CMR INSTITUTE OF TECHNOLOGY	USN	

Internal Assesment Test - 1									
Sub:	Microwave and Antennas							**Code:**	18EC63
Date:	30/04/2022	**Duration:**	90 mins	**Max Marks:**	50	**Sem:**	6	**Branch:**	ECE

Answer All Questions

	Questions	Marks	CO	RBT
1.	What are microwaves? Mention the applications of microwaves. Explain briefly the microwave system with the help of a suitable diagram.	[10]	CO1	L2
2.	With the help of a suitable diagram, derive the expression for the instantaneous voltage and current on a two wire transmission line.	[10]	CO1	L2
3.	What are standing waves? Derive expression for the voltage maximum and minimum for the standing waves.	[10]	CO1	L3
4.	Define and derive an expression for reflection coefficient when the transmission line is terminated by load impedance (Z_L).	[10]	CO1	L2
5.	A transmission line has the following primary constants per km of the line: R=8Ω/m, G=0.1μ℧/m, L=3.5mH/m, C=9nF/m. Calculate Zo, α, β, v_p and γ at ω=5000 rad/sec.	[10]	CO1	L3
6.a	Define standing wave ratio. Derive the relationship between the standing wave ratio and reflection coefficient.	[06]	CO1	L3
6.b	A transmission line working at RF has following constants: L=9μH/m, C=16pF/m, the line is terminated in a resistive load of 1000 Ω. Find the reflection coefficient and standing wave ratio.	[04]	CO1	L3
7	A load impedance of Z_l=60-j80 Ω is required to be matched to a 50 Ω coaxial line, by using short circuited stub of length `l' located at a distance `d' from the load. The wavelength of operation is 1m. Design the single stub impedance matching system using Smith Chart.	[10]	CO1	L3

Solutions

1. Microwaves are alternating current signals characterized by shortest wavelengths (1cm to 1m) and highest frequencies (300 MHz to 30 GHz).

Applications of Microwave:

- To cook food as it cause water and fat molecules to vibrate, which makes the substances hot.
- Mobile phones use microwaves, as they can be generated by a small antenna.
- Wifi also uses microwaves.
- Fixed traffic speed cameras.
- For radar, which is used by aircraft, ships and weather forecasters.

MICROWAVE SYSTEMS

A microwave system normally consists of a transmitter subsystem, including a microwave oscillator, waveguides, and a transmitting antenna, and a receiver subsystem that includes a receiving antenna, transmission line or waveguide, a microwave amplifier, and a receiver. Figure 0-1 shows a typical microwave system.

In order to design a microwave system and conduct a proper test of it, an adequate knowledge of the components involved is essential. Besides microwave devices, the text therefore describes microwave components, such as resonators, cavities, microstrip lines, hybrids, and microwave integrated circuits.

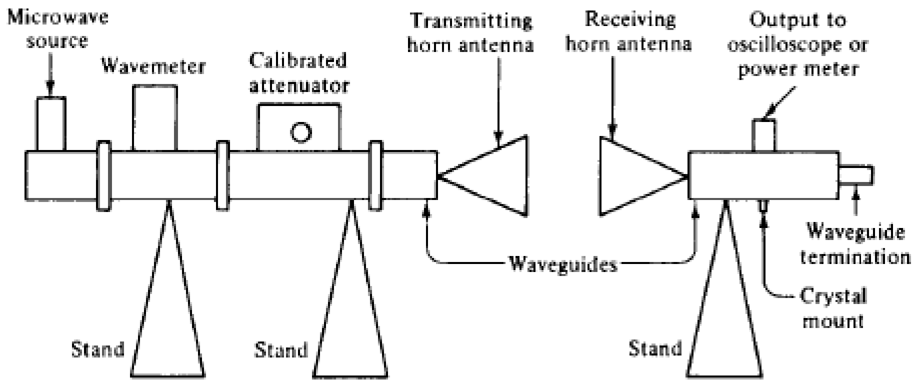


Figure 0-1 Microwave system.

2.

TRANSMISSION-LINE EQUATIONS AND SOLUTIONS

3-1-1 Transmission-Line Equations

A transmission line can be analyzed either by the solution of Maxwell's field equations or by the methods of distributed-circuit theory. The solution of Maxwell's equations involves three space variables in addition to the time variable. The distributed-circuit method, however, involves only one space variable in addition to the time variable. In this section the latter method is used to analyze a transmission line in terms of the voltage, current, impedance, and power along the line.

Based on uniformly distributed-circuit theory, the schematic circuit of a conventional two-conductor transmission line with constant parameters R, L, G, and C is shown in Fig. 3-1-1. The parameters are expressed in their respective names per unit length, and the wave propagation is assumed in the positive z direction.

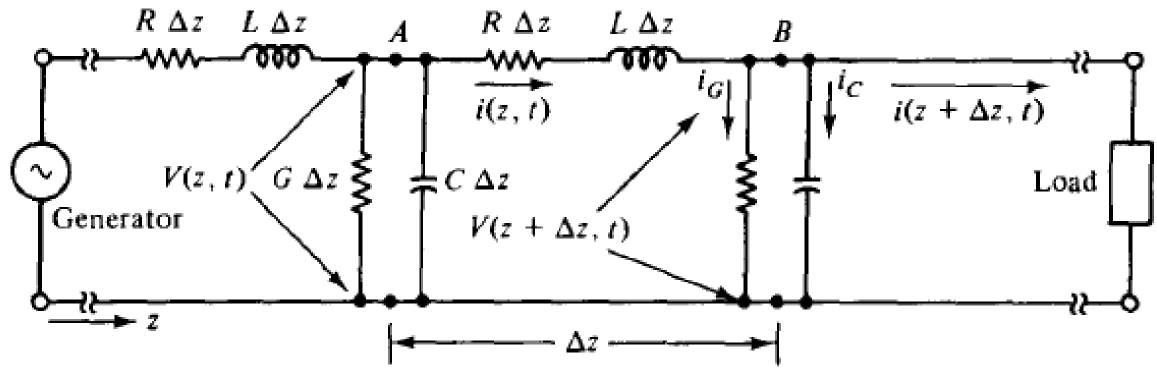


Figure 3-1-1 Elementary section of a transmission line.

By Kirchhoff's voltage law, the summation of the voltage drops around the central loop is given by

$$v(z, t) = i(z, t)R\,\Delta z + L\,\Delta z\frac{\partial i(z, t)}{\partial t} + v(z, t) + \frac{\partial v(z, t)}{dz}\,\Delta z \tag{3-1-1}$$

Rearranging this equation, dividing it by Δz, and then omitting the argument (z, t), which is understood, we obtain

$$-\frac{\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t} \tag{3-1-2}$$

Using Kirchhoff's current law, the summation of the currents at point B in Fig. 3-1-1 can be expressed as

$$\begin{aligned} i(z, t) &= v(z + \Delta z, t)G\,\Delta z + C\,\Delta z\,\frac{\partial v(z + \Delta z, t)}{\partial t} + i(z + \Delta z, t) \\ &= \left[v(z, t) + \frac{\partial v(z, t)}{\partial z}\,\Delta z\right]G\,\Delta z \\ &\quad + C\,\Delta z\,\frac{\partial}{\partial t}\left[v(z, t) + \frac{\partial v(z, t)}{\partial z}\,\Delta z\right] + i(z, t) + \frac{\partial i(z, t)}{\partial z}\,\Delta z \end{aligned} \tag{3-1-3}$$

By rearranging the preceding equation, dividing it by Δz, omitting (z, t), and assuming Δz equal to zero, we have

$$-\frac{\partial i}{\partial z} = Gv + C\frac{\partial v}{\partial t} \tag{3-1-4}$$

Then by differentiating Eq. (3-1-2) with respect to z and Eq. (3-1-4) with respect to t and combining the results, the final transmission-line equation in voltage form is found to be

$$\frac{\partial^2 v}{\partial z^2} = RGv + (RC + LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2} \tag{3-1-5}$$

Also, by differentiating Eq. (3-1-2) with respect to t and Eq. (3-1-4) with respect to z and combining the results, the final transmission-line equation in current form is

$$\frac{\partial^2 i}{\partial z^2} = RGi + (RC + LG)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2} \tag{3-1-6}$$

All these transmission-line equations are applicable to the general transient solution. The voltage and current on the line are the functions of both position z and time t. The instantaneous line voltage and current can be expressed as

$$v(z, t) = \text{Re}\, \mathbf{V}(z)e^{j\omega t} \tag{3-1-7}$$

$$i(z, t) = \text{Re}\, \mathbf{I}(z)e^{j\omega t} \tag{3-1-8}$$

where Re stands for "real part of." The factors $\mathbf{V}(z)$ aand $\mathbf{I}(z)$ are complex quantities of the sinusoidal functions of position z on the line and are known as *phasors*. The phasors give the magnitudes and phases of the sinusoidal function at each position of z, and they can be expressed as

$$\mathbf{V}(z) = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{\gamma z} \tag{3-1-9}$$

$$\mathbf{I}(z) = \mathbf{I}_+ e^{-\gamma z} + \mathbf{I}_- e^{\gamma z} \tag{3-1-10}$$

$$\gamma = \alpha + j\beta \qquad \text{(propagation constant)} \tag{3-1-11}$$

where $\mathbf{V}_+$ and $\mathbf{I}_+$ indicate complex amplitudes in the positive z direction, $\mathbf{V}_-$ and $\mathbf{I}_-$ signify complex amplitudes in the negative z direction, α is the attenuation constant in nepers per unit length, and β is the phase constant in radians per unit length.

If we substitute $j\omega$ for $\partial/\partial t$ in Eqs. (3-1-2), (3-1-4), (3-1-5), and (3-1-6) and divide each equation by $e^{j\omega t}$, the transmission-line equations in phasor form of the frequency domain become

$$\frac{d\mathbf{V}}{dz} = -\mathbf{ZI} \tag{3-1-12}$$

$$\frac{d\mathbf{I}}{dz} = -\mathbf{YV} \tag{3-1-13}$$

$$\frac{d^2\mathbf{V}}{dz^2} = \gamma^2\mathbf{V} \tag{3-1-14}$$

$$\frac{d^2\mathbf{I}}{dz^2} = \gamma^2\mathbf{I} \tag{3-1-15}$$

in which the following substitutions have been made:

$$\mathbf{Z} = R + j\omega L \qquad \text{(ohms per unit length)} \tag{3-1-16}$$

$$\mathbf{Y} = G + j\omega C \qquad \text{(mhos per unit length)} \tag{3-1-17}$$

$$\gamma = \sqrt{ZY} = \alpha + j\beta \qquad \text{(propagation constant)} \tag{3-1-18}$$

For a lossless line, $R = G = 0$, and the transmission-line equations are expressed as

$$\frac{d\mathbf{V}}{dz} = -j\omega L\,\mathbf{I} \tag{3-1-19}$$

$$\frac{d\mathbf{I}}{dz} = -j\omega C\,\mathbf{V} \tag{3-1-20}$$

$$\frac{d^2\mathbf{V}}{dz^2} = -\omega^2 LC\,\mathbf{V} \tag{3-1-21}$$

$$\frac{d^2\mathbf{I}}{dz^2} = -\omega^2 LC\,\mathbf{I} \tag{3-1-22}$$

It is interesting to note that Eqs. (3-1-14) and (3-1-15) for a transmission line are similar to equations of the electric and magnetic waves, respectively. The only difference is that the transmission-line equations are one-dimensional.

3-1-2 Solutions of Transmission-Line Equations

The one possible solution for Eq. (3-1-14) is

$$\mathbf{V} = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{\gamma z} = \mathbf{V}_+ e^{-\alpha z} e^{-j\beta z} + \mathbf{V}_- e^{\alpha z} e^{j\beta z} \tag{3-1-23}$$

The factors $\mathbf{V}_+$ and $\mathbf{V}_-$ represents complex quantities. The term involving $e^{-j\beta z}$ shows a wave traveling in the positive z direction, and the term with the factor $e^{j\beta z}$ is a wave going in the negative z direction. The quantity βz is called the *electrical length of the line* and is measured in radians.

Similarly, the one possible solution for Eq. (3-1-15) is

$$\mathbf{I} = \mathbf{Y}_0(\mathbf{V}_+ e^{-\gamma z} - \mathbf{V}_- e^{\gamma z}) = \mathbf{Y}_0(\mathbf{V}_+ e^{-\alpha z} e^{-j\beta z} - \mathbf{V}_- e^{\alpha z} e^{j\beta z}) \tag{3-1-24}$$

In Eq. (3-1-24) the characteristic impedance of the line is defined as

$$\mathbf{Z}_0 = \frac{1}{\mathbf{Y}_0} \equiv \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 \pm jX_0 \tag{3-1-25}$$

3.

STANDING WAVE AND STANDING-WAVE RATIO

3-3-1 Standing Wave

The general solutions of the transmission-line equation consist of two waves traveling in opposite directions with unequal amplitude as shown in Eqs. (3-1-23) and (3-1-24). Equation (3-1-23) can be written

$$\begin{aligned} \mathbf{V} &= \mathbf{V}_+ e^{-\alpha z} e^{-j\beta z} + \mathbf{V}_- e^{\alpha z} e^{j\beta z} \\ &= \mathbf{V}_+ e^{-\alpha z}[\cos(\beta z) - j\sin(\beta z)] + \mathbf{V}_- e^{\alpha z}[\cos(\beta z) + j\sin(\beta z)] \qquad (3\text{-}3\text{-}1) \\ &= (\mathbf{V}_+ e^{-\alpha z} + \mathbf{V}_- e^{\alpha z})\cos(\beta z) - j(\mathbf{V}_+ e^{-\alpha z} - \mathbf{V}_- e^{\alpha z})\sin(\beta z) \end{aligned}$$

With no loss in generality it can be assumed that $\mathbf{V}_+ e^{-\alpha z}$ and $\mathbf{V}_- e^{\alpha z}$ are real. Then the voltage-wave equation can be expressed as

$$\mathbf{V}_s = \mathbf{V}_0 e^{-j\phi} \tag{3-3-2}$$

This is called the *equation of the voltage standing wave*, where

$$\mathbf{V}_0 = [(\mathbf{V}_+ e^{-\alpha z} + \mathbf{V}_- e^{\alpha z})^2 \cos^2(\beta z) + (\mathbf{V}_+ e^{-\alpha z} - \mathbf{V}_- e^{-\alpha z})^2 \sin^2(\beta z)]^{1/2} \tag{3-3-3}$$

which is called the *standing-wave pattern* of the voltage wave or the amplitude of the standing wave, and

$$\phi = \arctan\left(\frac{\mathbf{V}_+ e^{-\alpha z} - \mathbf{V}_- e^{\alpha z}}{\mathbf{V}_+ e^{-\alpha z} + \mathbf{V}_- e^{\alpha z}} \tan(\beta z)\right) \tag{3-3-4}$$

which is called the *phase pattern of the standing wave*. The maximum and minimum values of Eq. (3-3-3) can be found as usual by differentiating the equation with respect to βz and equating the result to zero. By doing so and substituting the proper values of βz in the equation, we find that

1. The maximum amplitude is

$$\mathbf{V}_{\max} = \mathbf{V}_+ e^{-\alpha z} + \mathbf{V}_- e^{\alpha z} = \mathbf{V}_+ e^{-\alpha z}(1 + |\Gamma|) \tag{3-3-5}$$

   and this occurs at $\beta z = n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$.
2. The minimum amplitude is

$$\mathbf{V}_{\min} = \mathbf{V}_+ e^{-\alpha z} - V_- e^{\alpha z} = \mathbf{V}_+ e^{-\alpha z}(1 - |\Gamma|) \tag{3-3-6}$$

   and this occurs at $\beta z = (2n - 1)\pi/2$, where $n = 0, \pm 1, \pm 2, \ldots$.
3. The distance between any two successive maxima or minima is one-half wavelength, since

$$\beta z = n\pi \qquad z = \frac{n\pi}{\beta} = \frac{n\pi}{2\pi/\lambda} = n\frac{\lambda}{2} \qquad (n = 0, \pm 1, \pm 2, \ldots)$$

Then

$$z_1 = \frac{\lambda}{2} \tag{3-3-7}$$

It is evident that there are no zeros in the minimum. Similarly,

$$\mathbf{I}_{\max} = \mathbf{I}_+ e^{-\alpha z} + \mathbf{I}_- e^{\alpha z} = \mathbf{I}_+ e^{-\alpha z}(1 + |\Gamma|) \tag{3-3-8}$$

$$\mathbf{I}_{\min} = \mathbf{I}_+ e^{-\alpha z} - \mathbf{I}_- e^{\alpha z} = \mathbf{I}_+ e^{-\alpha z}(1 - |\Gamma|) \tag{3-3-9}$$

The standing-wave patterns of two oppositely traveling waves with unequal amplitude in lossy or lossless line are shown in Figs. 3-3-1 and 3-3-2.

A further study of Eq. (3-3-3) reveals that

1. When $\mathbf{V}_+ \neq 0$ and $\mathbf{V}_- = 0$, the standing-wave pattern becomes

$$\mathbf{V}_0 = \mathbf{V}_+ e^{-\alpha z} \tag{3-3-10}$$

5. A transmission line has the following primary constants per km of the line: $R = 8\ \Omega/m$, $G = 0.1\ \mu\mho/m$, $L = 3.5\ mH/m$, $C = 9\ nF/m$. Calculate Z_0, α, β, V_p and v at $\omega = 5000$ rad/sec. (10M)

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{8 + j\,5000 \times 3.5 \times 10^{-3}}{0.1 \times 10^{-6} + j\,5000 \times 9 \times 10^{-9}}}$$

$$= \sqrt{\frac{8 + j\,17.5}{0.1 \times 10^{-6} + j\,4.5 \times 10^{-5}}}$$

$$= \sqrt{\frac{19.24 \angle 65.43^\circ}{4.5 \times 10^{-5} \angle 89.87^\circ}}$$

$$= 653.87 \angle -12.22^\circ \ \Omega/km \rightarrow 3M$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(8 + j17.5)(0.1 \times 10^{-6} + j\,4.5 \times 10^{-5})}$$

$$= \sqrt{(19.24 \angle 65.43^\circ) \times (4.5 \times 10^{-5} \angle 89.87^\circ)}$$

$$= 0.0294 \angle 77.65^\circ$$

$$= 6.288 \times 10^{-3} + j\,0.0287 \rightarrow 3M$$

$$= \alpha + j\beta$$

$\alpha = 6.288$ nepers/km

$\beta = 0.0287$ radians/km $\Big\} \rightarrow 2M$

$$V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.5 \times 10^{-3} \times 9 \times 10^{-9}}} = 1.07817 \times 10^5 \ m/s \rightarrow 2M$$

3-3-2 Standing-Wave Ratio

Standing waves result from the simultaneous presence of waves traveling in opposite directions on a transmission line. The ratio of the maximum of the standing-wave pattern to the minimum is defined as the standing-wave ratio, designated by ρ. That is,

$$\text{Standing-wave ratio} \equiv \frac{\text{maximum voltage or current}}{\text{minimum voltage or current}}$$

$$\rho \equiv \frac{|\mathbf{V}_{max}|}{|\mathbf{V}_{min}|} = \frac{|\mathbf{I}_{max}|}{|\mathbf{I}_{min}|} \tag{3-3-16}$$

The standing-wave ratio results from the fact that the two traveling-wave components of Eq. (3-3-1) add in phase at some points and subtract at other points. The distance between two successive maxima or minima is $\lambda/2$. The standing-wave ratio of a pure traveling wave is unity and that of a pure standing wave is infinite. It should be noted that since the standing-wave ratios of voltage and current are identical, no distinctions are made between VSWR and ISWR.

When the standing-wave ratio is unity, there is no reflected wave and the line is called a *flat line*. The standing-wave ratio cannot be defined on a lossy line because the standing-wave pattern changes markedly from one position to another. On a low-loss line the ratio remains fairly constant, and it may be defined for some region. For a lossless line, the ratio stays the same throughout the line.

Since the reflected wave is defined as the product of an incident wave and its reflection coefficient, the standing-wave ratio ρ is related to the reflection coefficient $\mathbf{\Gamma}$ by

$$\rho = \frac{1 + |\mathbf{\Gamma}|}{1 - |\mathbf{\Gamma}|} \tag{3-3-17}$$

6b. A transmission line working at RF has following constants: L = 9 μH/m, C = 16 pF/m, the line is terminated in a resistive load of 1000 Ω. Find the reflection coefficient and standing wave ratio.

Assuming the line is lossless,

$(R = G = 0)$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{9 \times 10^{-6}}{16 \times 10^{-12}}} = 750\ \Omega$$

Given $Z_L = R_L = 1000\ \Omega$ → 2M

Reflection coefficient (Γ)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1000 - 750}{1000 + 750}$$

$$= 0.1428$$

→ 1M

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$= \frac{1 + 0.1428}{1 - 0.1428}$$

$$= 1.33$$ → 1M

REFLECTION COEFFICIENT AND TRANSMISSION COEFFICIENT

3-2-1 Reflection Coefficient

In the analysis of the solutions of transmission-line equations in Section 3-1, the traveling wave along the line contains two components: one traveling in the positive z direction and the other traveling the negative z direction. If the load impedance is equal to the line characteristic impedance, however, the reflected traveling wave does not exist.

Figure 3-2-1 shows a transmission line terminated in an impedance $\mathbf{Z}_\ell$. It is usually more convenient to start solving the transmission-line problem from the receiving rather than the sending end, since the voltage-to-current relationship at the load point is fixed by the load impedance. The incident voltage and current waves traveling along the transmission line are given by

$$\mathbf{V} = \mathbf{V}_+ e^{-\gamma z} + \mathbf{V}_- e^{+\gamma z} \tag{3-2-1}$$

$$\mathbf{I} = \mathbf{I}_+ e^{-\gamma z} + \mathbf{I}_- e^{+\gamma z} \tag{3-2-2}$$

in which the current wave can be expressed in terms of the voltage by

$$\mathbf{I} = \frac{\mathbf{V}_+}{\mathbf{Z}_0} e^{-\gamma z} - \frac{\mathbf{V}^-}{\mathbf{Z}_0} e^{\gamma z} \tag{3-2-3}$$

If the line has a length of ℓ, the voltage and current at the receiving end become

$$\mathbf{V}_\ell = \mathbf{V}_+ e^{-\gamma\ell} + \mathbf{V}_- e^{\gamma\ell} \tag{3-2-4}$$

$$\mathbf{I}_\ell = \frac{1}{\mathbf{Z}_0}(\mathbf{V}_+ e^{-\gamma\ell} - \mathbf{V}_- e^{\gamma\ell}) \tag{3-2-5}$$

The ratio of the voltage to the current at the receiving end is the load impedance. That is,

$$\mathbf{Z}_\ell = \frac{\mathbf{V}_\ell}{\mathbf{I}_\ell} = \mathbf{Z}_0 \frac{\mathbf{V}_+ e^{-\gamma\ell} + \mathbf{V}_- e^{\gamma\ell}}{\mathbf{V}_+ e^{-\gamma\ell} - \mathbf{V}_- e^{\gamma\ell}} \tag{3-2-6}$$

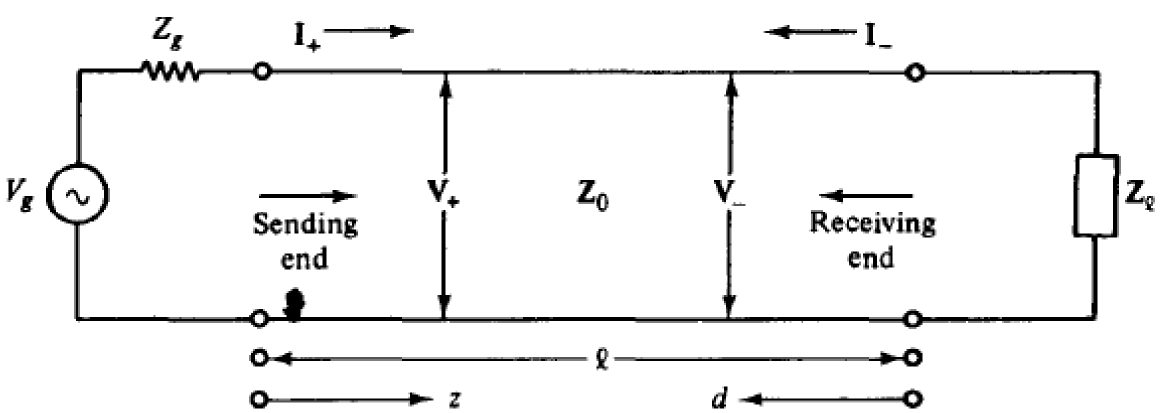


Figure 3-2-1 Transmission line terminated in a load impedance.

The reflection coefficient, which is designated by $\mathbf{\Gamma}$ (gamma), is defined as

$$\text{Reflection coefficient} \equiv \frac{\text{reflected voltage or current}}{\text{incident voltage or current}}$$

$$\mathbf{\Gamma} \equiv \frac{\mathbf{V}_{\text{ref}}}{\mathbf{V}_{\text{inc}}} = \frac{-\mathbf{I}_{\text{ref}}}{\mathbf{I}_{\text{inc}}} \tag{3-2-7}$$

If Eq. (3-2-6) is solved for the ratio of the reflected voltage at the receiving end, which is $\mathbf{V}_- e^{\gamma \ell}$, to the incident voltage at the receiving end, which is $\mathbf{V}_+ e^{\gamma \ell}$, the result is the reflection coefficient at the receiving end:

$$\mathbf{\Gamma}_\ell = \frac{\mathbf{V}_- e^{\gamma \ell}}{\mathbf{V}_+ e^{-\gamma \ell}} = \frac{\mathbf{Z}_\ell - \mathbf{Z}_0}{\mathbf{Z}_\ell + \mathbf{Z}_0} \tag{3-2-8}$$

7 A load impedance of $Z_L = 60 - j80\ \Omega$ is required to be matched to a $50\ \Omega$ coaxial line, by using short circuited stub of length 'l' located at a distance 'd' from the load. The wavelength of operation is 1m. Design the single stub impedance matching system using Smith chart.

10 M

$$Y_L = \frac{1}{Z_L} = \frac{1}{60 - j80} = 6 \times 10^{-3} + 8 \times 10^{-3}$$

$$Y_0 = \frac{1}{Z_0} = \frac{1}{50} = 0.02$$

$$\overline{y_L} = \frac{Y_L}{Y_0} = \frac{6 \times 10^{-3} + 8 \times 10^{-3}}{0.02}$$

$$= 0.3 + j0.4$$

$$d = (0.19 - 0.129)\lambda$$

$$= 0.061\lambda = 0.061 m$$

$$\overline{y}_{in} = \overline{y}_d + \overline{y}_s$$

$$1 = 1 + j2.1 + \overline{y}_s$$

$$\overline{y}_s = -j2.1$$

$$l = 0.32\lambda - 0.25\lambda$$

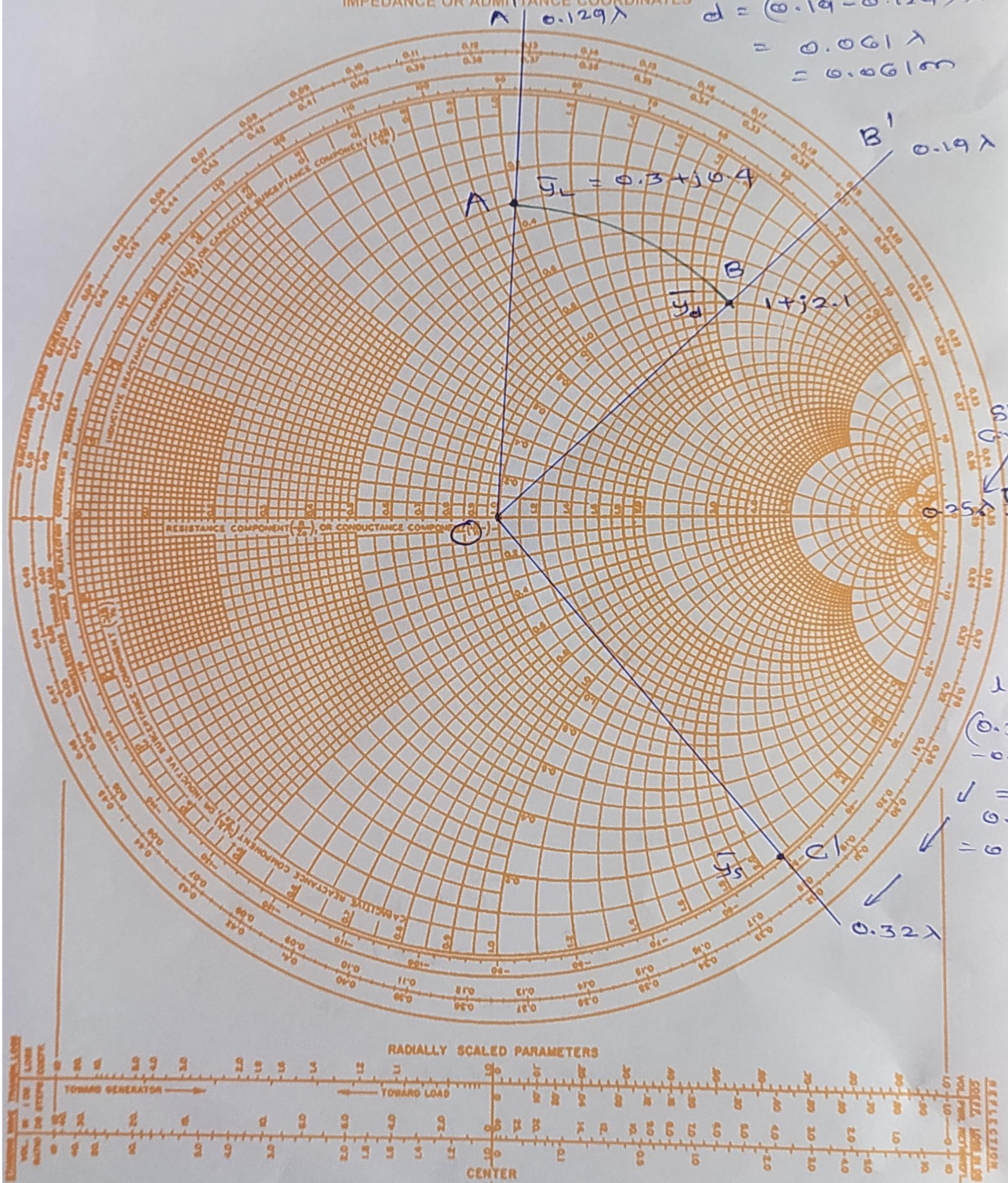
$$= 0.07\lambda$$

$$= 0.07 m$$

7.

VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELGAUM

IMPEDANCE OR ADMITTANCE COORDINATES



$d = (0.19 - 0.129)\lambda$
$= 0.061\lambda$
$= 0.061 m$

$l = (0.32 - 0.25)\lambda$
$= 0.07\lambda$
$= 0.07 m$