

TAT-3

- Q1. (a) Beam area :
- * The area of strip of antenna is $r d\theta$ extended along the spherical surface.
 - * The beam area is given by Ω_A as integral of power radiation is known as Beam area

$$\Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P(\theta, \phi) r \sin\theta d\theta d\phi$$

- (b) Radiation intensity :
- * The power radiated per unit area is known as Radiation Intensity
 - * The radiated power can be extended in the form of Radiation intensity

$$P = A_r W_{rad}$$

$$P = 4\pi r^2 W_{rad}$$

$$\frac{P}{4\pi} = r^2 W_{rad}$$

$$U = r^2 W_{rad}$$

- (c) Beam efficiency :
- * Beam area is given as the ratio product of ^{major} Beam area and minor beam. $\Omega_A = \Omega_m + \Omega_n$
 - * Beam efficiency is the ratio of Major beam to beam area.

$$\rho = \frac{R_M}{R_A} = \frac{R_M}{R_M + R_m}$$

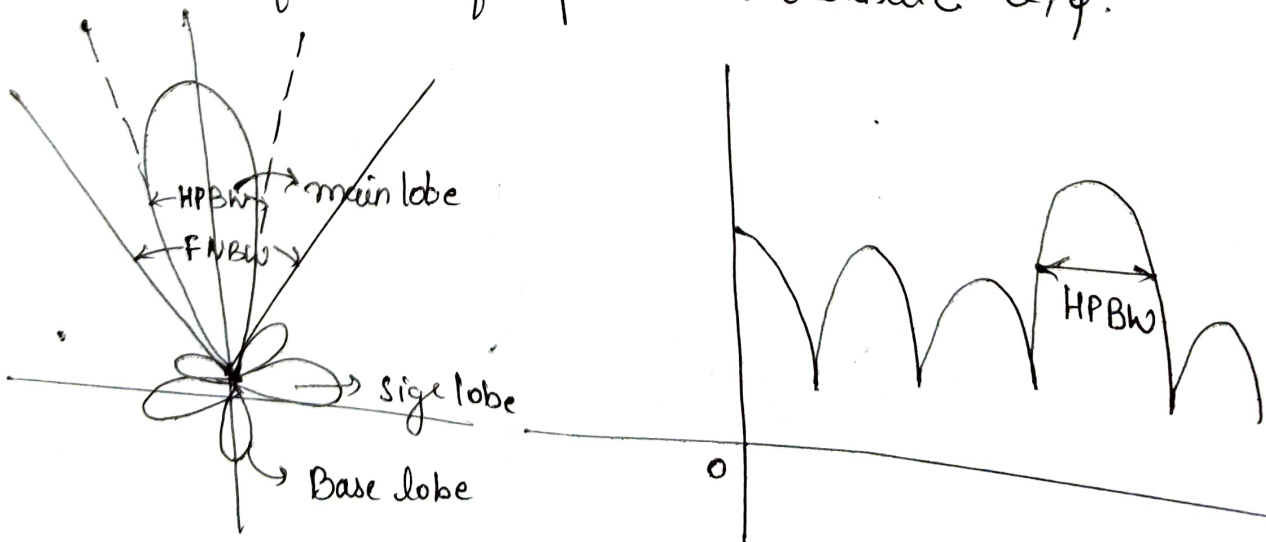
(d) Gain: Ratio of product of 4π to the efficiency of apparatus to the square of wave length of the beam. is known as gain of the antenna

$$A_{ep} = \frac{G \lambda^2}{4\pi} \Rightarrow G = \frac{A_{ep} (4\pi)}{\lambda^2}$$

(e) Radiation / Antenna aperture

Radiation pattern of the antenna is given as radiation characteristic or function of radiation pattern

It is the 3-dimensional representation quantity of the antenna with pattern varying field/power of the antenna function of spherical co-ordinate θ, ϕ .



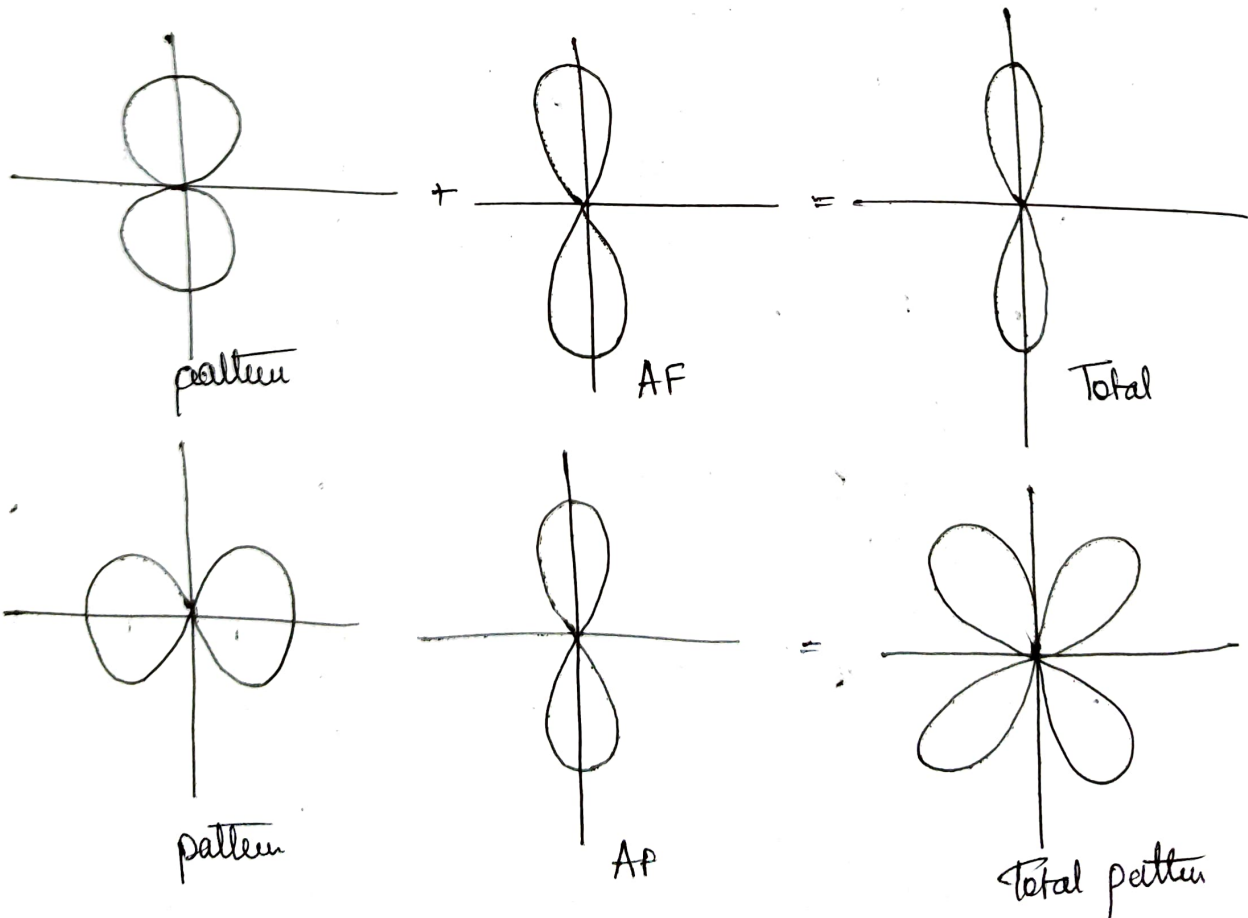
Pattern multiplication:

The principle of pattern multiplication states that the pattern of the antenna is the product of individual antenna pattern. The antenna of the pattern is situated (amplitude and phase) of the antenna

There are two ways where pattern multiplication can be shown:

1. Individual pattern multiplication
2. Array of pattern.

Individual pattern: The individual pattern of the antenna is multiplied. for example



Array of pattern : The function of pattern which varies with respect to the excited pattern (Amplitude pattern).

F_1 = array of two isotropic point source.

F_2 = array of line source.

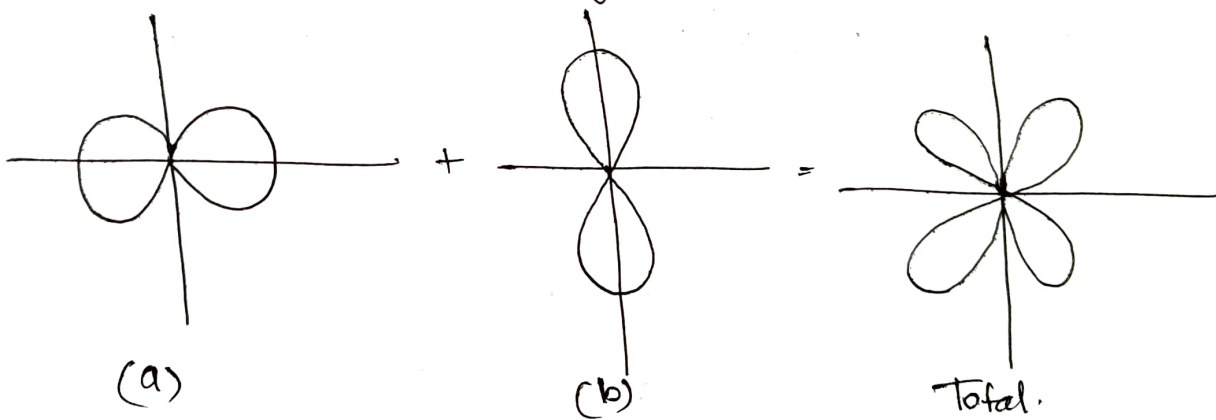


$$E = F_1 * F_2$$

Advantages \rightarrow pattern multiplication is easy.

dis-Advantages \rightarrow It is done only for similar pattern i.e. the size of the pattern should be same.

The total Radiation pattern of the given example:



Q9.

$$G_n \quad E(\theta) = \cos \theta \cos 2\theta$$

$$E(\theta) = 0.707$$

$$E(\theta) = \frac{1}{\sqrt{2}}$$

$$\cos \theta \cos 2\theta = \frac{1}{\sqrt{2}}$$

$$\cos \theta = \frac{1}{\sqrt{2} \cos 2\theta}$$

$$\theta = \cos^{-1} \left(\frac{1}{\sqrt{2} \cos 2\theta} \right)$$

$$\theta = 0 \Rightarrow \theta = 20.7^\circ$$

(a) where, Half power BW = 2θ
 $= 2 \times 20.7^\circ$
 $= 41.4^\circ$
 $\approx \underline{\underline{41^\circ}}$

(b) FNBW

$$E(\theta) = 0$$

$$\cos \theta \cdot \cos 2\theta = 0$$

$$\cos \theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$= \underline{\underline{45^\circ}}$$

$$\Rightarrow \text{FNBW} = 2\theta = 2(45^\circ) = \underline{\underline{90^\circ}}$$

$$f = 6 \text{ MHz}$$

$$= 6 \times 10^6 \text{ Hz}$$

$$r = 12 \text{ km}$$

$$= \underline{\underline{12 \times 1000 \text{ m}}}$$

$$P_T = 28 \text{ kW}$$

$$= 28 \times 10^3 \text{ W}$$

$$P_R = ?$$

$$P_R = \frac{P_T A_{re} A_{tn}}{r^2 \lambda^2}$$

$$= \frac{(28 \times 10^3)(10 \times 12) \lambda^2}{(12 \times 1000)^2}$$

$$= \frac{(28 \times 10^3)(10 \times 12)(50)^2}{(12 \times 1000)^2}$$

$$P_R = \underline{\underline{58.333W}}$$

$$\lambda = \frac{c}{f}$$

$$= \frac{3 \times 10^8}{26 \times 10^6}$$

$$= \underline{\underline{50m}}$$

(b) $U = U_m \sin^2 \theta \sin^2 \phi$ directivity = ?

$$D = \frac{4\pi}{\iint E(\theta, \phi) \cdot \sin \theta d\theta \cdot d\phi}$$

$$\int_{\phi=0}^{\pi} \left[\int_{\theta=0}^{\pi} (U_m \sin^2 \theta \sin^2 \phi) \sin \theta d\theta \right] d\phi$$

$$= \int_{\phi=0}^{\pi} (\sin^2 \phi \cdot d\phi) \int_{\theta=0}^{\pi} \sin^3 \theta \cdot d\theta \quad \text{--- (1)}$$

\Downarrow I_1 \Downarrow I_2

$$I_1 = \int_0^{\pi} \left(\frac{1 - \cos 2\phi}{2} \right) d\phi = \frac{\pi}{2} - 0$$

$$= \underline{\underline{\pi/2}} \quad \text{--- (2)}$$

$$I_2 = \int_0^{\pi} \sin^3 \theta \cdot d\theta$$

$$= \int_0^{\pi} \frac{1}{4} (3\sin^2 \theta - \sin 3\theta) \cdot d\theta$$

$$= \frac{4}{3} \text{ --- } \textcircled{3}$$

put $\textcircled{3}$ & $\textcircled{2}$ in $\textcircled{1}$

$$= \frac{\pi}{2} \times \frac{4}{3} = \frac{2\pi}{3}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} U \cdot \sin \theta \cdot d\theta \cdot d\phi}$$

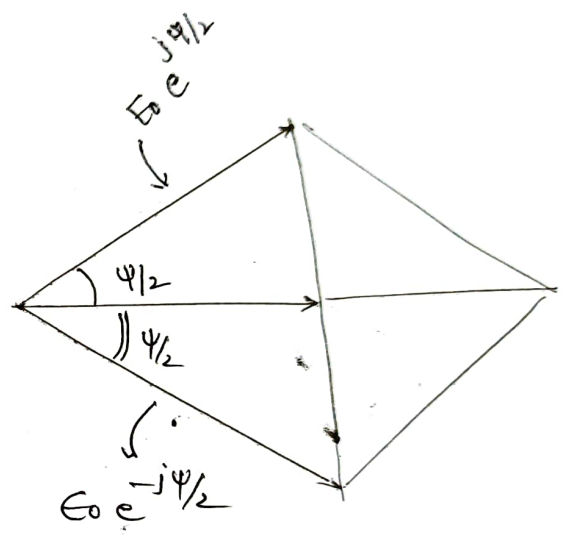
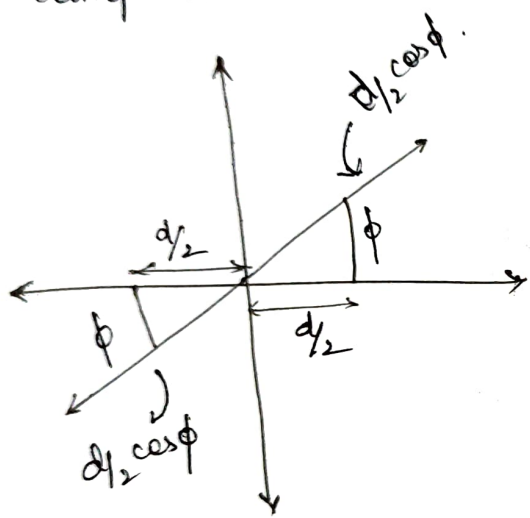
$$= \frac{\cancel{4\pi}^2}{\cancel{2\pi}/3}$$

$$\boxed{D = 6}$$

Exact dielectric Value = 5.1

Q7. point source : Antenna is known as two point source which have different source of operation that need to be performed it performs Radiation, transmitter, receiver. Antenna has different characteristics.

Consider, two pt. source which are placed symmetric to each other with respect to the origin. Assuming that the two point source have same amplitude and distance.



(a)

$$d_r \text{ (in radians)} = \frac{\lambda d}{2\pi} = \beta d.$$

Electric field can be given by

$$E = E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$$

$$= 2E_0 \left[\frac{e^{j\psi/2} + e^{-j\psi/2}}{2} \right]$$

$$\psi = d_r \cos \phi$$

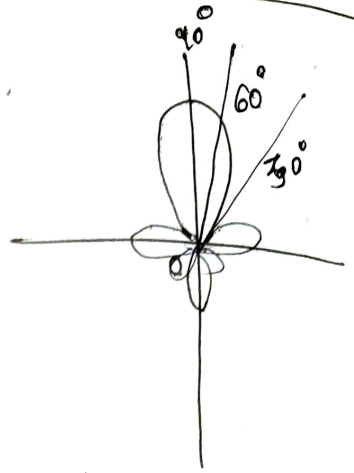
$$= 2E_0 \cos \psi/2$$

$$= 2E_0 \left[\cos \left(\frac{d_r \cos \phi}{2} \right) \right]$$

Condition $2E_0 = 1$; $E = \cos \phi / 2$

$d = \lambda/2$ then $\psi = \pi$

$$E = \cos\left(\frac{\pi}{2} \cos \phi\right)$$



$$S = \frac{1}{2} \operatorname{Re}(E \times H^*)$$

$$= \frac{1}{2} \operatorname{Re} E_0 H_0 H_\phi^*$$

$$= \frac{1}{2} \operatorname{Re} E_0 |H_\phi|^2$$

$$E_0 = |H_\phi|^2 = \sqrt{\frac{\mu}{\epsilon}}$$

$$P = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S \cdot d\theta \cdot d\phi$$

$$P = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \frac{B^2 I_0^2 L^2}{12\pi}$$

W.K.T

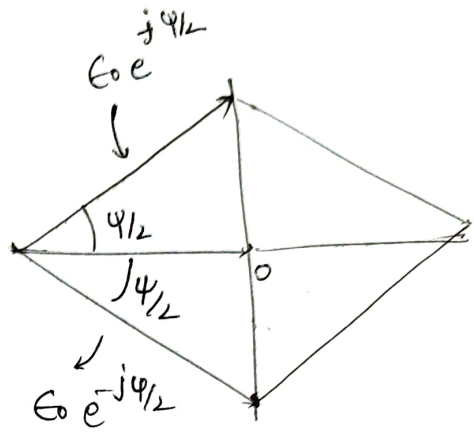
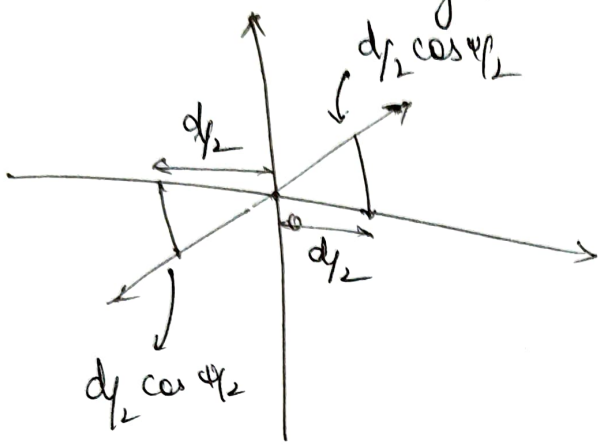
$$P = \left(\frac{I_0}{\sqrt{2}}\right)^2 \Rightarrow$$

$$P_e = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \frac{B^2 L^2}{6\pi}$$

$$P_e = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \frac{B^2 L^2}{6\pi}$$

Q4.
(a)

Consider, two isotropic point source, which have same amplitude and phase. They are placed at symmetric to each other they are placed with the origin.



$$dr \text{ (in radians) } = \frac{\lambda d}{2\pi} = \beta d //$$

$$E = E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$$

$$= 2 E_0 \left[\frac{e^{j\psi/2} + e^{-j\psi/2}}{2} \right]$$

$$= 2 E_0 \cos \psi/2$$

where, $\psi = dr \cos \phi$

$$= 2 E_0 \cos \left(\frac{dr}{2} \cos \phi \right)$$

$$2 E_0 = 1 ; dr = \lambda/2 ; \psi = \pi$$

$$\Rightarrow \boxed{E = \cos \left(\frac{\pi}{2} \cos \phi \right)}$$

$$S = \frac{1}{2} \operatorname{Re} [E \times H^*]^2$$

$$= \frac{1}{2} \operatorname{Re} E_0 |H_0| |H_\phi|^2$$

$$= \frac{1}{2} \operatorname{Re} E_0 |H_\phi|^2$$

$$E_0 = |H_\phi|^2 = \sqrt{\frac{\mu}{\epsilon}}$$

$$P = \int_{\phi} \int_{\theta} S \cdot \sin\theta \cdot d\theta \cdot d\phi$$

$$= \int_{\phi=0}^{2\pi} \left(\int_{\theta=0}^{\pi} \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \operatorname{Re} E_0 |H_\phi|^2 \sin\theta \cdot d\theta \right) d\phi$$

$$= \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \frac{B^2 I_0^2 L^2}{4\pi}$$

Wk. T $P = \left(\frac{I_0}{\sqrt{2}} \right)^2$

$$\Rightarrow \boxed{R_e = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} \frac{B^2 L^2}{6\pi}}$$

Hence proved

Pattern multiplication :-

Given:

$$U = \begin{cases} U_m \sin^2 \theta \sin^2 \phi, & 0 \leq \theta \leq \pi \\ & 0 \leq \phi \leq \pi \end{cases}$$

$$D = \frac{4\pi}{\Omega_A}$$

$$\Omega_A = \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) d\Omega$$

$$\begin{aligned} P_n(\theta, \phi) &= \frac{U_m}{U_{\max}} \\ &= \frac{U_m \sin^2 \theta \sin^2 \phi}{U_m} \end{aligned}$$

$$P_n(\theta, \phi) = \sin^2 \theta \sin^2 \phi$$

$$\Omega_A = \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} \sin^2 \theta \sin^2 \phi \sin \theta d\theta d\phi$$

$$\therefore (d\Omega = \sin \theta d\theta d\phi)$$

$$= \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} \sin^3 \theta \sin^2 \phi d\theta d\phi$$

$$= \int_{\theta=0}^{\pi} \sin^3 \theta d\theta \int_{\phi=0}^{\pi} \sin^2 \phi d\phi$$

$$= \int_{\theta=0}^{\pi} \sin^2 \theta \sin \theta \, d\theta \int_{\phi=0}^{\pi} \frac{1 - \cos 2\phi}{2} \, d\phi$$

$$= \int_{\theta=0}^{\pi} (1 - \cos^2 \theta) \sin \theta \, d\theta \left[\int_{\phi=0}^{\pi} \frac{1}{2} \, d\phi - \int_{\phi=0}^{\pi} \frac{\cos 2\phi}{2} \, d\phi \right]$$

$$= \int_{\theta=0}^{\pi} \quad \text{let } \cos \theta = t$$

$$\quad \quad \quad \rightarrow \sin \theta \, d\theta = -dt$$

$$= \int_{\theta=0}^{\pi} -(1 - t^2) \, dt \left[\int_{\phi=0}^{\pi} \frac{1}{2} \, d\phi - \left[\frac{\sin 2\phi}{4} \right]_{\phi=0}^{2\pi} \right]$$

$$= \int_0^{\pi} -dt + \int_0^{\pi} t^2 \, dt \left[\left[\phi \right]_0^{\pi} - 0 \right]$$

$$= \left[-t \right] + \left[\frac{t^3}{3} \right] \left[\pi \right]$$

$$\Rightarrow \left[-\cos \theta \right]_0^{\pi} + \left[\frac{\cos^3 \theta}{3} \right]_0^{\pi} \left[\pi \right]$$

$$\Rightarrow \frac{4}{3} \pi$$

$$\Omega_A \Rightarrow \frac{4\pi}{3} =$$

$$D = \frac{4\pi}{\Omega_A} \Rightarrow \frac{4\pi}{\frac{4\pi}{3}}$$

$$\boxed{D = 3}$$