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INTERNAL ASSESSMENT TEST – III

Sub:	DIGITAL COMMUNICATION							Code:	18EC61
Date:	08 / 07 / 2022	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	ECE

Answer any 5 full questions

		Marks	CO	RBT
1	Explain Binary Frequency Shift Keying (BFSK) with neat block diagrams of modulator and demodulator. Explain the decision logic at the demodulator.	[10]	CO2	L2
2	Draw the block diagram of Binary Phase Shift Keying (BPSK) demodulator. Explain the decision logic. Derive an expression for probability of error.	[10]	CO2	L3
3	Explain Quadrature Phase Shift Keying (QPSK) with neat block diagrams of modulator and demodulator. Explain the decision logic at the demodulator.	[10]	CO2	L3

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4a	Consider a signal space with the following basis functions. $\Phi_1(t) = \begin{cases} \frac{1}{\sqrt{2}} \text{ for } 0 \le t \le 2 \\ 0 \text{ otherwise} \end{cases}$ and $\Phi_2(t) = \begin{cases} \frac{1}{\sqrt{2}} \text{ for } 0 \le t < 1 \\ -\frac{1}{\sqrt{2}} \text{ for } 1 \le t \le 2 \\ 0 \text{ otherwise} \end{cases}$ Plot the signals with coordinates $\left(\mathbf{2\sqrt{2}, -2\sqrt{2}}\right)$ and $\left(\mathbf{-2\sqrt{2}, 2\sqrt{2}}\right)$.	[05]	CO3	L2
4b	Prove that the energy of a signal is equal to squared length of the signal vector representing it in the signal space diagram.	[05]	CO3	L2
5	Obtain a set of orthonormal basis functions for the following set of signals. $x_1(t) = \begin{cases} 3 \text{ from } 0 \le t \le 3 \\ 0 \quad \text{otherwise} \end{cases}$ and $x_2(t) = \begin{cases} 3 \text{ from } 0 \le t \le 1 \\ 0 \quad \text{otherwise} \end{cases}$ and $x_3(t) = \begin{cases} 3 \text{ from } 1 \le t \le 3 \\ 0 \quad \text{otherwise} \end{cases}$ Express the signals as a linear combination of basis functions. Draw the signal space diagram (Constellation Diagram).	[10]	CO3	L3
6	What is the need of matched filter in digital communication receiver? Derive the impulse response of a filter matched to the signal $x(t)$.	[10]	CO3	L3

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Scheme Of Evaluation

Internal Assessment Test III – July 2022

Sub:	DIGITAL COMMUNICATION							Code:	18EC61
Date:	08/07/2022	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	ECE

Note: Answer Any 5Questions

<table>
<tr><th colspan="2">Question #</th><th>Description</th><th colspan="2">Marks Distribution</th><th>Max Marks</th></tr>
<tr><td>1</td><td></td><td>Explain Binary Frequency Shift Keying (BFSK) with neat block diagrams of modulator and demodulator. Explain the decision logic at the demodulator.</td><td></td><td>10</td><td>10</td></tr>
<tr><td></td><td></td><td>Modulator
Demodulator
Basis Function
Constellation Diagram
Decision Logic</td><td>2
2
2
2
2</td><td></td><td></td></tr>
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<tr><td></td><td></td><td>Receiver
Decision Rule
Probability of Error</td><td>2
2
6</td><td></td><td></td></tr>
<tr><td>3</td><td></td><td>Explain Quadrature Phase Shift Keying (QPSK) with neat block diagrams of modulator and demodulator. Explain the decision logic at the demodulator.</td><td></td><td>10</td><td>10</td></tr>
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<tr><td></td><td></td><td>$x_1(t)$
$x_2(t)$</td><td>2.5
2.5</td><td>05</td><td></td></tr>
</table>

4	b	Prove that the energy of a signal is equal to squared length of the signal vector representing it in the signal space diagram.		05	
		• Basis functions and Coordinates • Expression for Energy	2 3		
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		• Basis Function $\phi_1(t)$ • Basis Function $\phi_2(t)$ • Linear Combination • Constellation Diagram	2 2 3 3		
6		What is the need of matched filter in digital communication receiver? Derive the impulse response of a filter matched to the signal $x(t)$.		10	10
		• Need of matched filter Decision Rule • Impulse Response of LTI system • Convolution • Impulse Response of Matched filter	2 2 3 3		

SOLUTIONS

1	In binary frequency shift keying (FSK), bit '1' and bit '0' are represented by the following symbols. Bit 1: $s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$, $0 \le t \le T_b$ $f_1 = \frac{n}{T_b}$ n - nonzero integer T_b - bit duration.

Bit 0:

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t), \quad 0 \le t \le T_b$$

$f_2 = \frac{m}{T_b}$

m - non zero integer

$m \neq n$.

To find basis functions

$$\int_0^{T_b} s_1(t)\, s_2(t)\, dt = 0$$

$\Rightarrow s_1(t)$ and $s_2(t)$ are orthogonal to each other.

$\therefore$ Basis function $\phi_1(t) = \frac{s_1(t)}{\sqrt{E_b}}$

$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t)$$

$$0 \le t \le T_b$$

Basis function $\phi_2(t) = \frac{s_2(t)}{\sqrt{E_b}}$

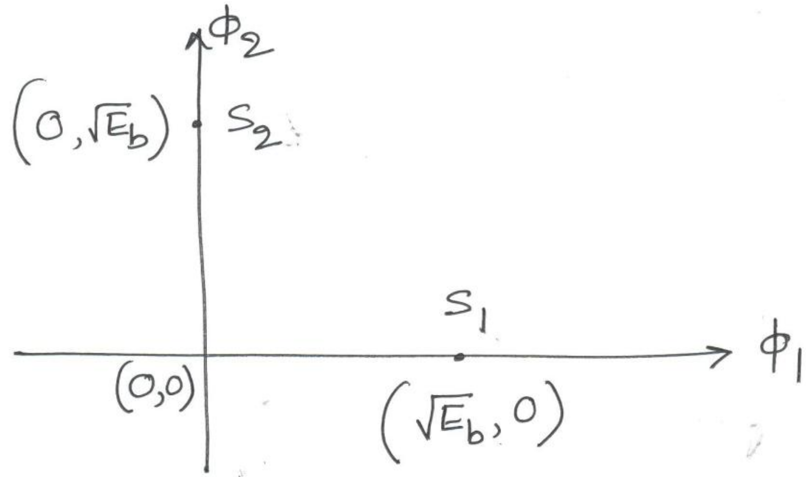
$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t)$$

$$0 \le t \le T_b$$

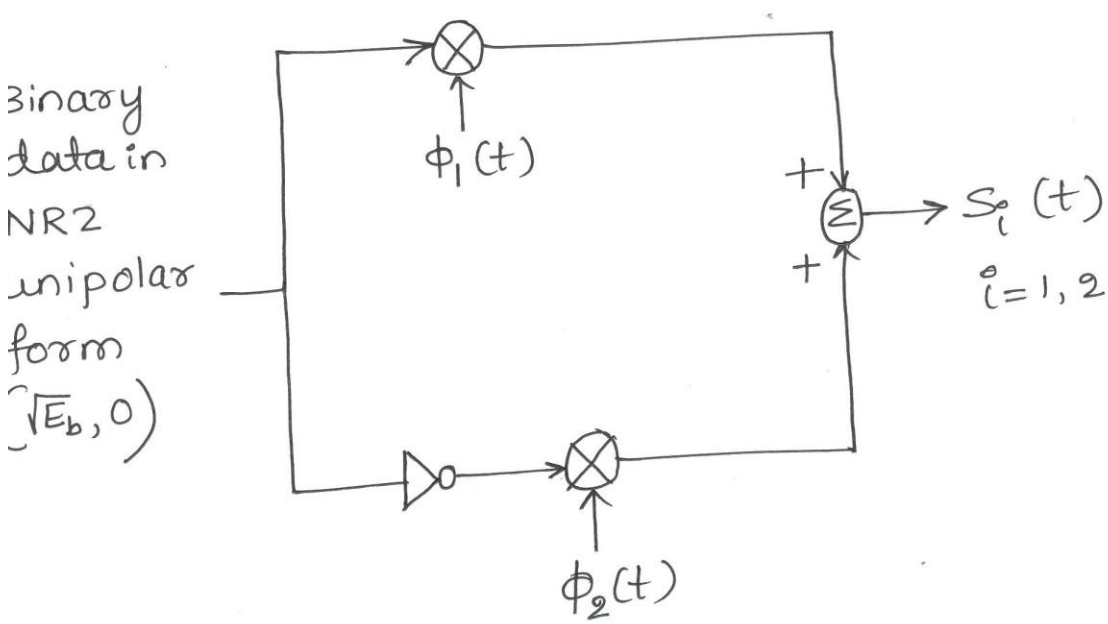
$\therefore \quad s_1(t) = \sqrt{E_b}\,\phi_1(t) + 0\,\phi_2(t) \quad , 0 \le t \le T_b$

$\quad s_2(t) = 0\,\phi_1(t) + \sqrt{E_b}\,\phi_2(t) \quad , 0 \le t \le T_b$

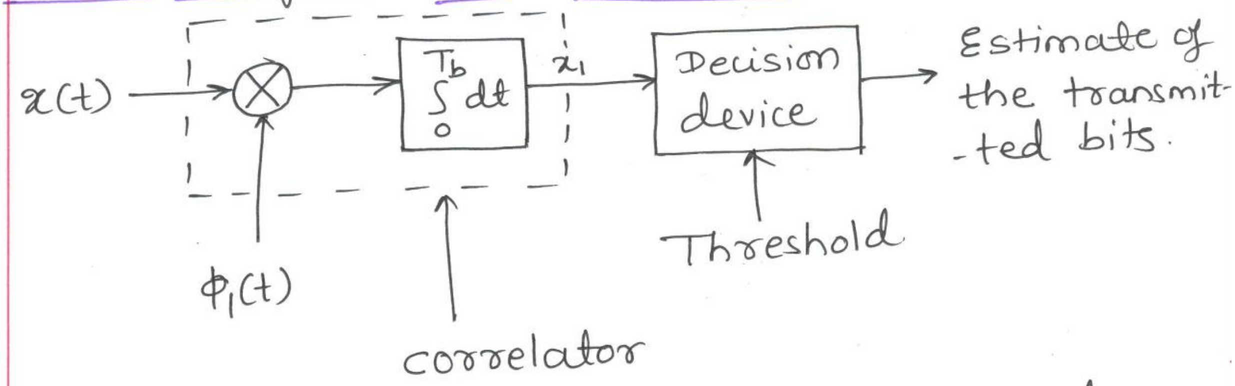
Constellation diagram



Block diagram of transmitter



Block diagram of receiver



Let $x(t)$, $0 \leq t \leq T_b$ be the received signal.

$$x(t) = S_i(t) + w(t), \quad 0 \leq t \leq T_b \qquad \ldots (1)$$
$$i = 1, 2$$

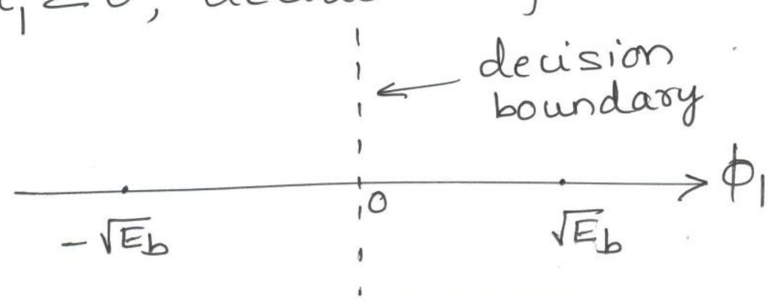
where $w(t)$ represents additive, white Gaussian noise with zero mean and PSD $\frac{N_0}{2}$ ie. variance $\frac{N_0}{2}$.

Decision logic

Let x_1 be the output of the correlator.

If $x_1 > 0$, decide in favor of bit '1'

If $x_1 < 0$, decide in favor of bit '0'



Probability of error

Suppose that bit '0' was transmitted.
ie, $s_2(t)$ was transmitted.
Then, from the block diagram of the receiver, we may write,

output of the correlator,

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} \left[s_2(t) + w(t)\right] \phi_1(t)\,dt$$

$$= \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt + \int_0^{T_b} w(t)\,\phi_1(t)\,dt$$

$$= -\sqrt{E_b} + w_1 \quad \cdots (2)$$

coordinate of $s_2(t)$.

Mean of x_1 when '0' was transmitted.

$$\mu = E[x_1]$$

$$= -\sqrt{E_b} \quad \cdots (3)$$

Variance of x_1 when '0' was transmitted,

$$\sigma^2 = VAR[W_1]$$
$$= \frac{N_0}{2} \quad \cdots (4)$$

($\because$ variance does not change by the addi-tion of a constant to a random varia-ble)

$\therefore$ Probability density function (PDF) of output of correlator when bit '0' was transmitted,

$$f_{X_1}(x_1/0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1+\sqrt{E_b})^2}{N_0}} \quad \cdots (5)$$

Wrong decision is made when $S_2(t)$ was transmitted and $x_1 > 0$.

$\therefore$ Probability of error when bit '0' was transmitted,

$$P_e(0) = P(X_1 > 0/0)$$
$$= \int_0^\infty f_{X_1}(x_1/0)\, dx_1$$

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$$= \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1+\sqrt{E_b})^2}{N_0}} dx_1 \quad \ldots (6)$$

We know that,

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-\frac{z^2}{2}} dz \quad \ldots (7)$$

Let us represent $P_e(0)$ in terms of Q function.

Put $$\frac{(x_1+\sqrt{E_b})^2}{N_0} = \frac{z^2}{2} \quad \ldots (8)$$

ie, $$\frac{x_1+\sqrt{E_b}}{\sqrt{N_0}} = \frac{z}{\sqrt{2}}$$

$$\therefore \frac{dx_1}{\sqrt{N_0}} = \frac{dz}{\sqrt{2}}$$

$$\therefore dx_1 = \sqrt{\frac{N_0}{2}} dz \quad \ldots (9)$$

when $x_1 = 0$, $z = \sqrt{\frac{2E_b}{N_0}} \quad \ldots (10)$

when $x_1 = \infty$, $z = \infty \quad \ldots (11)$

Using (8), (9), (10), (11), we may write (6) as

$$P_e(0) = \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{z^2}{2}} \sqrt{\frac{N_0}{2}}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} e^{-\frac{z^2}{2}}\, dz$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (12)$$

Similarly, we may prove that, probability of error when bit '1' was transmitted,

$$P_e(1) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (13)$$

$\therefore$ Average probability of error

$$= \frac{1}{2} P_e(0) + \frac{1}{2} P_e(1)$$

(Assuming equiprobable 0s & 1s)

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (14)$$

3

one of the four equally spaced values (30)

such as $\frac{\pi}{4}, 3\frac{\pi}{4}, 5\frac{\pi}{4}, 7\frac{\pi}{4}$.

For this set of values, we may define the transmitted signal as

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], \quad 0 \le t \le T$$

$$i = 1, 2, 3, 4$$

$$f_c = \frac{n}{T}$$

n-non-zero integer

Here, T is the symbol duration and E is the energy of each symbol.

Each possible value of the phase corre-sponds to a pair of bits (dibit).

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t)$$

$$0 \le t \le T$$

$E = 2E_b$ and $T = 2T_b$ $\quad i = 1, 2, 3, 4$.

Basis functions are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \quad 0 \le t \le T$$

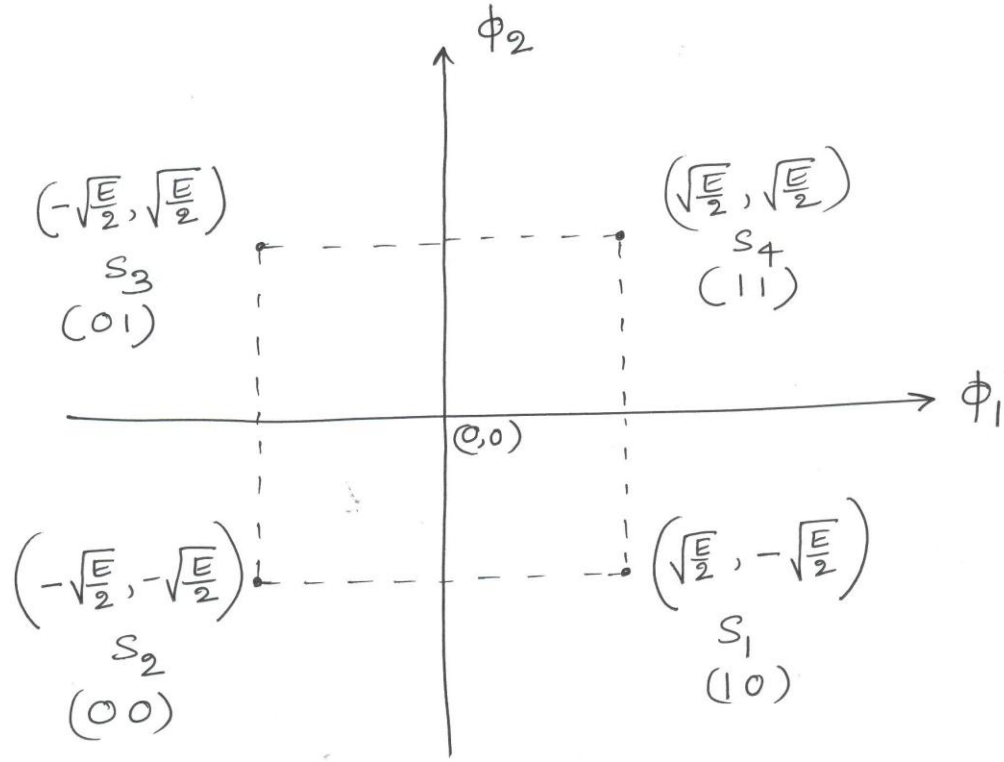
$$\therefore S_i(t) = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\phi_1(t) - \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\phi_2(t), \quad 0 \le t \le T$$

$\therefore$ The coordinates of message points are

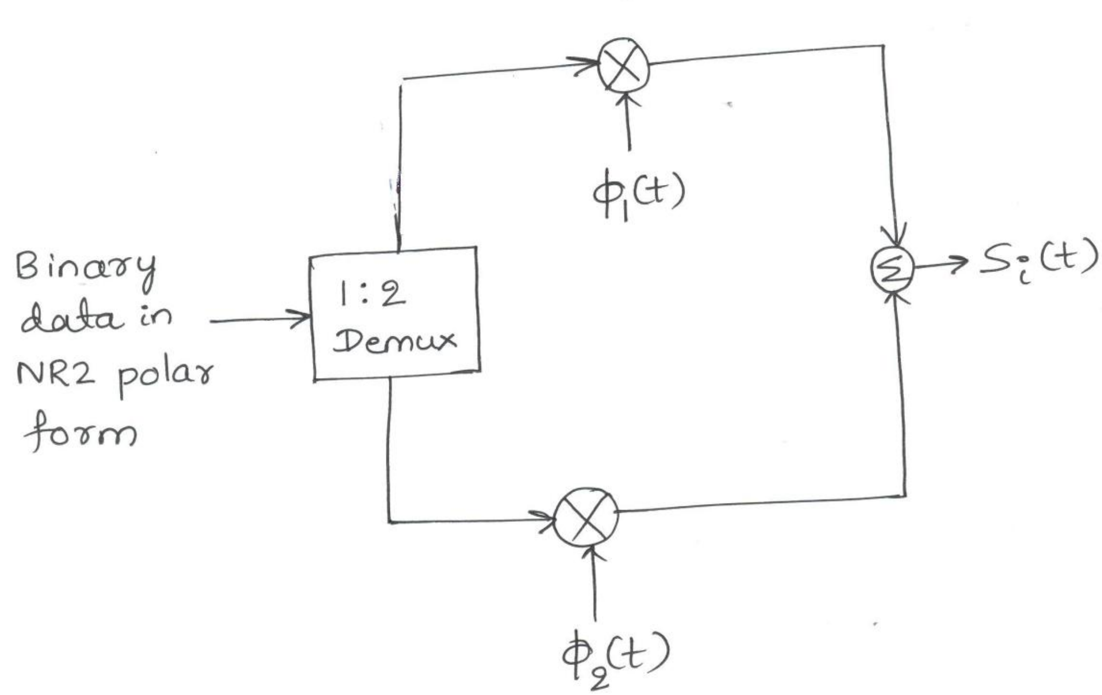
$$\begin{bmatrix} \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] \\ -\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] \end{bmatrix} \quad i = 1, 2, 3, 4.$$

i	phase	coordinates	dibits
1	$\frac{\pi}{4}$	$\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$	10
2	$\frac{3\pi}{4}$	$-\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$	00
3	$\frac{5\pi}{4}$	$-\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}$	01
4	$\frac{7\pi}{4}$	$\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}$	11

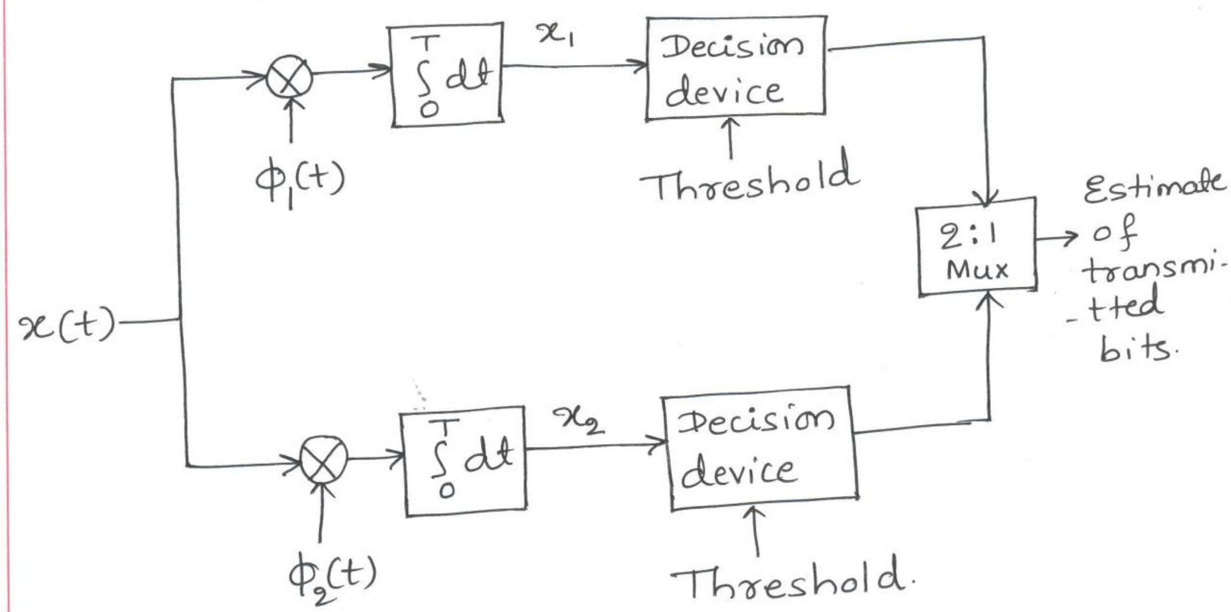
Based on these coordinates, signal space diagram of QPSK system may be drawn as follows.



Block diagram of transmitter



Block diagram of receiver



Let $x(t), 0 \leq t \leq T$ be the received symbol.

$$x(t) = s_i(t) + \omega(t), \quad 0 \leq t \leq T$$
$$i = 1, 2, 3, 4$$

where $\omega(t)$ represents additive, white Gaussian noise with zero mean and PSD $\frac{N_0}{2}$ ie, variance $\frac{N_0}{2}$.

Probability of error.

Suppose that $s_4(t)$ was transmitted.

From the block diagram of receiver, we have

$\therefore$ $x_2(t)$ and $x_3(t)$ are orthogonal to each other from 0 to 4.

Hence, an appropriate set of basis funct-ions may be found as follows.

step i) Energy of $x_2(t)$,

$$E_2 = \int_0^2 3^2\, dt$$

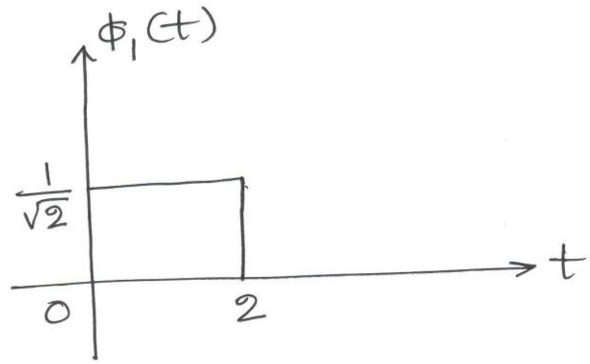
$$= 9t \Big|_0^2$$

$$= 9[2-0]$$

$$= 18$$

step ii) Basis function, $\phi_1(t) = \dfrac{x_2(t)}{\sqrt{18}}$

$$= \frac{x_2(t)}{3\sqrt{2}}$$



step iii) Energy of $x_3(t)$,

$$E_3 = \int_2^4 3^2 \, dt$$

$$= 9t \Big|_2^4$$

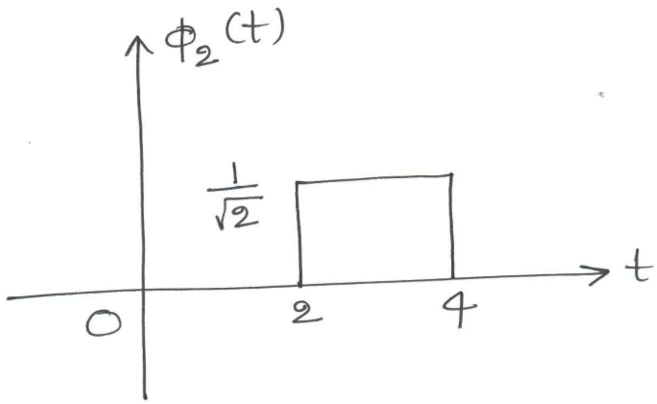
$$= 9[4-2]$$

$$= 18$$

step iv) Basis function, $\phi_2(t) = \frac{x_3(t)}{\sqrt{E_3}}$

$$= \frac{x_3(t)}{\sqrt{18}}$$

$$= \frac{x_3(t)}{3\sqrt{2}}$$



Expressing the signals as a linear combination of basis functions.

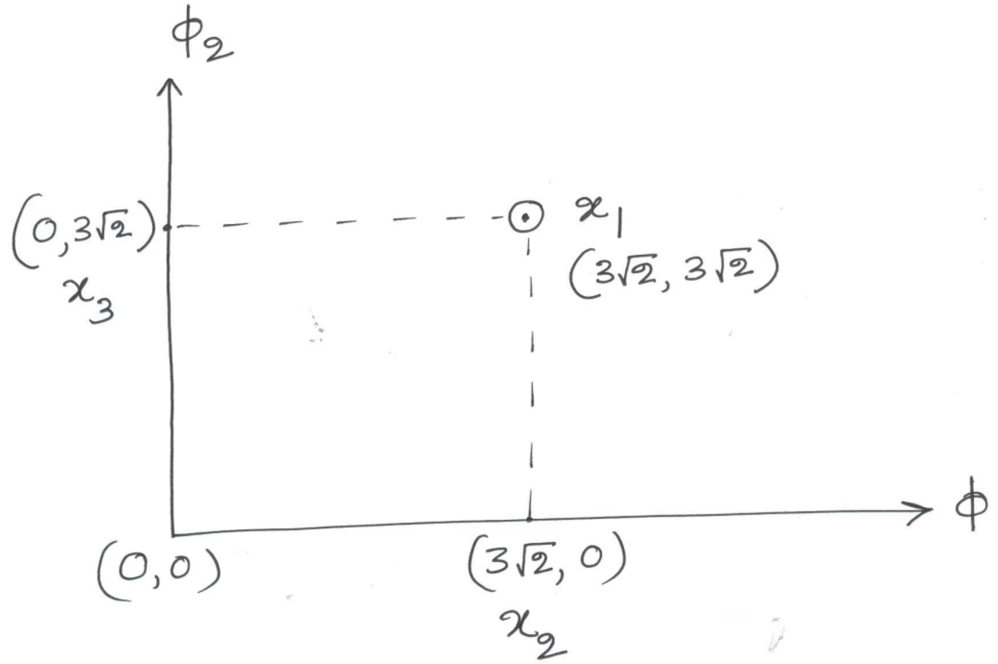
$$x_1(t) = 3\sqrt{2}\,\phi_1(t) + 3\sqrt{2}\,\phi_2(t)$$

$$x_2(t) = 3\sqrt{2}\,\phi_1(t) + 0\,\phi_2(t)$$

$$x_3(t) = 0\,\phi_1(t) + 3\sqrt{2}\,\phi_2(t)$$

(16)

Constellation diagram
(signal-space diagram)



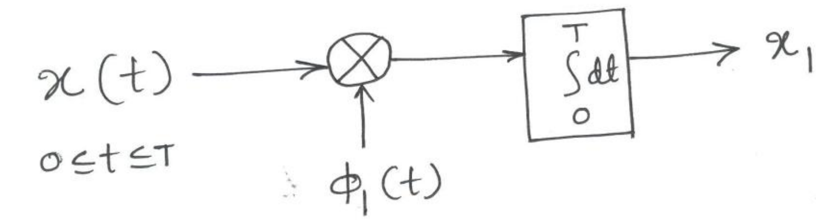
6 Explain matched filter receiver.

Correlation receiver consists of multiple correlators which involve multipliers and integrators.

Analog multipliers are hard to build.

Matched filter is an alternative to correlator which avoids the use of multipliers.

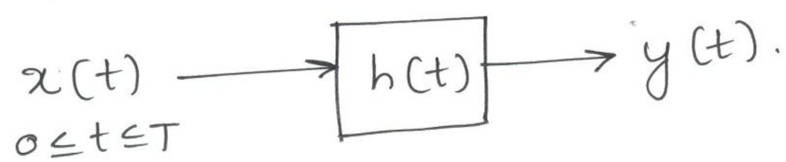
Consider the following correlator.



Output of the correlator,

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt \quad \ldots (1)$$

Consider the following LTI system with impulse response $h(t)$.



$$y(t) = x(t) * h(t)$$

$$= \int_0^T x(\tau)\,h(t-\tau)\,d\tau \quad \ldots (2)$$

Sampling $y(t)$ @ $t = T$, we get

$$y(T) = \int_0^T x(\tau)\,h(T-\tau)\,d\tau \quad \ldots (3)$$

(1) may also be written as,

$$x_1 = \int_0^T x(\tau)\,\phi_1(\tau)\,d\tau \quad \cdots (4)$$

Comparing (3) and (4), we may state that for $y(T)$ to be equal to x_1, $h(T-\tau)$ should be equal to $\phi_1(\tau)$.

ie, $h(T-\tau) = \phi_1(\tau) \quad \cdots (5)$

put $T-\tau = t$. We get,

$$h(t) = \phi_1(T-t) \quad \cdots (6)$$

This is the impulse response of the filter matched to $\phi_1(t)$.

Correspondingly, the correlation receiver detector part may be implemented as follows