CMR INSTITUTE OF TECHNOLOGY	USN	

Internal Assesment Test – 2

Sub:	Microwave and Antennas							**Code:**	18EC63
Date:	09/06/2022	**Duration:**	90 mins	**Max Marks:**	50	**Sem:**	6	**Branch:**	ECE

Answer Any FIVE FULL Questions

	Questions	Marks	CO	RBT
1	(a) Prove that it is not possible to construct to perfectly matched lossless reciprocal three port junction. (b) Derive S-matrix of E-plane Tee junction	[10]	CO2	L4
2	What is S-parameter? State and prove unitary properties of S-matrix.	[10]	CO2	L2
3	(a) A 20 mW signal is fed into the collinear arm of a lossless H-Plane Tee. Calculate the Power delivered through each port when others ports are terminated with matched load. (b) Define the following losses in a microwave network in terms of S – parameters: (i) Insertion loss, (ii) Transmission loss, (iii) Reflection loss, (iv) Return loss.	[10]	CO2	L3
4	What is a magic tee? Derive the S-matrix of magic tee. Mention its applications.	[10]	CO2	L3
5	With the help of a neat diagram, explain the working of a precision phase shifter. **OR** With the help of a neat diagram, explain the working of an attenuator.	[10]	CO2	L2
6.	Drive the relation between [S] and [Z] Matrix	[10]	CO3	L3
7.	Drive the relation between [S] and ABCD Parameter Matrix	[10]	CO3	L3
8.	Two transmission line of characteristic impedance Z and Z2 are joined at plane pp’. Express S-parameter in terms of impedances when each line is matched terminated.	[10]	CO3	L3

CCI

Q1.

(a) Consider a S-matrix,

$$S = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

$$\xrightarrow[\text{matched}]{\text{Perfectly}} = \begin{pmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{pmatrix} \quad [\because \text{diagonal elements are zero}]$$

$$\xrightarrow[\text{Condition}]{\text{reciprocal}} \begin{pmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{pmatrix} \quad [\because S_{ij} = S_{ji}]$$

$$\therefore \quad S_{12} = S_{21} \; ; \; S_{13} = S_{31} \; ; \; S_{23} = S_{23}$$

$$\sum_{K=\phi}^{N} S_{Ki} S^{*}_{Kj} = 0$$

→ Consider any two columns

$$0 \times S_{12}^{*} + S_{12} \times 0 + S_{13} \times S_{23}^{*} = 0 \quad \text{——— } (1)$$

$$0 \times S_{13}^{*} + S_{12} \times S_{23}^{*} + S_{13} \times 0 = 0 \quad \text{——— } (2)$$

$$S_{12} \times S_{13}^{*} + 0 \times S_{23}^{*} + S_{23} \times 0 = 0 \quad \text{——— } (3)$$

$$S_{13} \times S_{23}^{*} = 0$$

$$\boxed{S_{13} = 0} \Rightarrow \boxed{S_{23}^{*} = 0}$$

from $\sum_{K=1}^{N} S_{ik} \cdot S_{jk}^* = 1$

$$S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1 \quad \text{---} \quad (4)$$

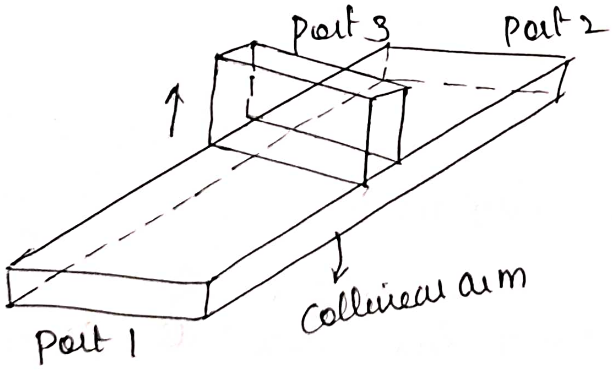
$$S_{12} \cdot S_{12}^* + S_{23} \cdot S_{23}^* = 1 \quad \text{---} \quad (5)$$

$$S_{13} \cdot S_{13}^* + S_{23} \cdot S_{23}^* = 1 \quad \text{---} \quad (6)$$

from above eq^ns we can say that it is not possible to construction perfectly matched lossless reciprocal three port junction.

Q 1.

(b) S-matrix of E-plane Tee junction:



$$[S] = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

1. $S_{31} = S_{32}$

2. $S_{33} = 0$ [perfectly matched port 3]

3. $S_{23} = S_{32} = -S_{13} = -S_{31}$; $S_{13} = S_{31}$.

$$[S] = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{pmatrix}$$

$$[S][S^*] = [U]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{--- (1)}$$

$$S_{12} \cdot S_{12}^* + S_{22} \cdot S_{22}^* + S_{13} \cdot S_{13}^* = 1$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{--- (2)}$$

$$S_{13} \cdot S_{13}^* + S_{13} \cdot S_{13}^* = 1$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{--- (3)}$$

$$2|S_{13}|^2 = 1$$

$$|S_{13}| = 1/\sqrt{2} \Rightarrow \boxed{S_{13} = 1/\sqrt{2}}$$

from eq^n ① & ②

$$|S_{11}| = |S_{22}| \Rightarrow \boxed{S_{11} = S_{22}}$$

$$2|S_{11}|^2 = 1 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1 - \frac{1}{2}$$

$$2|S_{11}|^2 = \frac{1}{2} \Rightarrow \boxed{S_{11} = \frac{1}{2}}$$

$$[S] = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{pmatrix}$$

Q2.

S-parameter: It is a matrix which has $n \times n$ matrix no of rows & no of columns are equal. The diagonal elements are 0, ~~matched~~ for perfectly matched condition. Symmetry property and Unitary property for loss less matrix.

Statement of Unitary property: The property states that the sum of items in the matrix and the its conjugate product is sum is equal to 1

$$\boxed{\sum_{K=1}^{N} S_{KI} S_{Kj}^{*} = 1}$$

proof:

→ Assume the impedence of all the ports are identical

→ $Z_{ON} = 1$ (To be made)

→ The current and voltage eq^n is given by,

$$V_n = V_n^+ + V_n^- \quad \text{——— (1)}$$

$$I_n = I_n^+ - V_n^- \quad \text{——— (2)}$$

add eq^n (1) & (2)

$$V_n + I_n = 2V_n^+$$

$$2[V^+] = [V] + [I]$$

$$[V^+] = \frac{1}{2}\{[V] + [I]\}$$

$$[V^+] = \frac{1}{2}\{[Z] + [U]\}[I] \quad \text{——— (3)}$$

Subtract eq^n (2) & (1)

$$[V^-] = \frac{1}{2}\{[Z] - [U]\}[I] \quad \text{——— (4)}$$

divide eq^n (4)/(3)

$$\frac{[V^-]}{[V^+]} = \{[Z] - [U]\}\{[Z] + [U]\}^{-1}$$

$$[V^-] = [V^+]\{[Z] - [U]\}\{[Z] + [U]\}^{-1} \quad \text{——— (5)}$$

$$[S] = \{[z]+[U]\}\{[z]-[U]\}^{-1}$$

taking transpose on both side

$$[S]^t = \{([z]+[U])^t\}\{([z]-[U])^t\}^{-1}$$

Consider power

$$P_{av} = \frac{1}{2} Re\{[V^+]^t [I]^*\}$$

$$= \frac{1}{2} Re\{([V^+]^t + [V^-]^t)([V^+]^* - [V^-]^*)\}$$

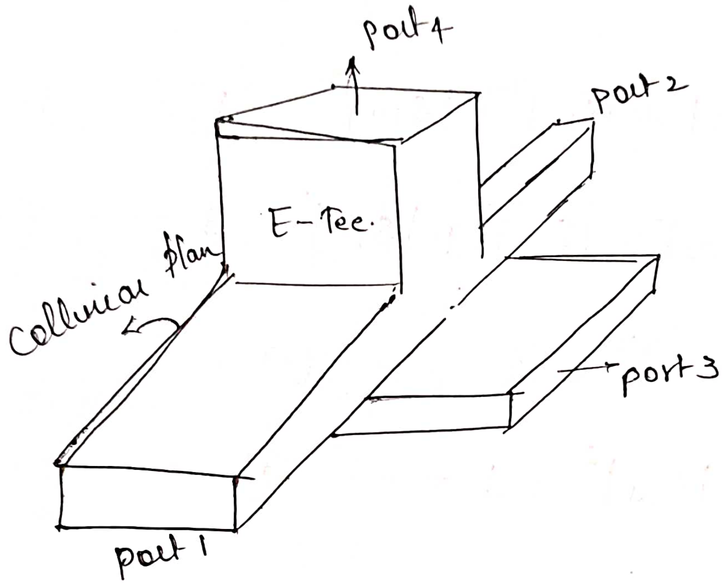
$$= \underbrace{\frac{1}{2}[[V^+]^t [V^+]^*]}_{\text{power produced but incident}} - \underbrace{\frac{1}{2}[V^-]^t [V^-]^*}_{\text{reflected power.}}$$

$$\sum_{K=1}^{N} S_{Ki} S_{Kj}^* = \delta_{ij}$$

where $\delta_{ij} = \begin{cases} 1 ; & i=j \\ 0 ; & i \neq j \end{cases}$

$$\Rightarrow \underbrace{\sum_{K=1}^{N} S_{Ki} S_{Kj}^* = 0}_{i \neq j} \quad \& \quad \underbrace{\sum_{K=1}^{N} S_{Ki} S_{Kj}^* = 1}_{i=j}$$

Q4. <u>Magic tee</u> : It is a 4-port network Combination of H tee and E-Tee Junction.. They are Symmetric, perfectly matched System. Is Known as <u>Magic Tee</u>.



1. [S] matrix is 4X4 = $\begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix}$

2. $S_{31} = S_{32}$ (H-Tee)

3. $S_{41} = -S_{41}$ (E-Tee)

4. $S_{33} = S_{44} = 0$ [perfectly matched cond^n].

5. Symmetric $S_{ij} = S_{ji}$

$$[S] = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{pmatrix}$$

$$[S][S^*] = [U]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \text{---} \quad (1)$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \text{---} \quad (2)$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{---} \quad (3)$$

$$|S_{14}|^2 + |S_{14}|^2 = 1 \quad \text{---} \quad (4)$$

from eq^n (3) $\Rightarrow$

$$2|S_{13}|^2 = 1$$

$$\boxed{S_{13} = 1/\sqrt{2}}$$

from eq^n (4) $\rightarrow$

$$2|S_{14}|^2 = 1$$

$$|S_{14}|^2 = 1/2$$

$$\Rightarrow \boxed{S_{14} = 1/\sqrt{2}}$$

from eq^n ① & ②

$$|S_{11}|^2 = |S_{22}|^2 \Rightarrow \boxed{S_{11} = S_{22}}$$

Eq^n ① ⇒

$$2|S_{11}|^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$\boxed{S_{11} = 0} \Rightarrow \boxed{S_{22} = 0}$$

$\boxed{S_{12} = 0}$ &

$$\Rightarrow S\,\text{matrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

Applications :

→ Used in Communication laboratory
→ Digital Communication
→ To transmitte the Signal.

Q6. [S] & [Z] matrix relation.

W.K.T.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \text{ ——— ①}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \text{ ——— ②}$$

} Z-matrix

&

$$V_1 = A V_2 - B I_2 \text{ ——— ③}$$
$$I_1 = C V_2 - D I_2 \text{ ——— ④}$$

} ABCD matrix

rearranging eq^n 2

$$Z_{21} I_1 = V_2 - Z_{22} I_2$$

$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \quad \text{—— } 5$$

Compare eq^n 5 & 4

$$C = \frac{1}{Z_{21}} \quad \& \quad D = \frac{Z_{22}}{Z_{21}}$$

replace 5 in 1 $\Rightarrow$

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left(\frac{Z_{11} Z_{22}}{Z_{21}} + Z_{12}\right) I_2 \quad \text{—— } 6$$

Compare eq^n 6 with. 3

$$A = \frac{Z_{11}}{Z_{21}} \quad \& \quad B = \frac{Z_{11} Z_{22}}{Z_{21}} + Z_{12}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—— } 7$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—— } 8$$

Q7. [S] and [ABCD] matrix

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—— (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—— (2)}$$

$$V_1 = AV_2 - BI_2 \quad \text{—— (3)}$$

$$I_1 = CV_2 - DI_2 \quad \text{—— (4)}$$

rearrange eq^n (2) ⇒

$$V_2 - Z_{22} I_2 = Z_{21} I_1$$

$$I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2 \quad \text{—— (5)}$$

$$\Rightarrow \quad C = \frac{1}{Z_{21}} \quad \& \quad D = \frac{Z_{22}}{Z_{21}}$$

replace (5) in eq^n (1)

$$V_1 = Z_{11}\left[\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right] + Z_{12} I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left(\frac{Z_{11} Z_{22}}{Z_{21}} + Z_{12}\right) I_2 \quad \text{——}$$

Compare (6) & (3)

$$A = \frac{Z_{11}}{Z_{21}} \quad \& \quad B = \frac{Z_{11} Z_{22}}{Z_{21}}$$

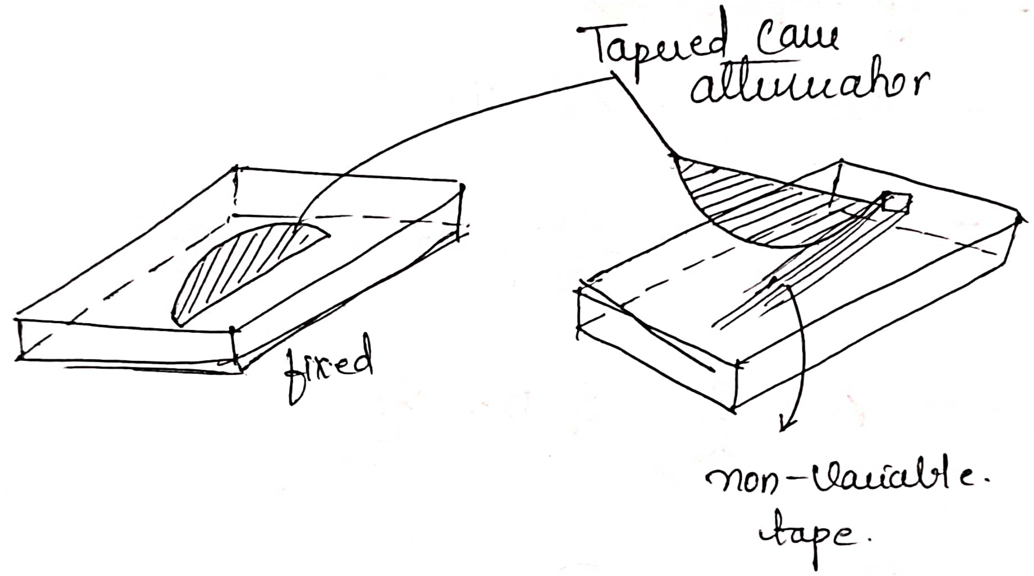
Q5. Attenuator : work as dissipator / variator.

There are two types of attenuators:

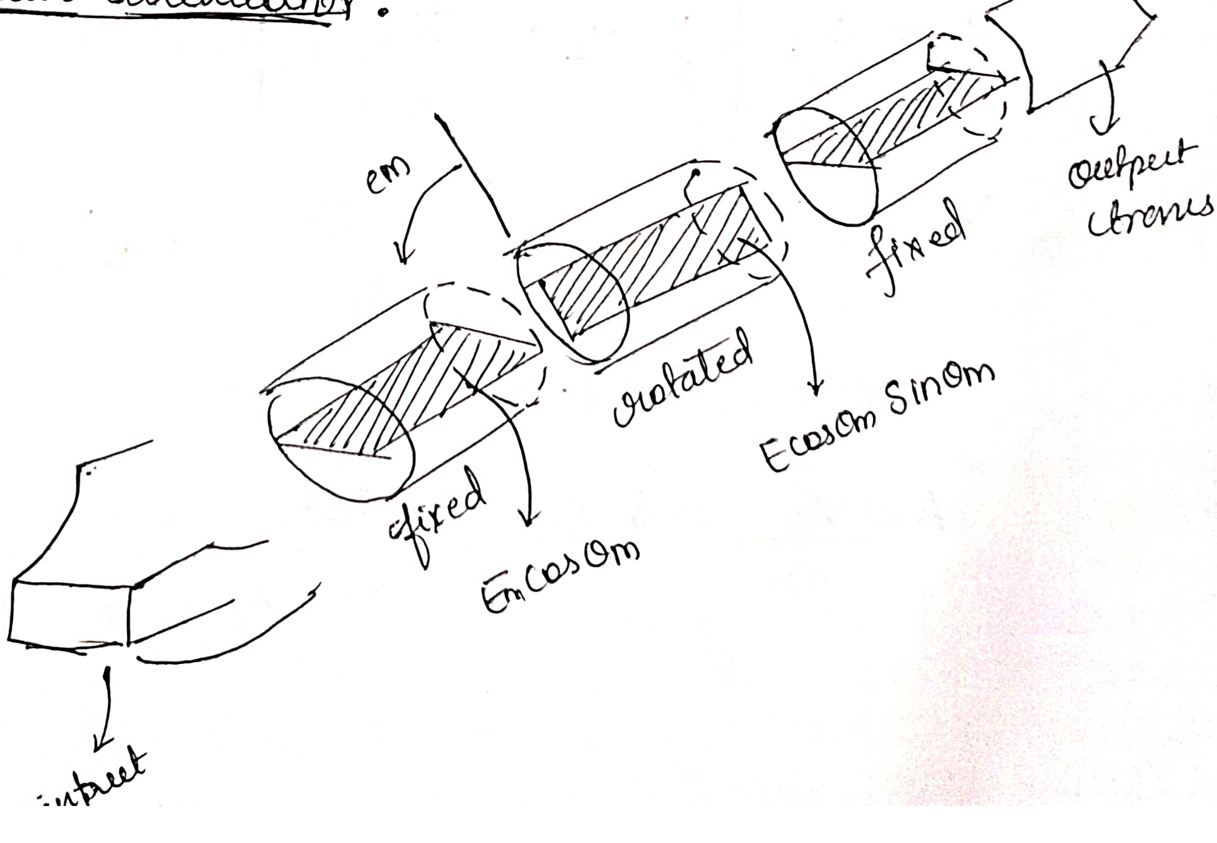
→ Resistive card attenuator.

→ Rotate card attenuator

Resistive card attenuator : It can be fixed / variable.



Rotate card attenuator :



→ There are three main successive parts two fixed and one rotated

→ There are two ~~rec~~ trans. input trans and the output trans where there will less SWR.

→ Attenuation can be controlled by rotating.

→ The mini attenuation can be obtained when $\theta_m = 0°$.

→ The max attenuation can be obtained when $\theta_m = 90°$.

→

part	attenuation value (dB)
fixed	40
Rotated	80.

→ Attenuation can be obtained by formula

$$A_t = 20 \log \left(\frac{1}{\cos^4 \theta_m} \right)$$

—— x ——