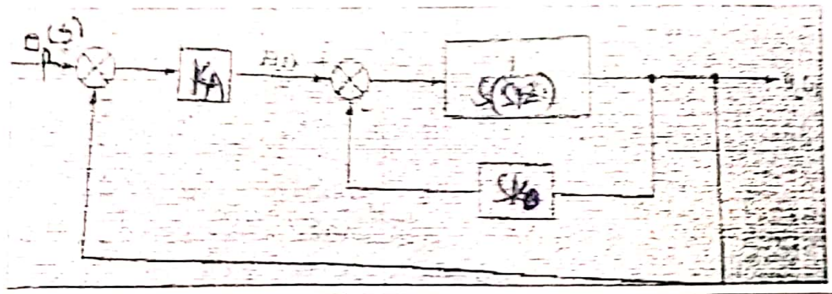
USN

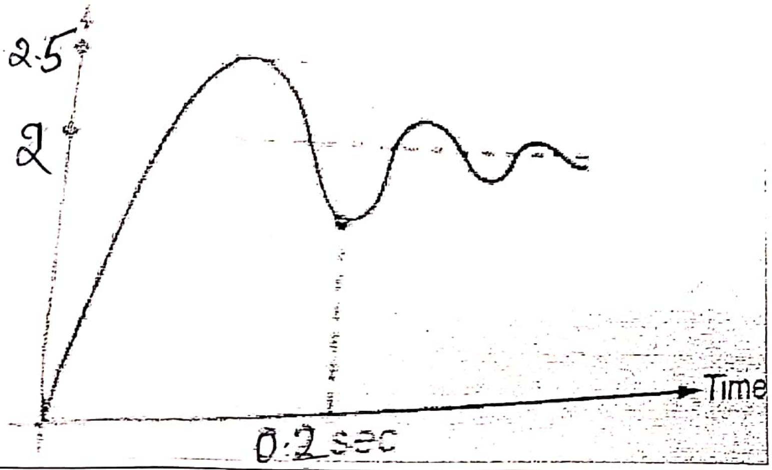
CMRIT

Internal Assessment Test 2 – August 2022

Sub:	Control Systems					Sub Code:	18 EC 43	Branch:	ECE
Date:	04-08-22	Duration:	90 min's	Max Marks:	50	Sem / Sec:	4 - A B C D		OBE

Answer any FIVE FULL Questions

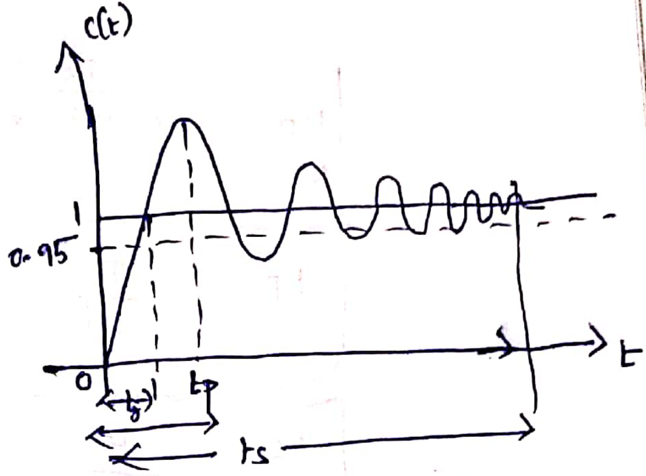
		MARKS	CO	RBT
1.	Starting from the output equation C(t) derive expressions for Rise time, Peak time, Settling Time for 5% tolerance band of an under damped second order system subjected to unit step input with neat diagram.	[10]	CO3	L4
2.	Derive an expression for C(t) of an under damped second order system for a unit step input.	[10]	CO3	L2
3.	A unity feedback system is characterized by an open loop transfer function $G(S)=K/S(S+20)$ find the value of K so that the system will have the damping ratio of 0.5. for this value of K find MP, tP & tS for a unit step input	[10]	CO3	L3
4.	A unity feedback system has $G(S)=\frac{K}{S(S+4)(S^2+2S+10)}$ (i) For a unit ramp input, it is desired that eSS ≤ 0.4. Find K. (ii) Find eSS if $r(t)= 2+ 4t+t^2/2$	[10]	CO3	L3
5.	For a Servomechanism system $G(S) =\frac{K1}{S^2}$, $H(S)=1+K_2S$. Determine the value of K1, K2 so that peak overshoot is 0.25 and peak time is 2 seconds for a unit step input.	[10]	CO3	L3
6.	For the block diagram shown in fig. Find the values of KA and K0, so that δ (zeta)=0.6 and steady state error for ramp input is 0.2 rad 	[10]	CO3	L2
7.	Find the open loop transfer function of a unity feedback system having second order, whose step response is shown in the fig. Also find Rise time and settling time.	[10]	CO3	L3


2.5
2
Time
0.2 sec

Control Systems

IAT 2 Solutions

1. **Derivation for Rise time**



$$t_r = \frac{\pi - \theta}{\omega_d} \text{ sec}$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} \text{ rad.}$$

Peak Time $t_p = \dfrac{\pi}{\omega_d}$ sec

settling time $t_s = \dfrac{3}{\zeta\omega_n}$ for $\pm 5\%$ Tolerance band.

2. For underdamped second order systems

$$\boxed{c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)}$$ derivation.

3. $G(s) = \dfrac{K}{s(s+20)}$ $\quad H(s) = 1$

Given that $\zeta = 0.5$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + 20s + \cancel{K}}$$

$\omega_n^2 = K.$

$2\zeta\omega_n = 20$

$\omega_n = \dfrac{20}{2 \times 0.5} = 20$

$\therefore K = \omega_n^2 = (20)^2 = 400$

$\boxed{K = 400}$

$$2\xi\omega_n = 20$$

$$\xi = \frac{20}{2 \times 20} = 0.5$$

$$\therefore \frac{C(s)}{R(s)} = \frac{400}{s^2 + 20s + 400}$$

$$\boxed{M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100 = 16.3\,\%}$$

$$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \cdot \sqrt{1-\xi^2}} = \frac{\pi}{20 \times 0.866} = \frac{3.14}{17.32} = 0.18\,\text{sec}}$$

$$\boxed{t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5 \times 20} = \frac{4}{10} = 0.4\,\text{sec}}$$

4. i) $G(s) = \dfrac{K}{s(s+4)(s^2+2s+10)}$ $\qquad H(s) = 1$

For a unit ramp i/p $\quad e_{ss} = 0.4$

$$K_v = \lim_{s \to 0} s\, G(s) H(s)$$

$$= \lim_{s \to 0} \cancel{s} \cdot \frac{K}{\cancel{s}(s+4)(s^2+2s+10)}$$

$$K_v = \frac{K}{4 \times 10} = \frac{K}{40}$$

$$\boxed{K_v = \frac{K}{40}}$$

$$e_{ss} = \frac{1}{K_v} = \frac{40}{K} = 0.4$$

$$K = \frac{40}{0.4} = 100$$

$$\boxed{K = 100}$$

ii) $$r(t) = 2 + 4t + \frac{t^2}{2}$$

$$K_p = \underset{s \to 0}{Lt}\ G(s)H(s) = \infty$$

$$K_v = \frac{K}{40} = \frac{100}{40} = \frac{10}{4}$$

$$K_a = \underset{s \to 0}{Lt}\ s^2 G(s)H(s) = 0$$

$$e_{ss1} = \frac{2}{1+K_p} = \frac{2}{1+\infty} = 0$$

$$e_{ss2} = \frac{4}{K_v} = \frac{4}{\frac{10}{4}} = \frac{16}{10} = 1.6$$

$$e_{ss3} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

$$e_{ss} = e_{ss1} + e_{ss2} + e_{ss3}$$

$$= 0 + 1.6 + \infty$$

$$\boxed{e_{ss} = \infty}$$

5. $G(s) = \dfrac{K_1}{s^2}$ $H(s) = 1 + K_2 s$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)} = \frac{\frac{K_1}{s^2}}{1+\frac{K_1}{s^2}(1+K_2 s)}$$

$$\boxed{\frac{C(s)}{R(s)} : \frac{K_1}{s^2 + K_1 K_2 s + K_1}}$$

Peak over shoot $M_p = 0.25$

$t_p = \dfrac{\pi}{w_d} = 2$ sec.

$w_n^2 = K_1$

$w_n = \sqrt{K_1}$

$2\xi w_n = K_1 K_2$

$\xi = \dfrac{K_1 K_2}{2 w_n} = \dfrac{K_1 K_2}{2 \cdot \sqrt{K_1}}$

$= \dfrac{1}{2}\sqrt{K_1} K_2$

$M_p = 0.25 = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$$\boxed{\xi = 0.4037}$$

$t_p = \dfrac{\pi}{w_n \cdot \sqrt{1-\xi^2}} = 2.$

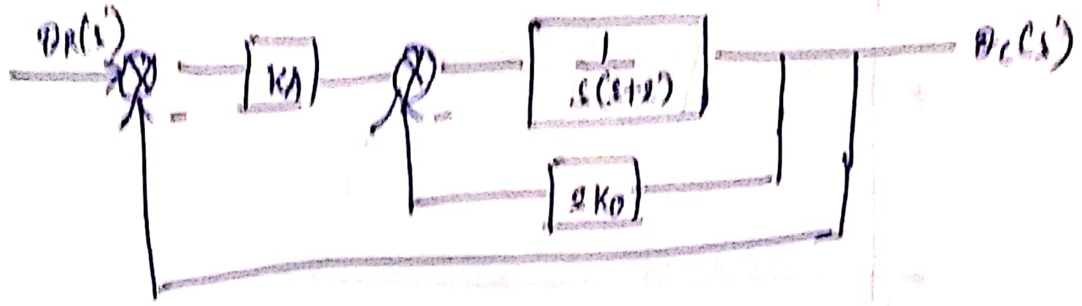
$=$ $w_n = 1.716$ rad/sec.

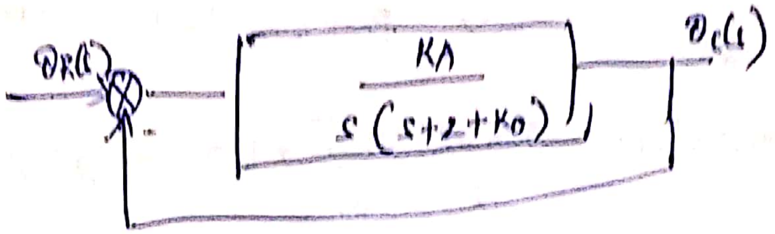
$$\boxed{K_1 = w_n^2 = 2.944}$$

$K_2 = \dfrac{2\xi w_n}{K_1} = \dfrac{2 \times 0.4037 \times 1.716}{2.944}$

$$\boxed{K_2 = 0.4706}$$

6.





$$\frac{\frac{1}{s(s+2)}}{1+\frac{1}{s(s+2)}\cdot sK_0}$$

$$= \frac{1}{s(s+2)+sK_0}$$

$\xi = 0.6$ $\qquad e_{ss}$ (unit ramp) $= 0.2$

$$K_v = \lim_{s\to 0} s\,G(s)H(s) = \frac{K_A}{2+K_0}$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{\frac{K_A}{2+K_0}} = 0.2$$

$$\boxed{\frac{K_A}{2+K_0} = 5.} \rightarrow (1)$$

$$\frac{C(s)}{R(s)} = \frac{K_A}{s^2+(2+K_0)s+K_A}.$$

$\omega_n^2 = K_A$ $\qquad \omega_n = \sqrt{K_A}.$

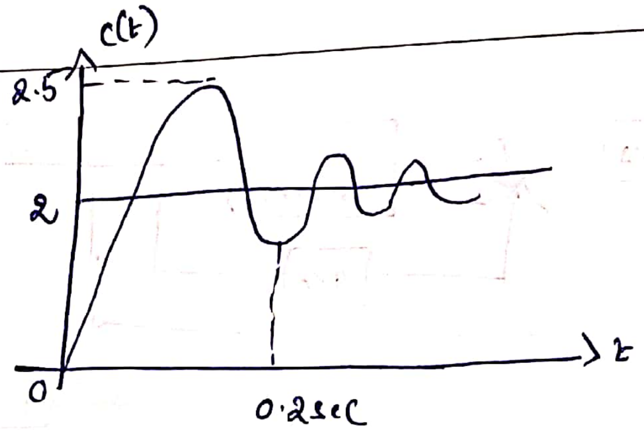
$2\xi\omega_n = 2+K_0$

$2\times 0.6\times\sqrt{K_A} = 2+K_0.$

$2\times 0.6\times\sqrt{K_A} = \frac{K_A}{5}$ $\qquad$ From (1)

$\boxed{K_A = 36}$ $\qquad \boxed{K_0 = 5.2}$

7.



$r(t) = 2.$

$\%$ Peak overshoot $= \dfrac{c(t_p) - c(\infty)}{c(\infty)} \times 100$

$\%$ Peak overshoot $= \dfrac{2.5-2}{2} = \dfrac{0.5}{2} \times 100 = 25\%.$

$\% M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100$

$0.25 = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$

$$\boxed{\zeta = 0.4036}$$

Time for first undershoot is 0.2 sec

$t_p = \dfrac{n\pi}{w_d} = \dfrac{2\pi}{w_d} = 0.2\text{sec}$

$$\boxed{w_n = 34.3367 \text{ rad/sec}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{w_n^2}{s^2+2\zeta w_n s + w_n^2} = \frac{1179.0134}{s^2 + 27.7165s + 1179.0134}}$$

$$\boxed{T_r = \frac{\pi-\theta}{w_d} = 0.063 \text{ sec}}$$

$$\boxed{T_s = \frac{4}{\zeta w_n} = 0.29 \text{ sec}}$$

open loop T.F $\boxed{G(s) = \dfrac{1179.0134}{s(s+27.7165)}}$ $H(s) = 1$