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CMRIT CMR INSTITUTE OF TECHNOLOGY, BENGALURU. ACCREDITED WITH A+ GRADE BY NAAC

Internal Assessment Test II– Aug' 2022

Sub:	Advanced Calculus and Numerical Methods					Sub Code:	21MAT21	
Date:	05/08/2022	Duration:	90 mins	Max.marks	50	Sem/Sec:	I to O (PHY CYCLE)	OBE

ATTEMPT ANY FIVE

		MARKS	CO	RBT
1.	Find y(1.02) correct to four decimal places, given **dy = (xy-1) dx** and y(1) = 2, using Taylor's series method. Consider up to 4^{th}degree term.	[10]	CO5	L3
2.	Given $\frac{dy}{dx} = 1 + \frac{y}{x}$, y = 2 at x = 1, find approximate value of y (1.4) taking h = 0.2 using Modified Euler's method.	[10]	CO5	L3
3.	Using Runge-Kutta method of fourth order, find y (0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y (0) = 1 taking h =0.2	[10]	CO5	L3

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4. Apply Mine’s method to compute y(1.4) corrected to four decimal places, given $\mathbf{\frac{dy}{dx} = x^2 + \frac{y}{2}}$ and following data: y(1) = 2, y(1.1) =2.2156, y(1.2) = 2.4649, y (1.3) = 2.7514 [10] CO5 L3

5. Using Lagrange’s interpolation formula, Fit a polynomial to the data and hence find y at x = 2 [10] CO4 L3

x	0	1	3	4
Y	-12	0	6	12

6. Using Regula- Falsi method find the real root of $x \log_{10} x - 1.2 = 0$ correct to four decimal places. [10] CO4 L3

7. Evaluate $\int_0^{0.6} e^{-x^2}$dx, by (i) Simpson’s 1/3rd and (ii) Simpson’s 3/8th rule taking appropriate number of intervals [10] CO4 L3

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Internal Assessment Test - 02

1. <u>Given</u>: $dy = (xy-1)\,dx$, $y(1) = 2 \Rightarrow y_0 = 2,\ x_0 = 1$

$\frac{dy}{dx} = (xy-1)$ $\quad y(1.02) = ?$, $x = 1.02$

>> w.k.t.,

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{\lfloor 2}y''(x_0) + \frac{(x-x_0)^3}{\lfloor 3}y'''(x_0) + \frac{(x-x_0)^4}{\lfloor 4}y^{iv}(x_0) \longrightarrow (1)$$

Now,

$y' = xy - 1$ $\qquad y'(x_0) = x_0y_0 - 1 = (1\times 2) - 1 = 1$

$y'' = xy' + y$ $\qquad y''(x_0) = x_0y'_0 + y_0 = (1\times 1) + 2 = 3$

$y''' = xy'' + y' + y'$ $\qquad y'''(x_0) = x_0y''_0 + 2y'_0$

$= xy'' + 2y'$ $\qquad = (1\times 3) + (2\times 1) = 5$

$y^{iv} = xy''' + y'' + 2y''$ $\qquad y^{iv}(x_0) = x_0y'''_0 + 3y''_0$

$= xy''' + 3y''$ $\qquad = (1\times 5) + (3\times 3)$

$= 14$

$\therefore y(x_0) = 2,\ y'(x_0) = 1,\ y''(x_0) = 3,\ y'''(x_0) = 5,\ y^{iv}(x_0) = 14$

By (1),

$$y(1.02) = 2 + (1.02-1).1 + \frac{(1.02-1)^2}{2}\times 3 + \frac{(1.02-1)^3}{6}\times 5 + \frac{(1.02-1)^4}{24}\times 14$$

$y(1.02)$ ~~$= 2 + 0.0204\ 0.0006 +$~~

$$= 2 + (0.02)1 + \frac{(0.02)^2}{2}\times 3 + \frac{(0.02)^3}{6}\times 5 + \frac{(0.02)^4}{24}\times 14$$

$y(1.02) = 2 + 0.02 + 0.0006 + 0.0000067 + 0.000000093$

$\therefore$ $y(1.02) = \underline{\underline{2.0206}}$.

2. Given : $\frac{dy}{dx} = 1 + \frac{y}{x}$, $y_0 = 2$, $x_0 = 1$, $h = 0.2$

$f(x,y)$ ~~$f(x)$~~ $= 1 + \frac{y}{x}$ $\quad y(1.4) = ?$

Ist stage: $x_0 = 1$, $y_0 = 2$, $h = 0.2$, $x_1 = 1.2$, $y(1.2) = ? = y_1^{(0)}$

By applying Euler's formula,

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$
$$= 2 + (0.2) \cdot f(1,2)$$
$$= 2 + 0.2 \left[1 + \frac{2}{1}\right]$$
$$= 2.6$$

Now, applying Modified Euler's formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$$
$$= 2 + \frac{0.2}{2}[f(1,2) + f(1.2, 2.6)]$$
$$= 2 + 0.1\left[\left(1 + \frac{2}{1}\right) + \left(1 + \frac{2.6}{1.2}\right)\right]$$
$$= 2.6167$$

$$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$$
$$= 2 + \frac{0.2}{2}[f(1,2) + f(1.2, 2.6167)]$$

$$y_1^{(3)} = 2 + 0.1\left[\left(1+\frac{2}{1}\right) + \left(1 + \frac{2.6167}{1.2}\right)\right]$$

$$= 2.6180$$

II stage $y_0 = 2.6180$, $x_0 = 1.2$, $h = 0.2$, $x_1 = 1.4$,

$y(1.4) = ? = y_1^{(0)}$

By applying Euler's formula,

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 2.6180 + (0.2) f(1.2, 2.6180)$$

$$= 2.6180 + 0.2\left[1 + \frac{2.6180}{1.2}\right]$$

$$= 3.2543$$

Now, applying Modified Euler's formula.

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 2.6180 + \frac{0.2}{2}\left[f(1.2, 2.6180) + f(1.4, 3.2543)\right]$$

$$= 2.6180 + 0.1\left[\left(1 + \frac{2.6180}{1.2}\right) + \left(1 + \frac{3.2543}{1.4}\right)\right]$$

$$= 3.2686$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 2.6180 + \frac{0.2}{2}\left[f(1.2, 2.6180) + f(1.4, 3.2686)\right]$$

$$= 2.6180 + 0.1\left[\left(1 + \frac{2.6180}{1.2}\right) + \left(1 + \frac{3.2686}{1.4}\right)\right]$$

$$= 3.2696$$

~~a $y(1.4) \approx 3.26$~~

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_2^{(1)})\right]$$

$$= \cancel{y_0}\ 2.6180 + 0.1\left[f(1.2, 2.6180) + f(1.4, 3.2696)\right]$$

$$= 2.6180 + 0.1\left[\left(1 + \frac{2.6180}{1.2}\right) + \left(1 + \frac{3.2696}{1.4}\right)\right]$$

$$= 3.2697$$

$$\therefore\ y(1.4) = \underline{3.2697}$$

3. Given: $\frac{dy}{dx} = \frac{y-x}{y+x} \Rightarrow f(x, y) = \frac{y-x}{y+x}$, $y(0) = 1$

$y(0.2) = ?$, $h = 0.2$, $y_0 = 1$, $x_0 = 0$

By R-K Method,

$$K_1 = hf(x_0, y_0) = 0.2f(0, 1) = 0.2\left[\frac{1-0}{1+0}\right] = 0.2$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = 0.2f(0.1, 1.1) = 0.2\left[\frac{1.1-0.1}{1.1+0.1}\right]$$

$$= 0.1667$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) = 0.2f(0.1, 1.08335) = 0.2\left[\frac{1.08335-0.1}{1.08335+0.1}\right]$$

$$= 0.1662$$

$$K_4 = hf(x_0 + h, y_0 + K_3) = 0.2f(0.2, 1.1662) = 0.2\left[\frac{1.1662-0.2}{1.1662+0.2}\right]$$

$$= 0.1414$$

$\longrightarrow$ P.T.O.

$\therefore k_1 = 0.2, \quad k_2 = 0.1667, \quad k_3 = 0.1662, \quad k_4 = 0.1414$

$$\therefore y(x) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$y(0.2) = 1 + \frac{1}{6}[0.2 + (2 \times 0.1667) + 2(0.1662) + 0.1414]$$

$$= 1 + \frac{1}{6}[1.0072]$$

$$= 1.16787$$

$$\therefore y(0.2) = \underline{\underline{1.1679}}$$

4. $\underline{\text{Given}}$: $\frac{dy}{dx} = x^2 + \frac{y}{2} \Rightarrow y' = x^2 + \frac{y}{2}$

$y(1.4) = ?$

By data, $h = 0.1$

x	y	$y' = x^2 + \frac{y}{2}$
$x_0 = 1$	$y_0 = 2$	$y_0' = 1^2 + \frac{2}{2}$ $= 2$
$x_1 = 1.1$	$y_1 = 2.2156$	$y_1' = (1.1)^2 + \frac{2.2156}{2}$ $= 2.3178$
$x_2 = 1.2$	$y_2 = 2.4649$	$y_2' = (1.2)^2 + \frac{2.4649}{2}$ $= 2.67245$
$x_3 = 1.3$	$y_3 = 2.7514$	$y_3' = (1.3)^2 + \frac{2.7514}{2}$ $= 3.0657$
$x_4 = 1.4$		

w.k.t.

Milne's Predictor formula,

$$y_P^{(4)} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 2 + \frac{4 \times 0.1}{3}\left[(2 \times 2.3178) - 2.6724 + 2(3.0657)\right]$$

$$= 3.0793$$

Now, $y_4' = (x_4)^2 + \frac{(y_P^{(4)})}{2} = (1.4)^2 + \frac{3.0793}{2} = 3.49965$

By Milne's corrector formula,

$$y_C^{(4)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 2.4649 + \frac{0.1}{3}\left[2.67245 + (4 \times 3.0657) + 3.49965\right]$$

$$= 3.0794$$

Now $y_4' = (x_4)^2 + \frac{(y_C^{(4)})}{2} = (1.4)^2 + \frac{3.0794}{2} = 3.4997$

$$y_C^{(4)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 2.4649 + \frac{0.1}{3}\left[2.67245 + (4 \times 3.0657) + 3.4997\right]$$

$$= 3.0794$$

$$\therefore y(1.4) = 3.0794$$

5. <u>Given:</u> $y(2) = ?$

x	y
$x_0 = 0$	$y_0 = -12$
$x_1 = 1$	$y_1 = 0$
$x_2 = 3$	$y_2 = 6$
$x_3 = 4$	$y_3 = 12$

By Lagarange's Interpolation Method,

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y(x) = \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)} \cdot (-12) + \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)} \cdot (0)$$

$$+ \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)} \cdot (6) + \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)} \cdot 12$$

$$y(x) = \frac{+\cancel{12}(x-1)(x-3)(x-4)}{(+1)(+\cancel{3})(+\cancel{4})} + 0$$

$$+ \frac{\cancel{6}x(x-1)(x-4)}{(\cancel{3})(\cancel{2})(-1)} + \frac{\cancel{12}x(x-1)(x-3)}{(\cancel{4})(\cancel{3})(1)}$$

$$y(x) = (x-1)(x-3)(x-4) - x(x-1)(x-4) + x(x-1)(x-3)$$

$$= (x-1)\left[x^2 - 7x + 12 - \cancel{x^2} + 4x + \cancel{x^2} - 3x\right]$$

$y(x) = (x-1)[x^2-6x+12]$

$y(x) = x^3-6x^2+12x-x^2+6x-12$

$\therefore \boxed{y(x) = x^3-7x^2+18x-12}$

$\therefore y(2) = 2^3 - 7(2)^2 + (18\times 2) - 12$

$= 8 - 28 + 36 - 12$

$= -20+24$

$\therefore y(2) = 4$

6. Given: $f(x) = x\log_{10}x - 1.2 \longrightarrow (1)$

Now, we put $x = 0, 1, 2, 3, \ldots$ in (1)

$f(0) = 0\log_{10}0 - 1.2 = -1.2$

$f(1) = 1\log_{10}1 - 1.2 = -1.2$

$f(2) = 2\log_{10}2 - 1.2 = -0.5979 < 0$

$f(3) = 3\log_{10}3 - 1.2 = 0.2314 > 0$

$\therefore$ The real root lies b/w $(2,3)$.

Now, we put $x = 2.9, 2.8, 2.7, \ldots$ in (1)

$f(2.9) = 2.9\log_{10}2.9 - 1.2 = 0.14095$

$f(2.8) = 2.8\log_{10}2.8 - 1.2 = 0.05204 > 0$

$f(2.7) = 2.7\log_{10}2.7 - 1.2 = -0.03530 < 0$

1st approximation: $a = 2.7$, $b = 2.8$

$f(a) = -0.0353$, $f(b) = 0.0520$

$$x_1 = \frac{af(b) - bf(a)}{f(b)-f(a)} = \frac{(2.7\times 0.0520) - (2.8\times(-0.0353))}{0.0520 + 0.0353} = 2.7404$$

Now,

$f(2.7404) = 2.7404 \log_{10} 2.7404 - 1.2 = -0.0002146 < 0$

IInd approximation : $a = 2.7404$, $b = 2.8$

$f(a) = -0.0002146$, $f(b) = 0.0520$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(2.7404 \times 0.0520) - (2.8 \times (-0.0002146))}{(0.0520 + 0.0002146)}$$

$= 2.7406$

Now,

$f(2.7406) = 2.7406 \log_{10} 2.7406 - 1.2 = -0.0000402 < 0$

IIIrd approximation :

$a = 2.7406$, $b = 2.8$

$f(a) = -0.0000402$, $f(b) = 0.0520$

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(2.7406 \times 0.0520) - (2.8 \times (-0.0000402))}{(0.0520 + 0.0000402)}$$

$= 2.7406$

$\therefore$ the real root of given ~~fun~~ equation is $x = 2.7406$

7. Given: $I = \int_0^{0.6} e^{-x^2} dx$, $y = f(x) = e^{-x^2}$, $a = 0$, $b = 0.6$, $n = 6$

$$h = \frac{b - a}{n} = \frac{0.6 - 0}{6} = 0.1$$

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$y = e^{-x^2}$	1	0.9900	0.9608	0.9139	0.8521	0.7788	0.6977
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(i) By Simpson's $\frac{1}{3}$rd rule,

$$I = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{0.1}{3}\left[(1+0.6977) + 4(0.9900 + 0.9139 + 0.7788) + 2(0.9608 + 0.8521)\right]$$

$$= \frac{0.1}{3}\left[16.0543\right]$$

$$\therefore \quad I = \underline{\underline{0.5351}}$$

(ii) By Simpson's $\frac{3}{8}$th rule,

$$I = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$= \frac{3 \times 0.1}{8}\left[(1+0.6977) + 3(0.9900 + 0.9608 + 0.8521 + 0.7788) + 2(0.9139)\right]$$

$$= \frac{3 \times 0.1}{8}\left[14.2706\right]$$

$$\therefore \quad I = \underline{\underline{0.5351}}$$