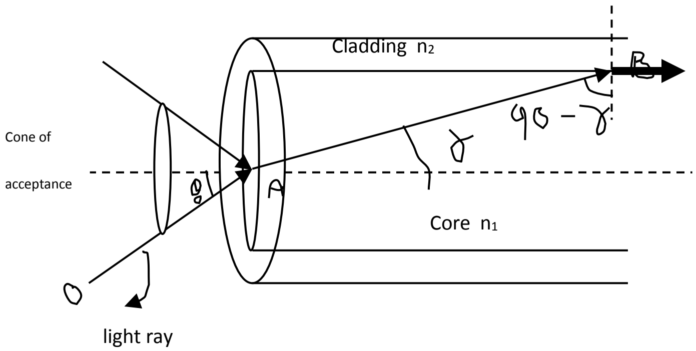
IAT-2 PHYSICS SCHEME

1.a) **Expression for condition for propagation :**



From Snell's Law:

For the ray OA $n_0 \sin\theta_0 = n_1 \sin r = n_1\left(\sqrt{1-\cos^2 r}\right)$ …………. (1)

For the ray AB

$$n_1 \sin(90 - r) = n_2 \sin 90$$

$$n_1 \cos r = n_2$$

$$\cos r = \frac{n_2}{n_1}$$

[here the angle of incidence is (90 - θ_1) for which angle of refraction is 90^0].

Substituting for cosr in equation (1)

$$n_0 \sin\theta_0 = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If the medium surrounding the fiber is air then $n_0 = 1$,

Numerical aperture = $\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$

The total internal reflection will take place only if the angle of incidence $\theta_i < \theta_0$

$\therefore \sin\theta_i < \sin\theta_0$

$\sin\theta_i <$ $\sqrt{n_1^2 - n_2^2}$

This is the condition for propagation.

1B

$$\frac{N\alpha}{3\varepsilon_o} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}$$

$$\varepsilon_r = 2.5$$

2A

Let F = F_o Sinω_ft be the oscillating applied force

The equation of motion is given by

$$F = ma = -kx - bv + F_o \sin\omega_f t$$

$$m\frac{d^2x}{dt^2} + b.\frac{dx}{dt} + kx = F_o \sin\omega_f t$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_o}{m}\sin\omega_f t$$

$$Let\ \frac{b}{m} = 2R; \frac{k}{m} = \omega^2; \frac{F_o}{m} = F$$

$$\frac{d^2x}{dt^2} + 2R\frac{dx}{dt} + \omega_o{}^2 x = F\sin\omega_f t \text{ …..(1)}$$

Let one particular solution be $x = A.\sin(\omega_f t - \phi)$

$$\frac{dx}{dt} = \omega_f A.\cos(\omega_f t - \phi)$$

$$\frac{d^2x}{dt^2} = -\omega_f^2 A.\sin(\omega_f t - \phi)$$

Also

$$F\sin\omega_f t = F.\sin(\omega_f t - \phi + \phi)$$

$$= F\sin(\omega_f t - \phi)\cos\phi + F\cos(\omega_f t - \phi)\sin\phi$$

Substituting in (1)

$$-\omega_f^2 A.\sin(\omega_f t - \phi) + 2RA\omega_f \cos(\omega_f t - \phi) + \omega_o^2 A\sin(\omega_f t - \phi$$

Comparing coefficients of $\sin(\omega_f t - \phi)$ *and* $\cos(\omega_f t - \phi)$ on both sides

$$A(.\omega_o^2 - \omega_f^2) = F\cos\phi$$
$$2RA\omega_f = F\sin\phi$$

$$\therefore F^2 = A^2(\omega_o^2 - \omega_f^2)^2 + 4R^2A^2\omega_f^2$$

$$A = \frac{F}{\sqrt{(\omega_o^2 - \omega_f^2)^2 + 4R^2\omega_f^2}}$$

$$\tan\phi = \frac{2R\omega_f}{\omega_o^2 - \omega_f^2}$$

Case 1: amplitude is infinity when at $\omega_0 = \omega_f$, damping is zero

Case 2: Amplitude is less when $\omega_0 \neq \omega_f$

2B

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$
$$E - E_F = -0.2eV = -0.2x1.6x10^{-19} J$$
$$T = 300K$$
$$f(E) = 0.5$$

3A

Expression for Electrical conductivity:

Imagine a conductor across which an electric field E is applied. The free electrons acquire drift velocity and the matter wave number change from k_1 to k_2 in time interval τ_F in the presence of electric field.

The force on the free electron is

$$F = \frac{dp}{dt} = eE$$
$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{2\pi p}{h}$$
$$p = \frac{hk}{2\pi}$$
$$\frac{dp}{dt} = \frac{h}{2\pi}\left(\frac{dk}{dt}\right)$$
$$dk = \frac{2\pi}{h} eEdt$$

On integration $k_2 - k_1 = \Delta k = \frac{2\pi.eE.\tau_F}{h}$ ....(1)

From quantum theory, conductivity $J = \Delta k.ne.\frac{h}{2\pi.m^*}$ ....(2)

Substituting (1) in (2)

We get $J = \frac{ne^2\tau_F}{m^*}E$ ...(3)

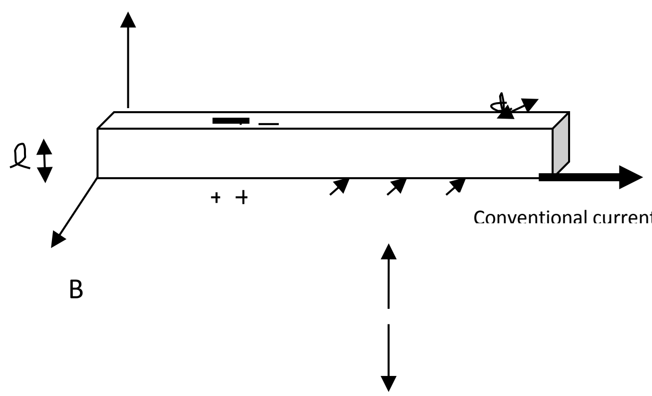
Since from Ohm's, J = σE, conductivity σ can be written as

$$\sigma = \frac{ne^2\tau_F}{m^*} = \frac{ne^2}{m^*}\frac{\lambda}{v_F}$$

3B

$$\sigma = ne(\mu_e + \mu_h)$$
$$\sigma = 1.32(\Omega m)^{-1}$$

4A



Hall effect: When a conductor carrying current is placed in transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.

Here B is along –X , V is along –Y axis

Lorentz force= $-e\left(-\hat{j}\ X -\hat{i}\right) = +\hat{k}$

So the electron is deflected along + Z axis

Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. This develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up.

Expression for Hall Coefficient:

At equilibrium, Lorentz force is equal to force due to applied electric field

$$Bev_d = eE_H$$

Hall Field $E_H = BV_d$

$$Current\ \ density\ \ J = n_e e v_d$$

$$v_d = \frac{J}{n_e e}$$

$$E_H = B\frac{J}{n_e e}$$

Hence

$$\frac{E_H}{JB} = \frac{1}{n_e e} = R_H$$

R_H is the Hall coefficient

Hall voltage $V_H = E_H.d = B.J.R_H.d$

Case 1: For P type semiconductor, the bottom surface will be at positive potential.

Case 2: For n type semiconductor, the bottom surface will be at positive potential

4B

$$E_F = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}$$

Fermi Energy $\frac{3n}{\pi} = \left[E_F.8m.\frac{1}{h^2}.\right]^{3/2}$

$$n = 2.4x10^{28}/m^3$$

5A

CLAUSIUS – MOSOTTI RELATION:

This expression relates dielectric constant of an insulator (ε) to the polarization of individual atoms(α) comprising it.

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

where N is the number of atoms per unit volume

α is the polrisability of the atom

ε_r is the relative permittivity of the medium

ε_o is the permittivity of free space.

Proof:

If there are N atoms per unit volume,the electric dipole moment per unit volume –known as polarization is given by

P = $N\alpha E_i$

By the definition of polarization P, it can be shown that

P =

$$\varepsilon_0 E_a(\varepsilon_r - 1) = N\alpha E_i$$

$$\varepsilon_0\varepsilon_r E_a - \varepsilon_0 E_a = N\alpha E_i$$

$$\varepsilon_r = 1 + \frac{N\alpha E_i}{\varepsilon_0 E_a}$$

……………………(1)

The internal field at an atom in a cubic structure(γ =1/3) is of the form

$$E_i = E_a + \frac{p}{3\varepsilon_0} = E_a + \frac{N\alpha E_i}{3\varepsilon_0}$$

$$\frac{E_i}{E_a} = \frac{1}{\left[1 - \left(\frac{N\alpha}{3\varepsilon_0}\right)\right]}$$

Substituting for $\frac{E_i}{E_a}$ in equation (1)

$$\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}\left[\frac{1}{\left(1 - \frac{N\alpha}{3\varepsilon_0}\right)}\right] = \frac{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right] + \frac{N\alpha\varepsilon_0}{\varepsilon_0}}{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right]} = \frac{1 + \frac{2}{3}\left(\frac{N\alpha}{\varepsilon_0}\right)}{1 - \frac{1}{3}\left[\frac{N\alpha}{\varepsilon_0}\right]}$$

$$\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right] = \frac{\dfrac{1 + (2/3)\frac{N\varepsilon}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} - 1}{\dfrac{1 - (2/3)\frac{N\alpha}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} + 2} = \frac{N\alpha}{3\varepsilon_0}$$

5B

Quantum free electron theory:

Assumptions:

1. The energy of conduction electrons in a metal is quantized.

2. The distribution of electrons amongst various energy levels is according to Pauli's exclusion principle and Fermi – Dirac statistical theory.

3. The average kinetic energy of an electron is equal to $\frac{3}{5}E_F$

4. The interaction between the electrons and ions, the repulsion between electrons are ignored. The electrons travel in a constant potential inside the metal but stay confined within its boundaries.

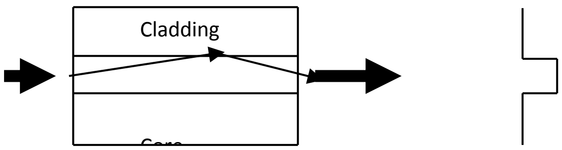
6A

Types:

1. Single mode fiber:

Core diameter is around 5-10 µm. The core is narrow and hence it can guide just a single mode.

Variation of refractive index

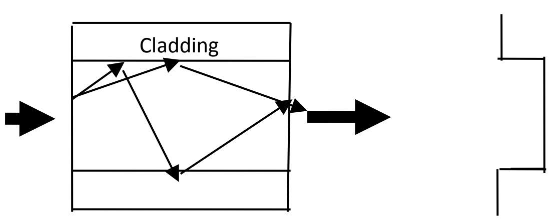


- No modal dispersion
- Difference between n_1& n_2 is less.Critical angle is high. Low numerical aperture.
- Low Attenuation -0.35db/km
- Bandwidth -100GHz
- Preferred for short range

Step index multimode fibre :

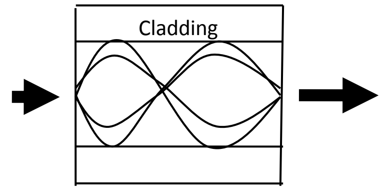
- Here the diameter of core is larger so that large number of rays can propagate. Core diameter is around 50. µm.
- High modal dispersion
- Difference between n_1& n_2 is high. Low Critical angle. Large numerical aperture.
- Losses high
- Bandwidth -500MHz
- Allows several modes to propagate
- Preferred for Long range

Variation of refractive index

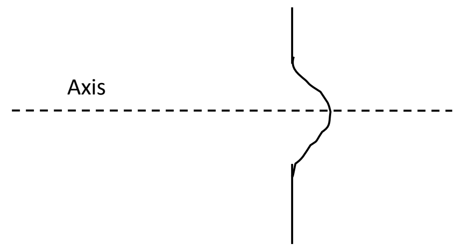


Graded index multimode fiber:

In this type, the refractive index decreases in the radially outward direction from the axis and becomes equal to that of the cladding at the interface. Modes travelling close to the axis move slower where as the modes close to the cladding move faster.As a result the delay between the modes is reduced. This reduces modal dispersion.



Variation of refractive index



- Low modal dispersion
- High data carrying capacity.
- High cost
- Many modes propagate
- Bandwidth -10GHz

6B

To show that energy levels below Fermi energy are completely occupied:

For E **<** E_F, at T = 0,

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 1$$

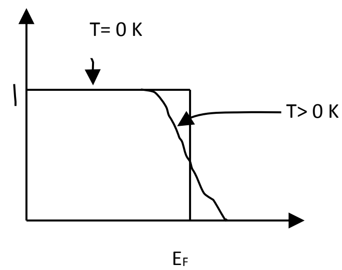
To show that energy levels above Fermi energy are empty:

For E > E_F , at T=0

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 0$$

At ordinary temperatures, for E = E_F,

$$f(E) = \frac{1}{2}$$



7A

ELECTRON DENSITY IN CONDUCTION BAND

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band is given by

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

Expression for Fermi Level in Intrinsic Semiconductor

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f + E_v + E_c}{kT}$$

$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

If m_e and m_h are equal,

$E_f = (Ec + Ev)/2$

So, Fermi level is said to be at the centre of energy gap

7B

Failures of Classical free electron theory:

1. Prediction of low specific heats for metals:

Classical free electron theory assumes that conduction electrons are classical particles similar to gas molecules. Hence, they are free to absorb energy in a continuously. Hence metals possessing more electrons must have higher heat content. This resulted in high specific heat given by the expression $C_V = \frac{3}{2}R$.

This was contradicted by experimental results which showed low specific heat for metals. $C_V = 10^{-4}R$.

2. Temperature dependence of electrical conductivity:

From the assumption of kinetic theory of gases

$$\frac{3}{2}kT = \frac{1}{2}mv^2$$

$$\therefore v \propto \sqrt{T}$$

Also mean collision time τ is inversely proportional to velocity,

$$\tau \propto \frac{1}{v}$$

$$\tau \propto \frac{1}{\sqrt{T}}$$

$$\therefore \sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

However experimental studies show that $\sigma \propto \frac{1}{T}$

3. Dependence of electrical conductivity on electron concentration:

As per free electron theory, $\sigma \propto n$

The electrical conductivity of Zinc and Cadmium are 1.09 x 10^7 /ohm m and .15 x 10^7 /ohm m respectively which are very much less than that for Copper and Silver for which the values are 5.88x10^7 /ohm m and 6.2 x 10^7 /ohm m. On the contrary, the electron concentration for zinc and cadmium are 13.1x10^{28}/m^3 and 9.28 x10^{28} /m^3 which are much higher than that for Copper and Silver which are 8.45x10^{28} /m^3 and 5.85 x10^{28} /m^3.

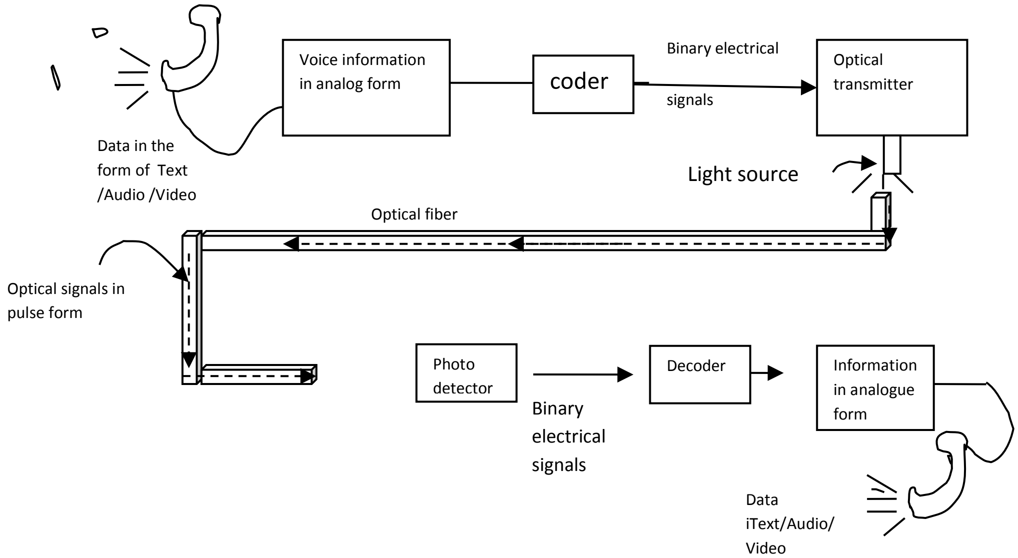
These examples indicate that $\sigma \propto n$ does not hold good.

4. Mean free path, mean collision time found from classical theory are incorrect.

8A

Point to point communication system using optical fibers

This system is represented through a block diagram as follows.



1.The information in the form of voice/ picture/text is converted to electrical signals through the transducers such as microphone/video camera.

2.The analog signal is converted in to binary data with the help of coder. The binary data in the form of electrical pulses are converted in to pulses of optical power using Semiconductor Laser.

3. This optical power is fed to the optical fiber. Only those modes within the angle of acceptance cone will be sustained for propagation by means of total internal reflection.

4 At the receiving end of the fiber, the optical signal is fed in to a photo detector where the signal is converted to pulses of current by a photo diode.

5 Decoder converts the sequence of binary data stream in to an analog signal . Loudspeaker/CRT screen provide information such as voice/ picture.

Merits and Demerits of optical fiber communication

Merits –

Large bandwidth (1000GHz)

Data security

No Electrical Interference (No cross talk)

Low loss (0.01dB/km)

Portable

Cheaper

Demerits

Repair costs high

Light emitting sources are limited to low power The distance between the transmitter and receiver should keep short or repeaters are needed to boost the signal.

8B

$$\alpha = \frac{10}{L}\log_{10}\left(\frac{P_{in}}{P_{out}}\right)$$

$$\frac{P_{in}}{P_{out}} = 10^{\frac{\alpha L}{10}} = 20.89$$