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INTERNAL ASSESSMENT TEST – III

Sub:	SIGNALS AND SYSTEMS							Code:	18EC45
Date:	29 / 08 / 2022	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	ECE

Answer any 5 full questions

		Marks	CO
1	Investigate the Causality, Stability and Memoryless properties of the following LTI Systems $(i) h(n) = 2\delta(n) - 3\,\delta(n-2) + 4\,\delta(n-3)$ $(ii)\ h(t) = e^{-t}u(t+100)$	[10]	CO2
2	Prove i) $x(n)*\{h_1(n)+h_2(n)\}= x(n)*h_1(n)+x(n)*h_2(n)$, ii) $x(t)*h(t)=h(t)*x(t)$.	[10]	CO3
3	Obtain the Fourier transform of the following signals. $(i)\ \ x(t) = 1$ for $-a \le t \le a$ $(ii) x(t) = e^{-2t}u(t-1)$	[10]	CO4
4	Obtain the DTFT of the following signal. $(i)\ x(n) = a^{\lvert n \rvert}\ \lvert a \rvert < 1$ $(ii)\ x(n) = \cos(\pi n)\left(\frac{1}{4}\right)^n u(n)$	[10]	CO4

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5	Find Inverse FT of the following $(i) X(j\omega) = \frac{j\omega}{6 + 7j\omega - \omega^2}$ $(ii)\ X(j\omega) = \frac{5j\omega - 9 - 2\omega^2}{(j\omega + 4)(3 + 4j\omega - \omega^2)}$	[10]	CO4	L1,L3
6	Find Inverse DTFT of the following $(i) X(e^{j\Omega}) = \frac{3 - \frac{1}{4}e^{-j\Omega}}{1 - \frac{1}{16}e^{-2j\Omega}}$ $(ii)\ X(e^{j\Omega}) = \frac{30 - 5e^{-j\Omega}}{-e^{-2j\Omega} - e^{-j\Omega} + 6}$	[10]	CO4	L1,L3
7	i) State and prove the time scaling property for FT. ii) State and prove the differentiation (Multiplication by ramp) property for DTFT.	[10]	CO4	L1
8	Determine the impulse response of the system described by $\frac{d^2y(t)}{dt} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$	[10]	CO2	L3

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1. a) $h(n)=2\delta(n)-3\,\delta(n-2)+4\,\delta(n-3)$

$$\therefore h(n)=\left\{\begin{matrix}2,0,-3,4\\ \uparrow\end{matrix}\right\}$$

- It's a R.H.S. sequence so causal. (2M)
- Exists for n≠0 so has memory(dynamic) (1M)
- $\sum_{n=0}^{3}|h(n)|$=2+3+4=9 < ∞ , So stable. (2M)

b) . d)$h(t)=e^{-t}u(t+100)$

$$h(t)=\begin{cases}e^{-t} & for\ t\geq -100\\ 0 & for\ t<-100\end{cases}$$

- It's double sided signal so not causal. (2M)
- Exists for t≠0 so has memory(dynamic) (1M)
- $\int_{-100}^{\infty}|e^{-t}|\,dt=e^{-t}|_{-100}^{\infty}$ =e^{100}< ∞ , So stable. (2M)

2. We write $h(t)=h_1(t)*h_2(t)$ (2M)

$$h(t) = \int_{-\infty}^{\infty} h_1(\tau)h_2(t-\tau)d\tau$$

Perform the change of variable,

$$\nu = t - \tau$$
$$d\nu = -d\tau$$

$$= -\int_{\infty}^{-\infty} h_1(t-\nu)h_2(\nu)d\nu$$

$$\int_{-\infty}^{\infty} h_1(t-\nu)h_2(\nu)d\nu$$

$$=$$

$$\int_{-\infty}^{\infty} h_2(\nu)h_1(t-\nu)\, d\nu$$

$= h_2(t) * h_1(t)$(3M)

ii) Statement (2M)

Proof (3M)

3. i). $x(t) = 1$ for $-a \le t \le$ a (5M)

$$\therefore X(j\omega) = \int_{-a}^{a} 1 \cdot e^{-j\omega t} dt$$

Now by definition of FT

$$(j\omega) = \frac{e^{-j\omega t}}{-j\omega}\Bigg|_{-a}^{a}$$

$$X(j\omega) = \frac{e^{-ja\omega}}{-j\omega} - \frac{e^{ja\omega}}{-j\omega}$$

$$= \frac{2}{\omega}\sin a\omega$$

For ω=0 use L'Hopital's rule

ii) $x(t) = e^{-2t}\mathrm{u}(\mathrm{t}-1)$ (5M)

$$\therefore x(t) = \begin{cases} e^{-2t} & for\; t \ge 1 \\ 0 & for\; t < 1 \end{cases}$$

Now by definition of FT

$$X(j\omega) = \int_{1}^{\infty} e^{-2t}\, e^{-j\omega t} dt = \int_{1}^{\infty} e^{-(2+j\omega)t}\, dt$$

$=\frac{e^{-t(2+j\omega)}}{-(2+j\omega)}\Big|_{1}^{\infty}$

$$\therefore X(j\omega) = \frac{e^{-\,(2+j\omega)}}{(2+j\omega)}$$

4. i) $x[n] = a^{|n|}$, $|a| < 1$ (5M)

$$x[n] = \begin{cases} a^n, & n \geq 0 \\ a^{-n}, & n < 0 \end{cases}$$

We have the DTFT as

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} x[n]\, e^{-j\Omega n} + \sum_{n=0}^{\infty} x[n]\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} a^{-n}\, e^{-j\Omega n} + \sum_{n=0}^{\infty} a^{n}\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} \left(ae^{j\Omega}\right)^{-n} + \sum_{n=0}^{\infty} \left(ae^{-j\Omega}\right)^{n}$$

$$= \sum_{m=1}^{\infty} \left(ae^{j\Omega}\right)^{m} + \sum_{n=0}^{\infty} \left(ae^{-j\Omega}\right)^{n}$$

In first summation, $m = -n$

$$n = -\infty, m = \infty,$$

$$n = -1, m = 1,$$

$$= \sum_{m=1}^{\infty} \left(ae^{j\Omega}\right)^{m} + \sum_{n=0}^{\infty} \left(ae^{-j\Omega}\right)^{n}$$

$$= \frac{ae^{j\Omega}}{1 - ae^{j\Omega}} + \frac{1}{1 - ae^{-j\Omega}}$$

$$= \frac{ae^{j\Omega}\left(1 - ae^{-j\Omega}\right) + \left(1 - ae^{j\Omega}\right)}{(1 - ae^{j\Omega})(1 - ae^{-j\Omega})}$$

$$= \frac{ae^{j\Omega} - a^2 + 1 - ae^{j\Omega}}{1 - a(e^{j\Omega} + e^{-j\Omega}) + a^2}$$

$$= \frac{1 - a^2}{1 - 2a\cos\Omega + a^2}$$

$$X(e^{j\Omega}) = \frac{1 - a^2}{1 - 2a\cos\Omega + a^2}$$

ii) Find FT of signal $x(\mathrm{n}) = \cos(\pi n)\left(\frac{1}{4}\right)^n u(n)$ (5M)

As we know that cos $(\pi n) = (-1)^n$

$$\therefore x(\text{n}) = (-1)^n \left(\frac{1}{4}\right)^n u(n)$$

Rewrite as $\therefore x(\text{n}) = \left(-\frac{1}{4}\right)^n u(n)$

We Know that $\left(\frac{1}{4}\right)^n u(n) \overset{DTFT}{\longleftrightarrow} \frac{1}{1-\frac{1}{4}e^{-j\Omega}}$

$$\therefore \left(-\frac{1}{4}\right)^n u(n) \overset{DTFT}{\longleftrightarrow} \frac{1}{1+\frac{1}{4}e^{-j\Omega}}$$

5. $X(j\omega) = \frac{(j\omega)}{(6+7(j\omega)-\omega^2)}$ (5M)

Replace variable jω with v

$X(v) = \frac{(v)}{(v^2+7v+6)}$

$A = \frac{(v)}{(v+1)}\Big|_{v=-6} = \frac{6}{5}$

$\therefore B = \frac{(v)}{(v+6)}\Big|_{v=-1} = \frac{-1}{5}$

$\therefore X(v) = \frac{6/5}{(v+6)} - \frac{1/5}{(v+1)}$

$= \frac{(v)}{(v+6)(v+1)}$

$= \frac{A}{(v+6)} + \frac{B}{(v+1)}$

Replace back variable v with jω

$\therefore X(j\omega) = \frac{6/5}{(j\omega+6)} - \frac{1/5}{(j\omega+1)}$

$$\frac{1/5}{(j\omega+1)} \overset{FT}{\longleftrightarrow} \frac{1}{5}e^{-t}u(t)$$

$$\frac{6/5}{(j\omega+6)} \overset{FT}{\longleftrightarrow} \frac{6}{5}e^{-6t}u(t)$$

$\therefore x(t) = \frac{6}{5}e^{-6t}u(t) - \frac{1}{5}e^{-t}u(t)$

ii) $X(j\omega) = \frac{5((j\omega)+1)}{(j\omega)^2+5(j\omega)+6}$ (5M)

Replace variable jω with v

$$X(v) = \frac{5(v+1)}{v^2+5v+6}$$

$$\therefore X(v) = \frac{5(v+1)}{(v+2)(v+3)}$$

$\therefore X(v) = \frac{A}{(v+2)} + \frac{B}{(v+3)}$

$\therefore A = \frac{5(v+1)}{(v+3)}\Big|_{v=-2} = \frac{5(-2+1)}{(-2+3)} = -5$

$\therefore B = \frac{5(v+1)}{(v+2)}\Big|_{v=-3} = \frac{5(-3+1)}{(-3+2)} = 10$

$\therefore X(v) = \frac{-5}{(v+2)} + \frac{10}{(v+3)}$

Replace back variable v with jω

$\therefore X(j\omega) = \frac{-5}{(j\omega+2)} + \frac{10}{(j\omega+3)}$

$$\frac{-5}{(j\omega+2)} \overset{IFT}{\longleftrightarrow} -5e^{-2t}u(t)$$

$$\frac{10}{(j\omega+3)} \overset{IFT}{\longleftrightarrow} 10e^{-3t}u(t)$$

AND

$\therefore x(t) = -5e^{-2t}u(t) + 10e^{-3t}u(t)$

6. i) b) $X(\Omega) = \frac{3-\frac{1}{4}e^{-j\Omega}}{\left(1-\frac{1}{16}e^{-j2\Omega}\right)}$ (5M)

Replace variable $e^{-j\Omega}$ with v

$X(v) = \frac{3-\frac{1}{4}v}{\left(1-\frac{1}{16}v^2\right)}$

$= \frac{3-\frac{1}{4}v}{\left(1-\frac{1}{4}v\right)\left(1+\frac{1}{4}v\right)}$

$= \frac{A}{1-\frac{1}{4}v} + \frac{B}{1+\frac{1}{4}v}$

$\therefore A = \frac{\left(3-\frac{1}{4}v\right)}{\left(1+\frac{1}{4}v\right)}\Bigg|_{v=4} = \frac{3-1}{1+1} = 1$

$\therefore B = \frac{\left(3-\frac{1}{4}v\right)}{\left(1-\frac{1}{4}v\right)}\Bigg|_{v=-4} = \frac{3+1}{1+1} = 2$

$\therefore X(v) = \frac{1}{1-\frac{1}{4}v} + \frac{2}{1+\frac{1}{4}v}$

$\therefore X\left(e^{-j\Omega}\right) = \frac{1}{1-\frac{1}{4}e^{-j\Omega}} + \frac{2}{1+\frac{1}{4}e^{-j\Omega}}$

Replace back variable v with $e^{-j\Omega}$

$$\frac{1}{1-\frac{1}{4}e^{-j\Omega}} \overset{\mathrm{IFT}}{\longleftrightarrow} \left(\frac{1}{4}\right)^n u(n)$$

$$\frac{2}{1+\frac{1}{4}e^{-j\Omega}} \overset{IFT}{\longleftrightarrow} 2\left(-\frac{1}{4}\right)^n u(n)$$

$$\therefore x(n) = \left(\frac{1}{4}\right)^n u(n) + 2\left(-\frac{1}{4}\right)^n u(n)$$

7. i) Consider the effect of scaling the time variable on the frequency domain representation of a signal. (2M)

Let $z(t) = x(at)$, where a is a constant

$$Z(j\omega) = \int_{-\infty}^{\infty} z(t)e^{-j\omega t}dt$$

$$= \int_{-\infty}^{\infty} x(at)e^{-j\omega t}dt$$

Change the variable $\tau = at$

$$Z(j\omega) = \frac{1}{a}\int_{-\infty}^{\infty} x(\tau)e^{-j\left(\frac{\omega}{a}\right)\tau}d\tau, \quad a > 0$$

$$\frac{1}{a}\int_{-\infty}^{\infty} x(\tau)e^{-j\left(\frac{\omega}{a}\right)\tau}d\tau, \quad a < 0$$

These two integrals may be combined into single integral (3M)

$$Z(j\omega) = \frac{1}{|a|}\int_{-\infty}^{\infty} x(\tau)e^{-j\left(\frac{\omega}{a}\right)\tau}d\tau, \quad a < 0$$

$$\boldsymbol{z(t) = x(at) \leftrightarrow \frac{1}{|a|}X\left(j\left(\frac{\omega}{a}\right)\right)}$$

ii) Statement-2M

Proof-3M

8. $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$

Take FT on both sides

$(j\omega)^2Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) = -j\omega Y(j\omega)$ (2M)

$$H(j\omega) = \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6}$$

Substitute $j\omega = v$

$$H(v) = \frac{-v}{(v)^2 + 5v + 6}$$

$$= \frac{-v}{(v+2)(v+3)}$$

$$= \frac{K_1}{(v+2)} + \frac{K_2}{(v+3)}$$

On solving $K_1 = 2$ and $K_2 = -3$

$H(j\omega) = \frac{2}{(j\omega+2)} - \frac{3}{(j\omega+3)}$ (4M)

Hence $x(t) = 2e^{-2t}u(t) - 3e^{-3t}u(t)$ (4M)