Internal Assessment Test 3 – Aug. 2022

Sub:	Control Systems			Sub Code:	18EC43	Branch:	ECE
Date:	27/08/2022	Duration:	90 Minutes	Max Marks: 50	Sem / Sec:	4/A, B, C, D	OBE

	Answer for 50 Marks	MARKS	CO	RBT
1	A closed loop control system has a characteristic equation given below 1. $S^3+4.5S^2+3.5S+1.5 = 0$ 2. $S^4+2S^3+3S^2+4S+5 = 0$ Investigate the stability using Routh-Hurwitz Criterion	10	CO4	L3
2	For the given characteristics equation determine the stability using Routh's array. $F(S)= S^6+3S^5+4S^4+6S^3+5S^2+3S+2=0$ $S = \pm j, \pm j$ Unstable	10	CO4	L3
3	Using Routh's criterion, calculate the range of k for which the system has its closed loop poles more negative than -1. $s'^3 + 7s'^2 + (4+K)s' - 12 + 12K = 0$ $G(S)= k(S+13) / S(S+3) (S+7)$ $0 < K < 8$	10	CO4	L3
4	Explain Routh – Hurwitz criterion for stability of the system and what its limitations are	10	CO4	L2
5	Draw the root locus sketch and comment on the stability of the given system. $\sigma = -2$ $G(S)= K / S(S+2) (S+4)$ BAP $= 0.845$ $60°, 180°, 300°,$ $\pm 2.8j$	20	CO4	L3
6	Draw the root locus sketch and comment on the stability of the given system. BAP $= -0.634$ $G(s)= K (S+2) (S+3) / S(S+1)$ -2.366	20	CO4	L3
7	Sketch the Bode plot for transfer function. $G(S) = 80 / S (S+2) (S+20)$ Comment the stability and determine the Gain Margin, Phase Margin, Gain & Phase Crossover Frequency.	20	CO4	L3
8	Sketch the polar plot for the given transfer function $G(S)H(S) = 1/ S (S+1) (S+1/2)$	10	CO5	L3
9	Obtain the state model for the given system. $Y(S) / U(S) = 24 / S^3 +9S^2+26S+ 24$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}$	10	CO5	L3
10	Find the state transition matrix for the given state equation. $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$	10	CO5	L3

HoD

CCI

IAT - III

2) $F(s) = s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	0	0		

s^3 row: $\boxed{0 \quad 0} \rightarrow$ Row of zeros

$\frac{12-6}{3} = \frac{6}{3}$

$\frac{15-3}{3} = \frac{12}{3}$

$\frac{12-12}{2} = 0$ $\quad \frac{6}{3}$

$\frac{6-6}{}$

Auxilary Equation from s^4 is

$$2s^4 + 4s^2 + 2 = A(s) \rightarrow (1)$$

$$\frac{dA(s)}{ds} = 8s^3 + 8s + 0 \rightarrow \text{New Co-efficients of } s^3\text{-row}$$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	
s^3	8	8	0	
s^2	2	2		
s^1	0	0		

s^1 row: $\boxed{0 \quad 0} \rightarrow$ Row of zeros.

$\frac{32-16}{8} = 2$

$\frac{16}{8} =$

$16 - 16$

$\frac{16-2}{8}$

$\frac{16-16}{2} =$

Auxilary Equation from s^2 =

$$A(s) = 2s^2 + 2 = 0 \rightarrow (2)$$

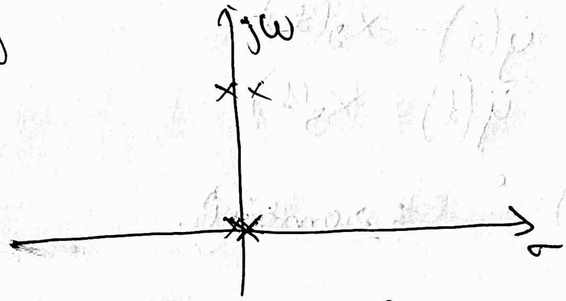
$$\frac{dA(s)}{dt} = 4s + 0$$

From ① & ② $s_{1,2} = \pm j$, $s_{3,4} = \pm j$

+	s^6	1	4	5	2
+	s^5	3	6	3	0
+	s^4	2	4	2	
+	s^3	8	8	0	
+	s^2	2	2		
+	s^1	4	0		
+	s^0	2			

$\frac{8-0}{4}$

Since there is no sign change it is stable but according to the root $s_{1,2,3,4}$ lies only on imaginary axis.



∴ The system is said to be "unstable"

Q. 3 ii) $s = s' - 1 \rightarrow$ Replace s in Eqⁿ (1).

$$(s'-1)^3 + 10(s'-1)^2 + (s'-1)(21+K) + 13K = 0$$

$$(s'-1)\left[s'^2+1-2s'\right] + 10\left[s'^2+1-2s'\right] + \left[21s'+Ks'-21-K\right] + 13K = 0$$

$$s'^3 + s' - \cancel{2s'^2} - s'^2 - 1 + 2s' + 10s'^2 + 10 - 20s' + 21s' + Ks' - 21 - K + 13K$$

$$s'^3 + 7s'^2 + \cancel{(4-K)s' - 12 + 14K = 0}$$

s'^3	$\cancel{1}$	$\cancel{4-K}$	0
$\cancel{s'^2}$	7	$\cancel{-12+14K}$	

$$\frac{\cancel{21-7K+12-14K}}{\cancel{7}}$$

$$\frac{33+21K}{7}$$

$$s'^3 + 7s'^2 + (4+K)s' - 12 + 12K = 0$$

$$\frac{28+7K+12-12K}{7}$$

$$\frac{40-5K}{7}$$

s'^3	1	$4+K$	0
s'^2	7	$-12+12K$	
s'^1	$\boxed{\frac{40-5K}{7} \quad 0}$		
s'^0	$-12+12K$		

$\rightarrow$ Row of zeros

$$\frac{40-5K_{max}}{7} = 0$$

$$K_{max} = \underline{\underline{8}}$$

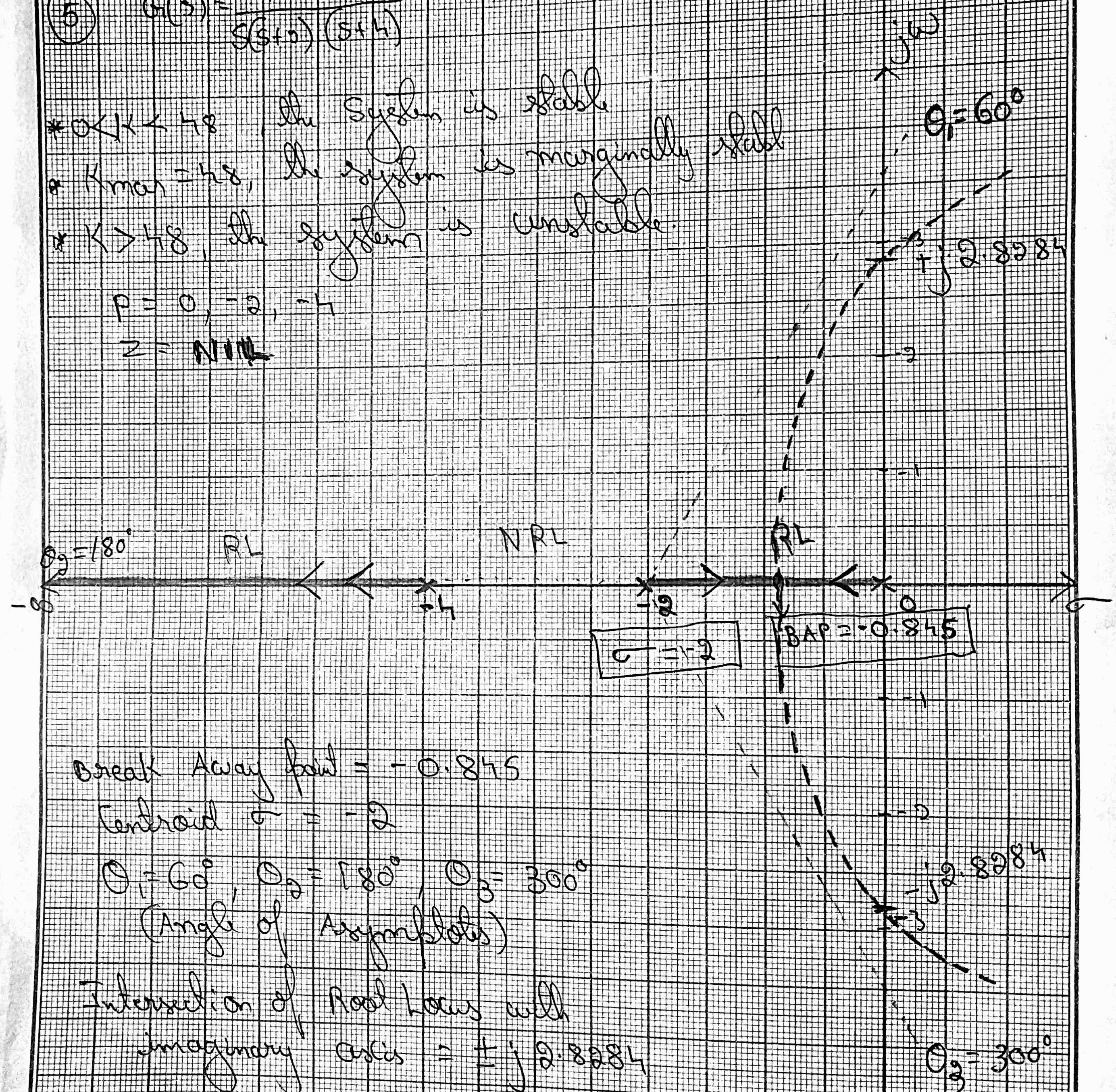
$$-12+12K > 0$$
$$K > 0$$

K range $\Rightarrow 0 < K < 8$

(5) $G(s) = \dfrac{K}{s(s+2)(s+4)}$

* $0 < K < 48$, the system is stable
* $K_{max} = 48$, the system is marginally stable
* $K > 48$, the system is unstable.

$P = 0, -2, -4$
$Z = \text{NIL}$



Break Away point $= -0.845$
Centroid $\sigma = -2$
$\theta_1 = 60°$, $\theta_2 = 180°$, $\theta_3 = 300°$
(Angle of Asymptotes)

Intersection of Root Locus with imaginary axis $= \pm j2.8284$

Batch No.

Date :

Q4) $\dfrac{Y(s)}{U(s)} = \dfrac{24}{s^3+9s^2+26s+24}$

soln. $s^3Y(s)+9s^2(Y(s))+26s(Y(s))+24Y(s)$

$=24U(s)$

$$\frac{d^3y(t)}{dt^3}+9\frac{d^2y(t)}{dt^2}+26\frac{dy}{dt}+24y(t)=24u(t)$$

$x_1(t) = y(t)$ — ①

$x_2(t) = \dot{y}(t)$ — ②

$x_3(t) = \ddot{y}(t)$ — ③

from ①, $\dot{x}_1(t) = \dot{y}(t) = x_2(t)$

from ②, $\dot{x}_2(t) = \ddot{y}(t) = x_3(t)$

from eqn, $\dot{x}_3(t)+9x_3(t)+26x_2(t)+24x_1(t)$

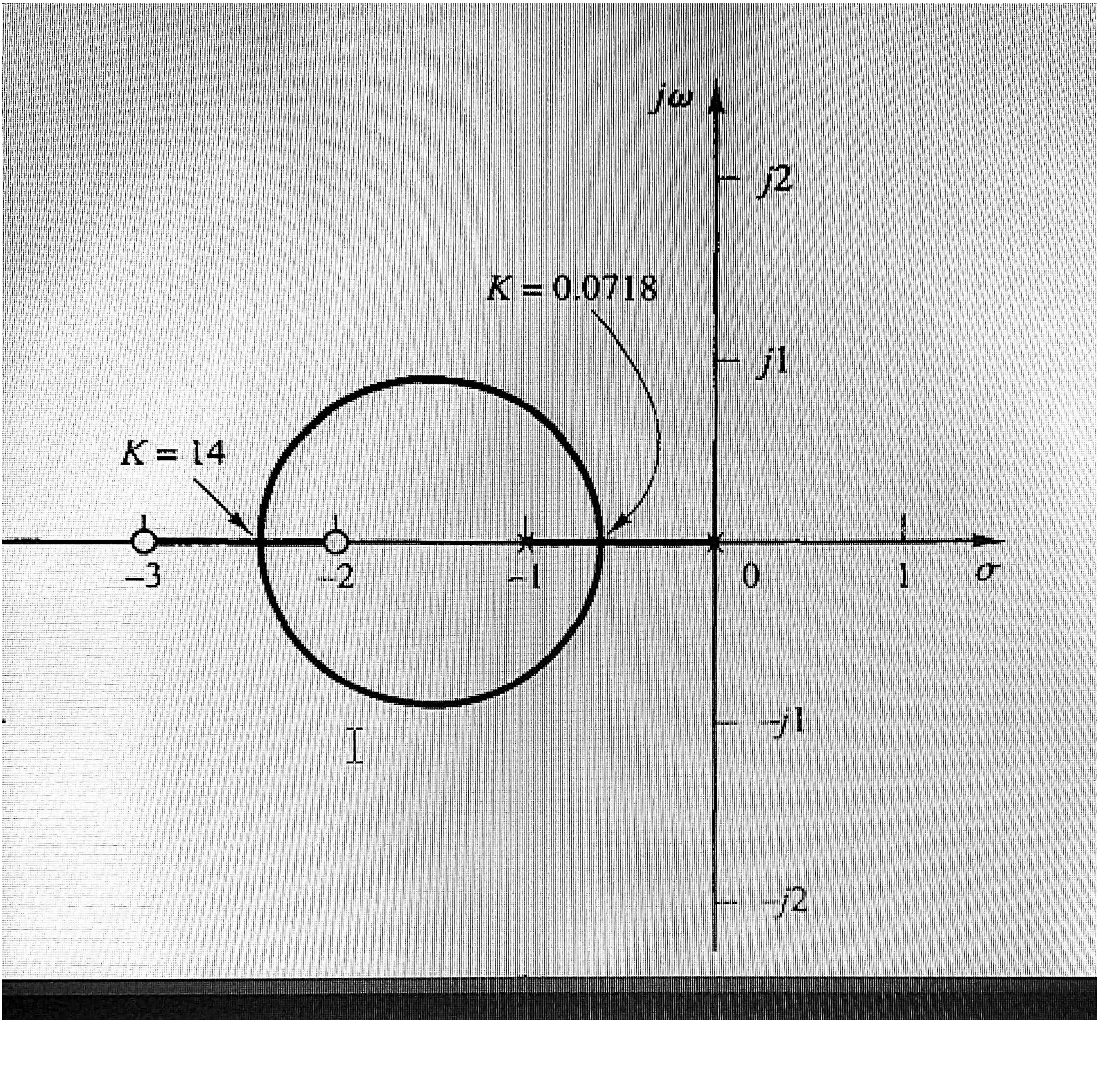
$=24u(t)$

$\dot{x}_3(t) = -9x_3(t)-26x_2(t)-24x_1(t)+24u(t)$

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-24&-26&-9\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\24\end{bmatrix}u(t)$$

$y(t) = x_1(t)$

$$y(t)=\begin{bmatrix}1&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$



jω
j2
K = 0.0718
j1
K = 14
−3
−2
−1
0
1
σ
−j1
−j2

Q.) For the given state equation, obtain state transition matrix and inverse transition matrix.

10

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Soln

$$e^{+At} = L^{-1}\{(SI - A)^{-1}\}$$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$\boxed{[SI - A]^{-1} = \mathrm{Adj}(SI - A) / |SI - A|}$$

$$e^{At} = \phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$\phi^{-1}(t) = \phi(-t) = e^{-At}$$

$$= \begin{bmatrix} 2e^{t} - e^{2t} & e^{t} - e^{2t} \\ -2e^{t} + 2e^{2t} & -e^{t} + 2e^{2t} \end{bmatrix}$$