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Internal Assesment Test - III

Sub:	Engineering Statistics and Linear Algebra							Code	18EC44
Date:	27.08..2022	Duration:	90 mins	Max Marks:	50	Sem:	I	Branch:	EC

	Answer any five questions	Marks	CO	RBT
1	The joint pmf of a bivariate random variable is given by $p(x,y)=\begin{cases} k(x+2y), x=1,2; y=1,2 \\ 0, otherwise \end{cases}$ Where k is a constant. Find the value of k. Find the marginal pmf of X and Y. Are X and Y independent?	10	CO2	L3
2	Prove that correlation coefficient $\rho_{XY} = \pm 1$	10	CO2	L3
3	Consider the two dimensional random variables X and Y, related to two dimensional random variables P and Q by P = 4X+2Y, Q=X+2Y, X and Y have zero means and $\sigma_X^2 = 9, \sigma_Y^2 = 4, \rho_{XY} = -0.5.$ Obtain ρ_{PQ}	10	CO2	L3

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4	Define autocorrelation and state the properties of ACF.	5+5	CO3	L3
5	Show that the random process $X(t) = 10\cos(100t + \theta)$ is wide sense stationary if it is uniformly distributed in the range $-\pi$ to π.	10	CO3	L3
6.	X(t), Y(t) are zero mean jointly WSS process. The random process $Z(t) = 4X(t) + 6Y(t)$. Find the correlation function $R_Z(\tau)$, $R_{ZX}(\tau)$ $R_{YZ}(\tau)$, $R_{XZ}(\tau)$,	10	CO3	L3
7.	.Consider the random experiment of tossing the fair coin twice. Let the random variables X and Y be defined as follows: X(s) is 1 when two coins behave in the same way and it is zero it it is otherwise. Y(s) is the number of tails observed. Find R_X, R_Y, joint pmf and conditional pmf.	10	CO2	L3

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Q 1 $p(x,y) = \begin{cases} k(x+2y) & x=1,2;\ y=1,2 \\ 0 & \text{otherwise} \end{cases}$ (1)

We K.T $\sum_{y=1}^{2}\sum_{x=1}^{2} k(x+2y) = 1$

$k\left[(1+2(1)) + (1+2(2)) + (2+2(1)) + (2+2(2))\right] = 1$

$k\left[3+5+4+6\right] = 1 \Rightarrow k = \frac{1}{18}$

$p(x,y) = \begin{cases} \frac{1}{18}(x+2y) & x=1,2 \quad y=1,2 \\ 0 & \text{otherwise} \end{cases}$

$p(1,1) = \frac{1}{18}(1+2)$ $p(1,2) = \frac{1}{18}(1+2(2))$

$p(2,1) = \frac{1}{18}(2+2(1))$ $p(2,2) = \frac{1}{18}(2+2(2))$

x \ y	1	2	
1	3/18	5/18	8/18
2	4/18	6/18	10/18
	7/18	11/18	1

M.D of X

X	1	2
P(X)	8/18 (P_1)	10/18 (P_2)

M.D of Y

Y	1	2
P(Y)	7/18 (Q_1)	11/18 (Q_2)

$p_{11} = P_1 Q_1$

$\frac{3}{18} \neq \frac{8}{18}\,\frac{7}{18}$

$p_{21} = P_2 Q_1$

$\frac{4}{18} \neq \frac{10}{18}\,\frac{7}{18}$

$p_{12} = P_1 Q_2$

$\frac{5}{18} \neq \frac{8}{18}\,\frac{11}{18}$

$p_{22} = P_2 Q_2$

$\frac{6}{18} \neq \frac{10}{18}\,\frac{11}{18}$

X & Y are not independent

Q2 ②

$Y = aX + b \Rightarrow E(Y) = aE(X) + b$

$XY = aX^2 + bX$

$E(XY) = E(aX^2 + bX)$

$= aE(X^2) + bE(X)$

$\text{cov}(XY) = \sigma_{XY} = E(XY) - E(X)E(Y)$

$= aE(X^2) + bE(X) - E(X)(aE(X) + b)$

$= aE(X^2) - a(E(X))^2 = a(E(X^2) - (E(X))^2)$

$= a\sigma_X^2$

$Y = aX + b \qquad \sigma_Y = |a|\sigma_X$

$$\rho_{XY} = \frac{\text{cov}(XY)}{\sigma_X \sigma_Y} = \frac{a\sigma_X^2}{\sigma_X |a| \sigma_X} = \frac{a}{|a|} = \pm 1$$

$$= \begin{cases} +1 & a > 0 \\ 0 & a = 0 \\ -1 & a < 0 \end{cases}$$

$\rho_{XY} = \pm 1$

$x(t) = 10\cos(100t + \theta)$

$(\ldots + \tau) + \theta)$

Q3 $P = 4X + 2Y \quad Q = X + 2Y \quad \sigma_X^2 = 9 \quad \sigma_Y^2 = 4$

$\rho_{XY} = -0.5$

$P = aX + bY \quad Q = cX + dY$

$a = 4 \quad b = 2 \quad c = 1 \quad d = 2$

$\mu_X = 0 \quad \mu_Y = 0 \qquad \sigma_X^2 = 9 \quad \sigma_Y^2 = 4 \quad \rho_{XY} = -0.5$

$\sigma_X = 3 \quad \sigma_Y = 4$

$$\rho_{XY} = \frac{\operatorname{cov}(XY)}{\sigma_X \sigma_Y} \Rightarrow \operatorname{cov}(XY) = \rho_{XY}\sigma_X\sigma_Y$$

$$= (-0.5)(3)(4)$$

$$= -6$$

Variance of P

$$\sigma_P^2 = a^2\sigma_X^2 + 2ab\operatorname{cov}(XY) + b^2\sigma_Y^2$$

$$= 16(9) + 2(4)(2)(-6) + 4(4)$$

$$= 144 - 96 + 16 = 64$$

Var of Q

$$\sigma_Q^2 = c^2\sigma_X^2 + 2cd\operatorname{cov}(XY) + d^2\sigma_Y^2$$

$$= 1^2(9) + 2(1)(2)(-6) + 2^2(4)$$

$$= 9 - 24 + 16 = 1$$

$$\operatorname{cov}(PQ) = ac\sigma_X^2 + (bc + ad)\operatorname{cov}(XY) + bd\sigma_Y^2$$

$$= (4)(1)(9) + ((2)(1) + (4)(2))(-6) + (2)(2)(4)$$

$$= 36 + (-48) + 16 = 4$$

$$\rho_{PQ} = \frac{\operatorname{cov}(PQ)}{\sigma_P\sigma_Q} = \frac{4}{\sqrt{64}\sqrt{1}} = \frac{4}{8} = \frac{1}{2}$$

$$= 0.5$$

(7)

5 $f(\theta) = \frac{1}{2\pi}, \quad 0 < \theta < 2\pi \qquad -\pi < \theta < \pi$

$$E[x(t)] = E[10\cos(100t+\theta)]$$

$$= 10\int_{-\pi}^{\pi} \cos(100t+\theta)\,d\theta$$

$$= 10\left[\frac{\sin(100t+\theta)}{1}\right]_{\theta=-\pi}^{\pi}$$

$$= 10\left[\sin(100t+\theta)\right]_{\theta=0}^{2\pi}$$

$$= 10\left[\sin(2\pi+100t) - \sin 100t\right]$$

$= 0$ a constant

$$E[x(t_1)\,x(t_2)] = E\left[10\cos(100t_1+\theta)\;10\cos(100t_2+\theta)\right]$$

$$= 100\,E\left[\cos(100t_1+\theta)\cos(100t_2+\theta)\right]$$

$$= 100\,E\,\frac{1}{2}\left[\cos(100(t_1+t_2)+2\theta) + \cos 100(t_1-t_2)\right]$$

$$= 50\,E\left[\cos(100(t_1+t_2)+2\theta) + \cos 100(t_1-t_2)\right]$$

$$= 50\int_{\theta=0}^{2\pi}\frac{1}{2\pi}\left[\cos(100(t_1+t_2)+2\theta) + \cos(100(t_1-t_2))\right]d\theta$$

$$= 50\cos 100(t_1-t_2)$$

$$50\cos\tau$$

$\{X(t)\}$ is a WSS process

(5)

6) $Z(t) = 4X(t) + 6Y(t)$

$Z(t+\tau) = 4X(t+\tau) + 6Y(t+\tau)$

$R_Z(\tau) = E[Z(t)\, Z(t+\tau)]$

$= E[\{4X(t) + 6Y(t)\}\{4X(t+\tau) + 6Y(t+\tau)\}]$

$= E[16X(t)X(t+\tau) + 36Y(t)Y(t+\tau) + 24Y(t)X(t+\tau) + 24X(t)Y(t+\tau)]$

$= 16E[X(t)X(t+\tau)] + 36E[Y(t)Y(t+\tau)] + 24E[Y(t)X(t+\tau)] + 24E[X(t)Y(t+\tau)]$

$= 16R_X(\tau) + 36R_Y(\tau) + 24R_{XY}(\tau) + 24R_{YX}(\tau)$

$R_{ZX}(\tau) = E[(4X(t) + 6Y(t))\, X(t+\tau)]$

$= E[4X(t)X(t+\tau) + 6Y(t)X(t+\tau)]$

$= 4E[X(t)X(t+\tau)] + 6E[Y(t)X(t+\tau)]$

$= 4R_{XX}(\tau) + 6R_{YX}(\tau)$

$R_{XZ}(\tau) = E[X(t)(4X(t+\tau) + 6Y(t+\tau))]$

$= 4E[X(t)X(t+\tau)] + 6E[X(t)Y(t+\tau)]$

$= 4R_X(\tau) + 6R_{XY}(\tau)$

(6)

$R_{zy}(\tau) = E[(4X(t) + 6Y(t))\, Y(t+\tau)]$

$= 4E[X(t)Y(t+\tau)] + 6E[Y(t)\, Y(t+\tau)]$

~~$= 4R_x(\tau) + 6R_{xy}(\tau)$~~

R $= 4R_{xy}(\tau) + 6R_y(\tau)$

$R_{yz}(\tau) = E[Y(t)(4X(t+\tau) + 6Y(t+\tau))]$

$= 4E[Y(t)X(t+\tau)] + 6E[Y(t)\, Y(t+\tau)]$

$= 4R_{yx}(\tau) + 6R_y(\tau)$

Q7 $S = \{HH, HT, TH, TT\}$

$X(s) = \begin{cases} 1 & s = HH, TT \\ 0 & s = HT, TH \end{cases}$

$Y(s) \rightarrow$ no. of tails $\quad R_x = \{0, 1\}$ $\quad R_y = \{0, 1, 2\}$

$p_{00} = P(X=0, Y=0) = P\{(HT, TH) \cap (HH)\}$
$= P(\phi) = 0$

$p_{01} = P(X=0, Y=1) = P\{(HT, TH) \cap (HT, TH)\}$
$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$p_{02} = P(X=0, Y=2) = P\{(HT, TH) \cap TT\}$
$= P(\phi) = 0$

(7)

$p_{10} = P(X=1, Y=0) = P\{(HH\ TT) \cap HH\}$
$= \frac{1}{4}$

$p_{11} = P(X=1, Y=1) = P\{(HH, TT) \cap (HT, TH)\}$
$= P(\phi) = 0$

$p_{12} = P(X=1, Y=2) = P\{(HH, TT) \cap TT\}$
$= \frac{1}{4}$

X \ Y	0	1	2	
0	0	1/2	0	1/2
1	1/4	0	1/4	1/2
	1/4	1/2	1/4	1

M.D of X

X	0	1	
P(X)	1/2 (P_1)	1/2 (P_2)	1

M.D of Y

Y	0	1	2	
P(Y)	1/4 (Q_1)	1/2 (Q_2)	1/4 (Q_3)	1

$p_{00} = 0 \neq \underset{P_1}{\frac{1}{2}} \cdot \underset{Q_1}{\frac{1}{4}}$

$p_{01} = \frac{1}{2} \neq \underset{P_1}{\frac{1}{2}} \cdot \underset{Q_2}{\frac{1}{2}}$

X & Y are not independent.

$p_{Y/X} = \frac{p_{XY}(x, y)}{p_X(x)} \qquad p_X(x) \geq 0$

Case 1 $x = 0$

$$p_{Y/X} = \frac{p_{XY}(0, y)}{p_X(0)} = \frac{p_{XY}(0, y)}{1/2}$$

$y = 1$ $p_{Y/X}$ $\frac{p_{XY}(0,1)}{1/2} = \frac{1/2}{1/2} = 1$

$y = 2$ $\frac{p_{XY}(0,2)}{1/2} = \frac{0}{1/2} = 0$

$y = 0$ $\frac{p_{XY}(0,0)}{1/2} = \frac{0}{1/2} = 0$

Case 2 $x = 1$

$$p_{Y/X} \quad \frac{p_{XY}(1, y)}{p_X(1)} = \frac{p_{XY}(1, y)}{1/2}$$

$y = 0$ $\frac{p_{XY}(1,0)}{p_X(1)} = \frac{1/4}{1/2} = 1/2$

$y = 1$ $\frac{p_{XY}(1,1)}{p_X(1)} = \frac{0}{1/2} = 0$

$y = 2$ $\frac{p_{XY}(1,2)}{p_X(1)} = \frac{1/4}{1/2} = 1/2$

$$p_{X/Y} = \frac{p_{XY}(x, y)}{p_Y(y)} \quad p_Y(y) > 0$$

Case 1 $y = 0$ $\frac{p_{XY}(x, 0)}{p_Y(0)} = \frac{p_{XY}(x, 0)}{1/4}$

$x=0$ $\quad \frac{p_{XY}(0,0)}{1/4} = \frac{0}{1/4} = 0$

$x=1$ $\quad \frac{p_{XY}(1,0)}{1/4} = \frac{1/4}{1/4} = 1$

Case 2 $\quad y=1$ $\quad \frac{p_{XY}(x,1)}{p_Y(1)}$

$x=0$ $\quad \frac{p_{XY}(0,1)}{1/2} = \frac{1/2}{1/2} = 1$

$x=1$ $\quad \frac{p_{XY}(1,1)}{1/2} = \frac{0}{1/2} = 0$

Case 3 $\quad y=2$ $\quad \frac{p_{XY}(x,2)}{p_Y(2)}$

$x=0$ $\quad \frac{p_{XY}(0,2)}{1/4} = \frac{0}{1/4} = 0$

$x=1$ $\quad \frac{p_{XY}(1,2)}{1/4} = \frac{1/4}{1/4} = 1$