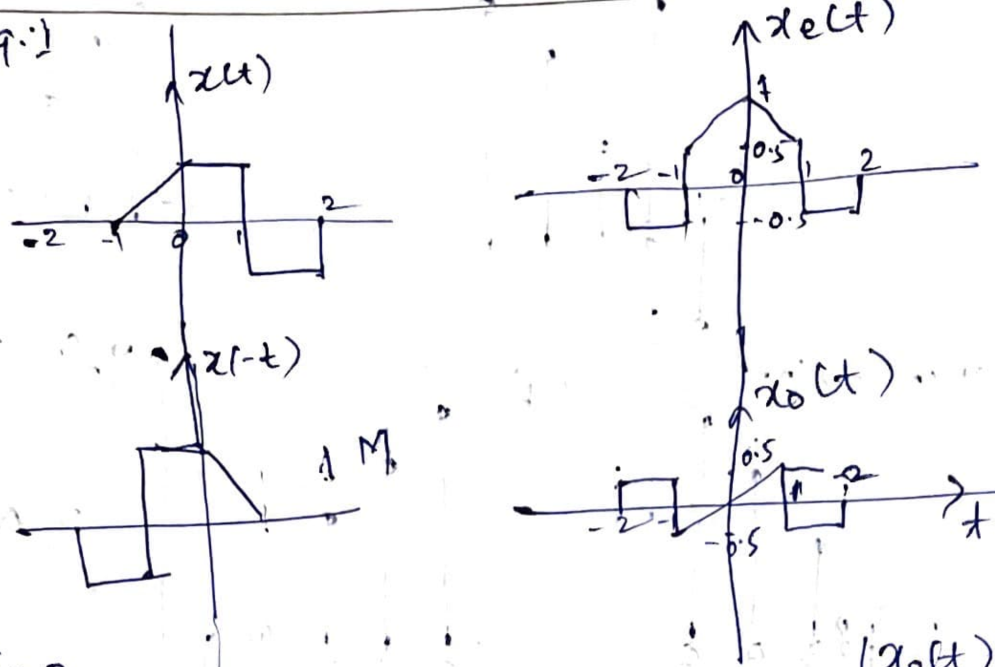
3M

c) $x(n) = u(n+3) - 2u(n) + u[n-2]$



3M

Q.5. Fig. 1

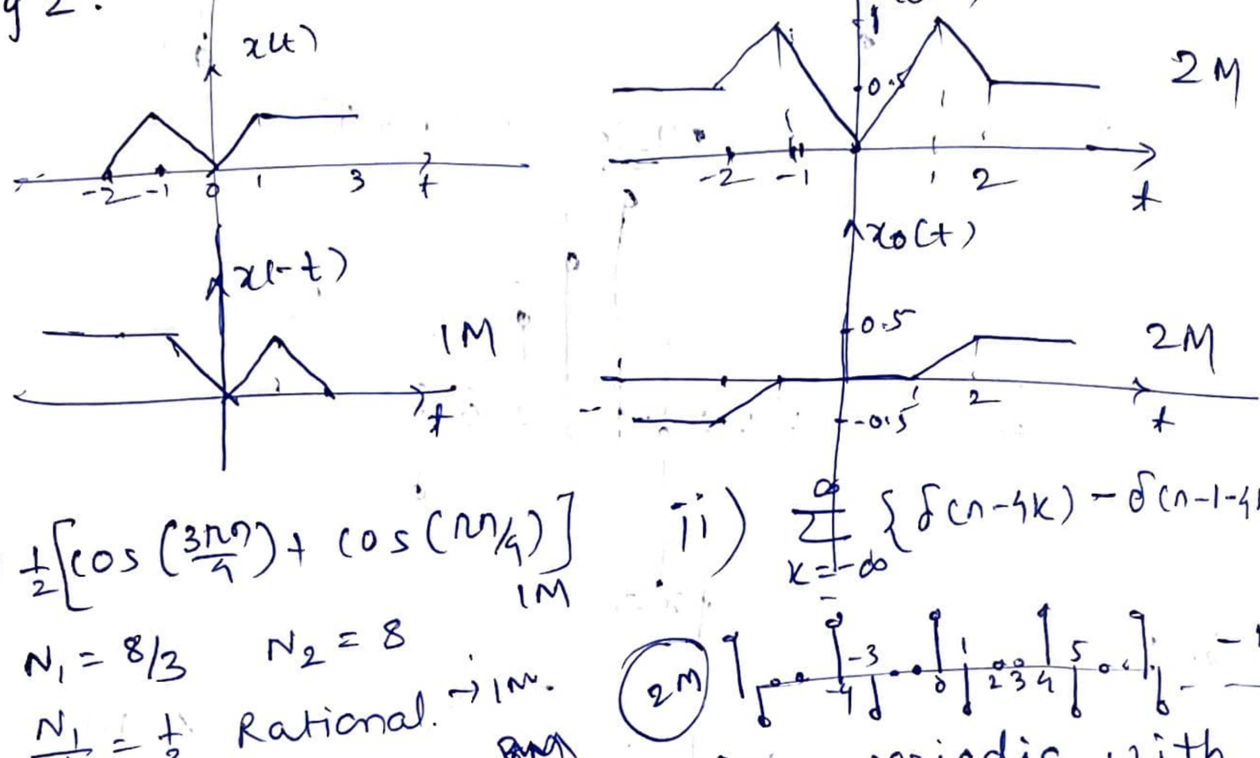


2 M

1 M

2 M

Fig 2.



2 M

1 M

2 M

Q.6 i) $\frac{1}{2}\left[\cos\left(\frac{3\pi n}{4}\right) + \cos\left(\pi n/4\right)\right]$ 1M

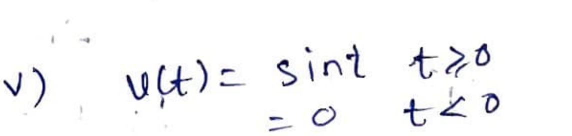
$N_1 = 8/3 \quad N_2 = 8$

(3M) $\frac{N_1}{N_2} = \frac{1}{3}$ Rational. → 1M.

$\therefore N = 8$ OR $N = 24$ for $m = 3$ 1M.

ii) $\sum_{k=-\infty}^{\infty} \{\delta(n-4k) - \delta(n-1-4k)\}$

(2M)



– 1M

periodic with period $N = 4$ – 1M

iii) $x(t) = \cos t + \sin\sqrt{2}t$

$T_1 = 2\pi, \quad T_2 = \sqrt{2}\pi$ 2M

(2M) $\frac{T_1}{T_2} = \frac{1}{\sqrt{2}}$ Not rational

Not periodic

iv) $u(t) = \sin t \quad t \geq 0$
$= 0 \quad t < 0$

$u(-t) = -\sin t \quad t < 0$
$= 0 \quad t \geq 0$

(2M) Not periodic

(Ingrace)

CMR

Q.7 i) $E_x = \int_0^{10} e^{-2t} dt = \left. \frac{e^{-2t}}{-2} \right|_0^{10} = \frac{1}{2}\left[1 - e^{-20}\right] < \infty$ ∴ Energy signal

$2\frac{1}{2}M$

$P_x = \lim_{T\to\infty} \frac{1}{T} \int_0^{10} e^{-2t} dt = \lim_{T\to\infty} \frac{1}{2T}\left[1 - e^{-20}\right] = 0$

ii) $E_x = \lim_{N\to\infty} \sum_{n=0}^{N-1} 1 = \lim_{N\to\infty} N = \infty$

$P_x = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} 1 = \lim_{N\to\infty} \frac{1}{N} N = 1 < \infty$ ∴ Power signal.

2M

iii) $E_x = \lim_{T\to\infty} A^2 \int_0^T \cos^2(2\pi f t + \theta) dt = \lim_{T\to\infty} \frac{A^2}{2} \int_0^T \{1 + \cos(4\pi f t + 2\theta)\} dt$

$= \lim_{T\to\infty} \frac{A^2}{2}\left[t\Big|_0^T + \left.\frac{\sin(4\pi f t + 2\theta)}{4\pi f}\right|_0^T \right]$

$= \infty$

$P_x = \lim_{T\to\infty} \frac{A^2}{2T}\left[t\Big|_0^T + \left.\frac{\sin(4\pi f t + 2\theta)}{4\pi f}\right|_0^T \right]$

3M

$= \lim_{T\to\infty} \frac{A^2}{2}\left[1 + \frac{1}{T}\frac{T}{4\pi}\left[\sin(4\pi + 2\theta) - \sin(2\theta)\right]\right]$

$= \frac{A^2}{2} < \infty$ if A is finite

∴ Power signal.

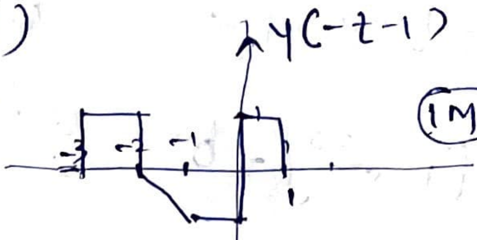
iv) $E_x = \lim_{N\to\infty} \sum_{n=1}^{N} \left((-1/2)^n\right)^2 = \sum_{n=1}^{\infty} (1/4)^n = \frac{1}{1-1/4} = 3/4$

$E_x < \infty$ ∴ Energy signal.

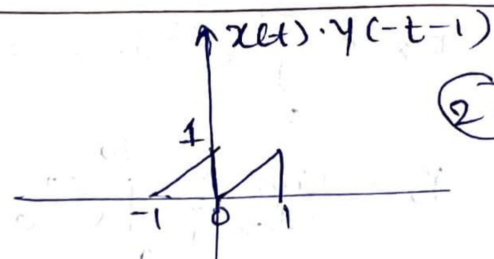
$2\frac{1}{2}M$.

$P_x = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{\infty} (1/4)^n = \lim_{N\to\infty} \frac{1}{N} \frac{1-(1/4)^{N+1}}{1-1/4} = 0$

Q 8 i)

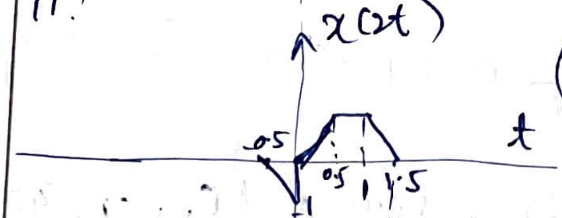


(1M)

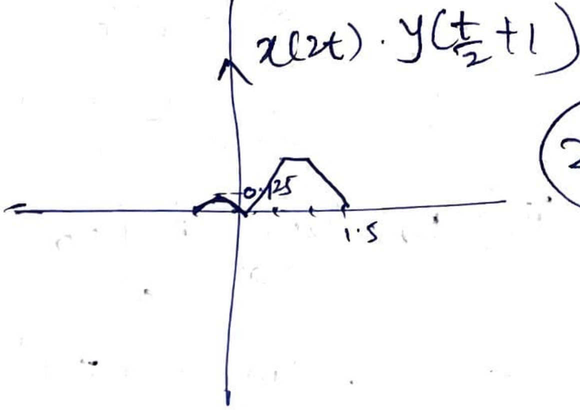


(2M)

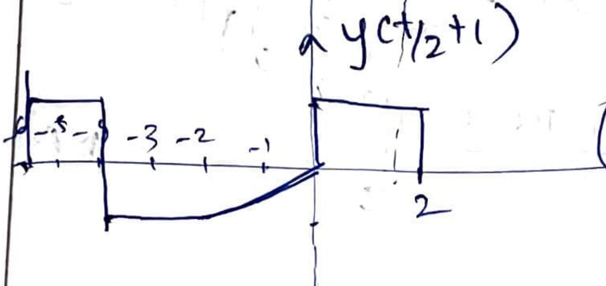
ii)



(1M)

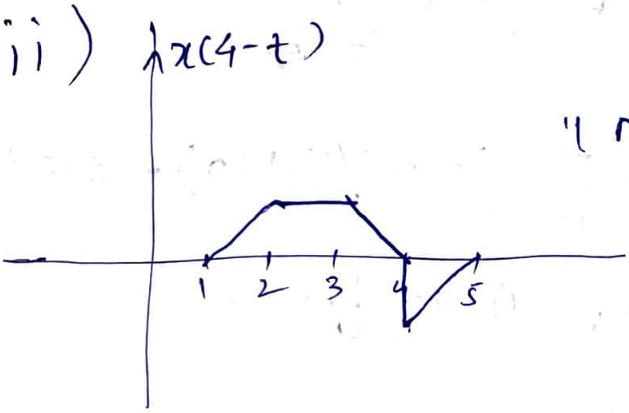


(2M)

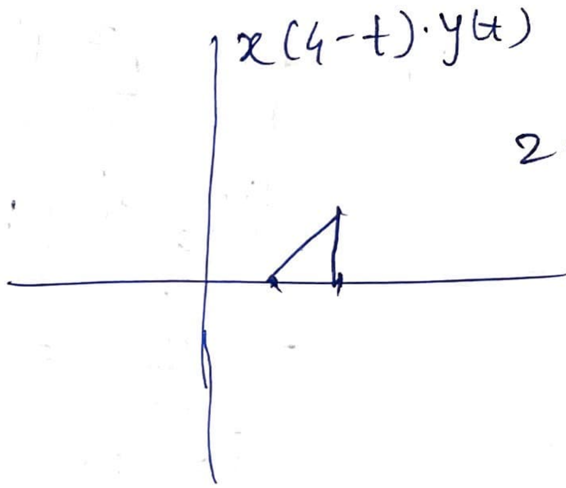


(1M)

iii)



1M



2M.