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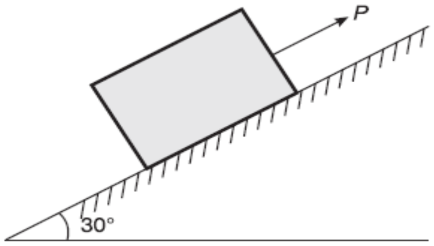
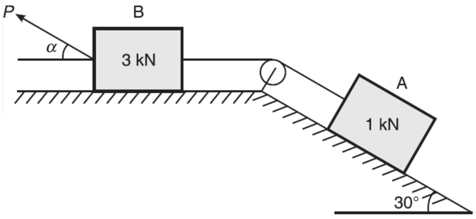
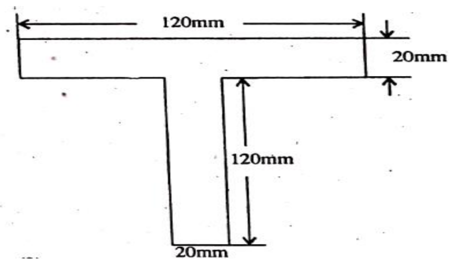
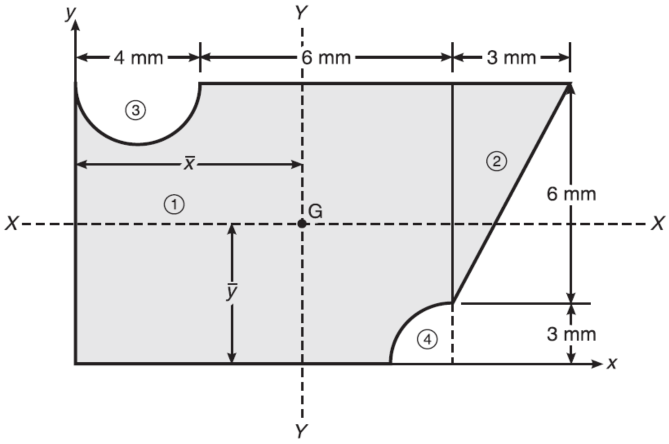
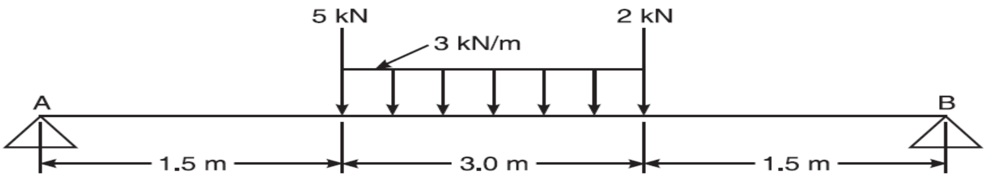
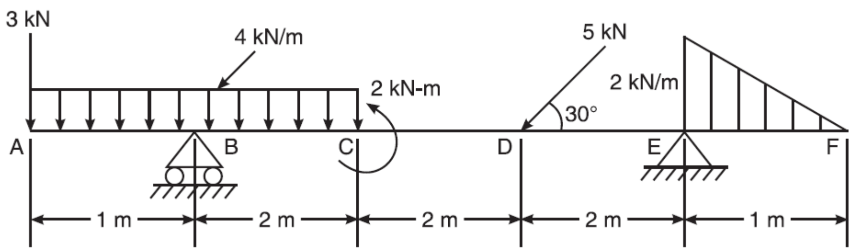
Internal Assessment Test II – Aug.2022

Sub:	Elements of Civil Engineering and Mechanics		Sub Code:	21CIV24	Branch	Civil Engg	
Date:	08.08.2022	Duration:	90 min's	Max Marks:	50	Sem/Sec:	2^{nd} sem /All sections	OBE

	Answer any FIVE FULL Questions	MARKS	CO	RBT
1 (a)	Discuss the various types of friction.	[05]	CO3	L1
(b)	State the laws of dry friction.	[05]	CO3	L1
2 (a)	Define angle of friction and angle of repose with diagram.	[03]	CO3	L1
(b)	A block of mass 20 kg placed on an inclined plane as shown in Figure 8.7, is subjected to a force P that is parallel to the plane. Taking the inclination of plane with respect to horizontal as 30° and the coefficient of friction as 0.24, determine the value of P for impending motion of the block. 	[07]	CO3	L3
3 (a)	Find the least value of P required to cause the system of blocks shown in Figure to have impending motion to the left. Take coefficient of friction as 0.2 for all contact surfaces. 	[10]	CO3	L3
4 (a)	Determine the x and y coordinates of the centroid of a triangular section.	[05]	CO4	L1
(b)	Locate the centroid of the T-section shown in Figure. 	[05]	CO4	L3
5 (a)	Locate the centroid of the shaded area shown in Figure.	[10]	CO4	L3

				
6 (a)	Explain the deferent types of beams and supports with diagram.	[05]	CO3	L1
	A simply supported beam of span 6 m is subjected to loading as shown in the figure. Determine the reactions at A and B. 	[05]	CO3	L3
7	Find the support reactions of the beam loaded as shown in Figure. 	[10]	CO3	L3

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Answers

1. Depending on the state of rest or motion, we can categorize friction into:
(i) Static friction
and
(ii) Dynamic friction
Sliding friction & Rolling friction

(i) **Static friction:** It is the friction experienced between two bodies when both bodies are at rest.
(ii) **Dynamic friction:** It is the friction experienced between two bodies when one body moves over the other body. It is of two types:
(a) *Sliding friction:* It is the resisting force which opposes the sliding motion of one body over another body, This force acts in a direction opposite to the direction of impending motion.
(b) *Rolling friction:* It is the friction between the two bodies when one body rolls over the other body.
Based on the surface of contact, there are two types of friction, namely:
(i) **Dry friction:** If the contact surfaces between the two bodies are dry, then the friction between such bodies is known as dry friction.
(ii) **Fluid friction:** The friction between two fluid layers or the friction between a solid and a fluid is known as fluid friction.

LAWS OF FRICTION
The laws of static friction are:
(i) The force of friction always acts in a direction, opposite to that in which the body tends to move.
(ii) The magnitude of the force of friction is exactly equal to the applied force which just moves the body.
(iii) The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces in contact, i.e.
F/N = constant
where F is the limiting friction and N is the normal reaction.
(iv) The force of friction is independent of the area of contact between the two surfaces.
(v) The force of friction depends upon the roughness of the surfaces in contact.
The laws of dynamic friction are:
(i) The force of friction always acts in a direction, opposite to that in which the body is moving.
(ii) The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces in contact. But this ratio is slightly less than that in the case of limiting friction.
(iii) The friction force remains constant for moderate speeds but decreases slightly with the increase in speed.

2 (a)	Define angle of friction and angle of repose with diagram.

Angle of Friction (ϕ)

Let us again consider a body of weight W which is placed over a rough surface and is subjected to an external force P as shown in Figure 8.3. The following forces are acting on the body:

(i) Self-weight, W
(ii) External force, P
(iii) Frictional force, F
(iv) Normal reaction, N

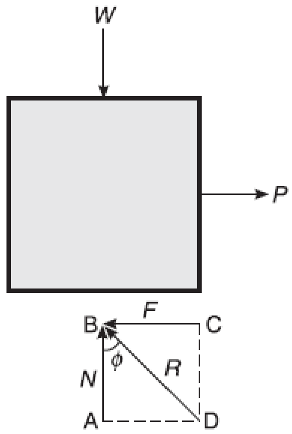

Figure 8.3 Angle of friction (ϕ).

The angle of friction for two contacting surfaces is the angle between the resultant R (of friction force F and the normal reaction N) and the normal reaction N. It is denoted by ϕ.

In triangle ABD,

$$\tan \phi = \frac{AD}{AB} = \frac{F}{N} = \mu \tag{8.1}$$

Angle of Repose (θ)

When a plane is inclined to the horizontal by a certain angle, the body placed on it will remain at rest up to a certain angle of inclination, beyond which the body just begins to move. This maximum angle made by the inclined plane with the horizontal, when the body placed on that plane is just at the point of sliding down the plane, is known as the angle of repose. Repose means sleep which is disturbed at that particular angle of inclination.

Let us consider a body of weight W which is placed on an inclined plane as shown in Figure 8.4. The body is just at the point of sliding down the plane when the angle of inclination is θ. The various forces acting on the body are self-weight, normal reaction, and frictional force.

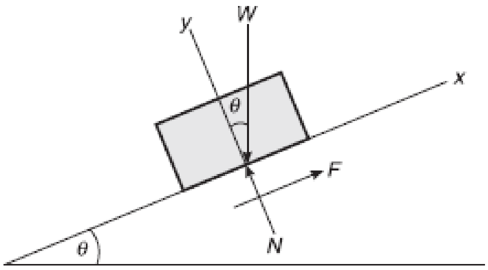

Figure 8.4 Angle of repose (θ).

Applying the conditions of equilibrium,

$$\Sigma F_x = 0; \ \Sigma F_y = 0$$

Resolving forces along the x-axis,

$$-F + W \sin \theta = 0$$

or $$F = W \sin \theta \tag{8.2}$$

Resolving forces along the y-axis,

$$N - W \cos \theta = 0$$

or $$N = W \cos \theta \tag{8.3}$$

We know that

$$\mu = \frac{F}{N}$$

$\Rightarrow$ $$\mu = \frac{W \sin \theta}{W \cos \theta} = \tan \theta \tag{8.4}$$

or $$\tan \phi = \tan \theta$$

or $$\phi = \theta$$

It is evident from Eqs. (8.1) and (8.4) that

Angle of friction = Angle of repose

2b) A block of mass 20 kg placed on an inclined plane as shown in Figure, is subjected to a force P that is parallel to the plane. Taking the inclination of plane with respect to horizontal as 30° and the coefficient of friction as 0.24, determine the value of P for impending motion of the block.

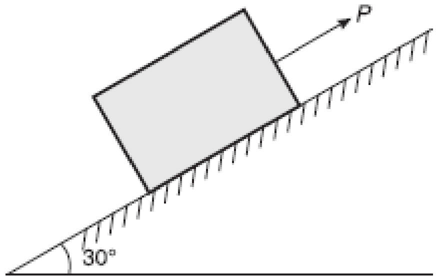

Figure 8.7 Example 8.2.

Solution (i) *The value of P for impending motion down the plane* (Figure 8.8):

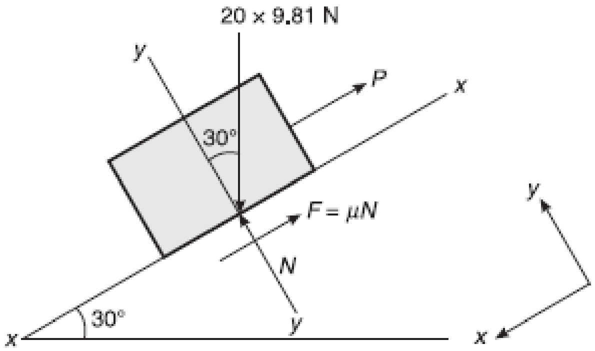

Figure 8.8 Example 8.2.

Consider the free body diagram of block

$$\Sigma F_y = 0$$

or $$20 \times 9.81 \times \cos 30° - N = 0$$

or $$N = 20 \times 9.81 \cos 30° = 169.914 \text{ N}$$

Also, $$\Sigma F_x = 0$$

or $$-P - 0.24 \times 169.914 + 20 \times 9.81 \sin 30°$$

∴ $$P = 57.320 \text{ N}$$ **Ans.**

(ii) *The value of P for impending motion up the plane* (Figure 8.9)

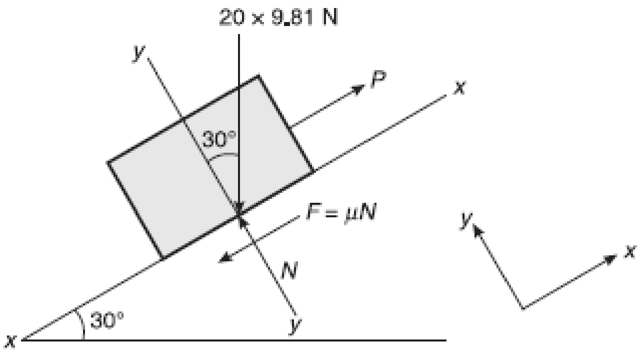

Figure 8.9 Example 8.2.

Consider the free body diagram of block

$$\Sigma F_y = 0$$

or $$N = 20 \times 9.81 \cos 30°$$

or $$N = 169.914 \text{ N}$$

Also, $$\Sigma F_x = 0$$

or $$-20 \times 9.81 \sin 30° + P - \mu N = 0$$

or $$P = 20 \times 9.81 \sin 30° + 0.24 \times 169.914$$

∴ $$P = 138.879 \text{ N}$$ **Ans.**

3) Find the least value of P required to cause the system of blocks shown in Figure to have impending motion to the left. Take coefficient of friction as 0.2 for all contact surfaces.

Example 8.13 Find the least value of P required to cause the system of blocks shown in Figure 8.39 to have impending motion to the left. Take $\mu = 0.2$.

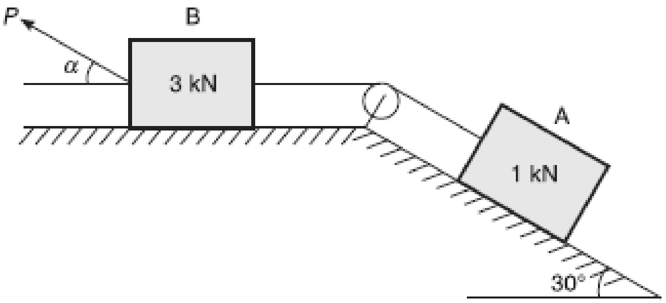

Figure 8.39 Example 8.13.

Solution Consider the free body diagram of block A (Figure 8.40):

$$\Sigma F_y = 0$$

or $$N_1 - 1 \times \cos 30° = 0$$

or $$N_1 = 0.866 \text{ kN}$$

Also, $$\Sigma F_x = 0$$

or $$T - 0.2(0.866) - 1 \times \sin 30° = 0$$

or $$T = 0.673 \text{ kN}$$

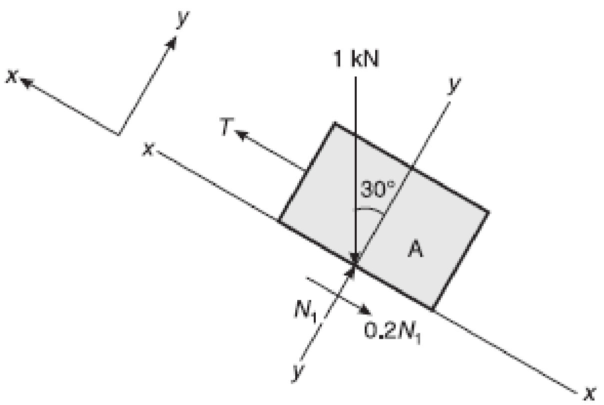

Figure 8.40 Example 8.13.

Consider the free body diagram of block B (Figure 8.41):

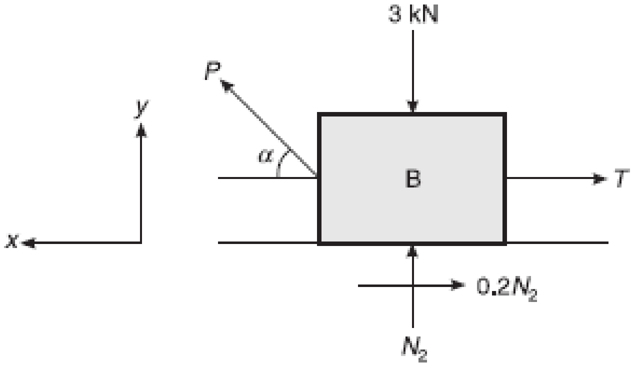

Figure 8.41 Example 8.13.

$$\Sigma F_y = 0$$

or $$N_2 = 3 - P \sin \alpha$$

Also, $$\Sigma F_x = 0$$

or $$P \cos \alpha - 0.673 - 0.2(3 - P \sin \alpha) = 0$$

or $$P \cos \alpha + 0.2P \sin \alpha - 1.273 = 0$$

or $$P \cos \alpha + 0.2P \sin \alpha = 1.273$$

or $$P(\cos \alpha + 0.2 \sin \alpha) = 1.273$$

or $$P = 1.273/(\cos \alpha + 0.2 \sin \alpha)$$

P is least, when the denominator is maximum.

Denominator is maximum when,

$$\frac{d}{d\alpha}(\cos \alpha + 0.2 \sin \alpha) = 0$$

or $$-\sin \alpha + 0.2 \cos \alpha = 0$$

or $$\sin \alpha = 0.2 \cos \alpha$$

or $$\tan \alpha = 0.2$$

or $$\alpha = \tan^{-1} (0.2)$$

$$= 11.31°$$

∴ $$P = 1.273/(\cos 11.31° + 0.2 \sin 11.31°)$$

$$= 1.248 \text{ kN}$$ **Ans.**

4 (a)
Determine the x and y coordinates of the centroid of a triangular section.

Triangle

Consider a triangular lamina of area (1/2) × b × d as shown in Figure 9.10.

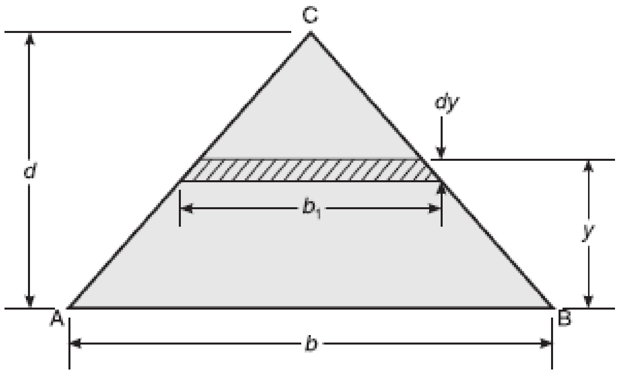

Figure 9.10 Triangular lamina.

Now consider an elementary strip of area $b_1 \times dy$ which is at a distance y from the reference axis AB.

Using the property of similar triangles, we have

$$\frac{b_1}{b} = \frac{d-y}{d}$$

or

$$b_1 = \frac{(d-y)b}{d}$$

$$\text{Area of the elementary strip} = b_1 \times dy = \frac{(d-y)b \cdot dy}{d}$$

Moment of area of elementary strip about AB

$$= \text{area} \times y$$

$$= \frac{(d-y)b \cdot dy \cdot y}{d}$$

$$= \frac{b \cdot dy \cdot d \cdot y}{d} - \frac{by^2 \cdot dy}{d}$$

$$= by \cdot dy - \frac{by^2 \cdot dy}{d}$$

Sum of moments of such elementary strips is given by

$$\int_0^d by \cdot dy - \int_0^d \frac{by^2}{d} \cdot dy$$

$$= b \times \left[\frac{y^2}{2}\right]_0^d - \frac{b}{d}\left[\frac{y^3}{3}\right]_0^d$$

$$= \frac{bd^2}{2} - \frac{bd^3}{3d}$$

$$= \frac{bd^2}{2} - \frac{bd^2}{3}$$

$$= \frac{bd^2}{6}$$

$$\text{Moment of total area about AB} = \frac{1}{2}bd \times \bar{y}$$

Applying the principle of moments,

$$\frac{bd^2}{6} = \frac{1}{2} \times bd \times \bar{y}$$

$$\therefore \quad \bar{y} = \frac{d}{3}$$

4b) Locate the centroid of the T-section shown in Figure.

Solution :

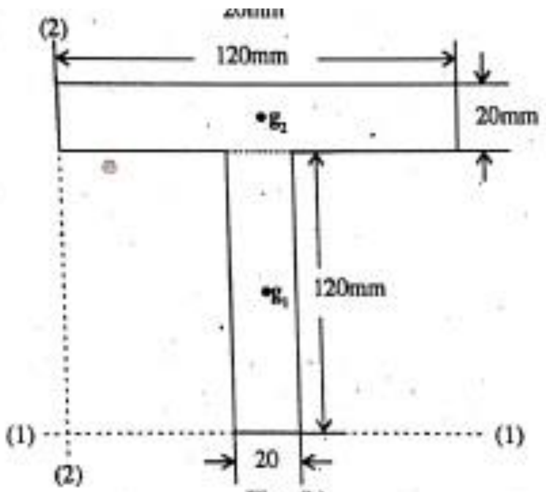

Fig. (b)

Consider reference (1)-(1) and (2)-(2) as shown. Divide the given figure into simple figure. The figure is symmetrical about YY-axis.

$\therefore$ Required to find $\bar{y} = \frac{\Sigma ay}{\Sigma a}$

$$\bar{x} = \frac{120}{20} = 60mm$$

Component	Area a mm^2	Distance of centroid of component from (1)-(1) y mm	Moment of area about (1)-(1) ay mm^3
g_1 (rectangle)	b × d 120 × 20 = 2400	$\frac{120}{2} = 60$	144000
g_2 (rectangle)	b × d 120 × 20 = 2400	$120 + \frac{20}{2} = 130$	312000
	$\Sigma a = 4800$		$\Sigma ay = 456000$

$$\therefore \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{456000}{4800}$$

$$\bar{y} = 95mm$$

5) Locate the centroid of the shaded area shown in Figure.

Example 9.14 Locate the centroid of the shaded area shown in Figure 9.30.

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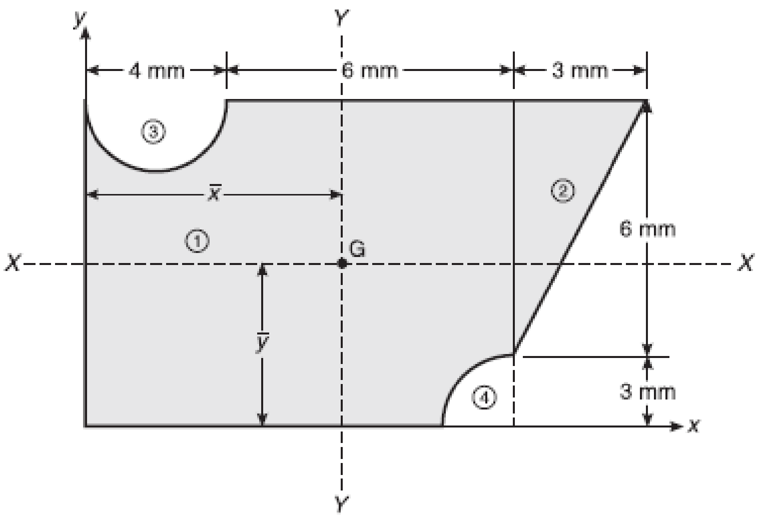


Figure 9.30 Example 9.14.

Solution

Component	*Area, a*	*x*	*y*	*ax*	*ay*
Rectangle 1	$10 \times 9 = 90$	$\frac{10}{2} = 5$	$\frac{9}{2} = 4.5$	450	405
Triangle 2	$\frac{1}{2} \times 3 \times 6 = 9$	$10 + \frac{1}{3} \times 3 = 11$	$3 + \frac{2}{3} \times 6 = 7$	99	63
Semicircle 3	$\frac{\pi \times (2)^2}{2} = -6.283$	$\frac{4}{2} = 2$	$9 - \frac{4 \times 2}{3\pi} = 8.151$	–12.566	–51.213
Quadrant 4	$\frac{\pi (3)^2}{4} = -7.068$	$10 - \frac{4 \times 3}{3\pi} = 8.727$	$\frac{4 \times 3}{3 \times \pi} = 1.273$	–61.682	–8.998
Sum	$\Sigma a = 85.649$			$\Sigma ax = 474.752$	$\Sigma ay = 407.789$

$$\bar{x} = \frac{\Sigma ax}{\Sigma a} = 5.543 \text{ mm}$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = 4.761 \text{ mm}$$

Ans.

6 (a) Explain the deferent types of beams and supports with diagram.

SUPPORT REACTIONS

The various structural members are connected to the surroundings by various types of supports. The structural members exert forces on supports known as *action*. Similarly, the supports exert forces on structural members known as *reaction*.

A beam is a horizontal member, which is generally placed on supports. The beam is subjected to vertical forces known as action. Supports exert forces, known as reaction, on the beam.

Types of Supports

The following types of supports are found in practice:

1. Simple supports
2. Roller supports
3. Hinged or pinned supports
4. Fixed supports

Simple supports

Simple supports (Figure 6.1) are those which exert reactions perpendicular to the plane of support. They restrict translation of the body in one direction only, but do not restrict rotation.

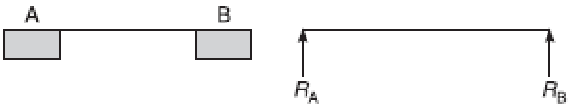


Figure 6.1 Simple supports.

Roller supports

Roller supports (Figure 6.2) are those which exert reactions perpendicular to the plane of the support. They restrict translation of the body along one direction only, and rotation is allowed.

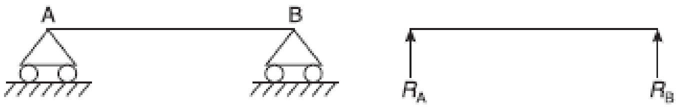


Figure 6.2 Roller supports.

Hinged or pinned supports

Hinged supports (Figure 6.3) are those which exert reactions in any direction, but from our convenient point of view we resolve these reactions into two components. Therefore, hinged supports restrict translation in both directions. But rotation is possible.

Figure 6.3 Hinged supports.

Fixed supports

Fixed supports (Figure 6.4) are those which restrict both translation and rotation of the body. Fixed supports develop an internal moment known as *restraint* moment to prevent the rotation of the body.

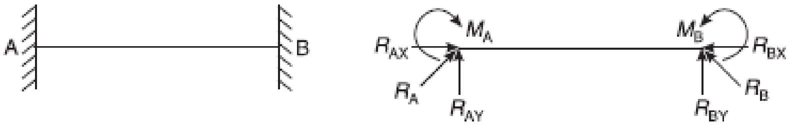


Figure 6.4 Fixed supports.

Types of Beams

Simply supported beam

It is a beam which consists of simple supports (Figure 6.5). Such a beam can resist forces normal to the axis of the beam.

Figure 6.5 Simply supported beam.

Continuous beam

It is a beam which consists of three or more supports (Figure 6.6).

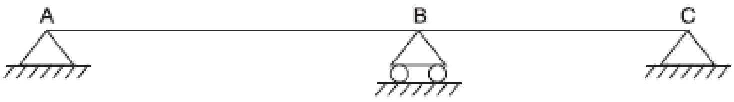


Figure 6.6 Continuous beam.

Cantilever beam

It is a beam whose one end is fixed and the other end is free (Figure 6.7).

Figure 6.7 Contilever beam.

Propped cantilever beam

It is a beam whose one end is fixed and the other end is simply supported (Figure 6.8).

Figure 6.7 Contilever beam.

Propped cantilever beam

It is a beam whose one end is fixed and the other end is simply supported (Figure 6.8).

Figure 6.8 Propped cantilever beam.

Overhanging beam

It is a beam which extends beyond support(s). In Figure 6.9, it is seen that the beam extends beyond support B up to C. The overhang portion is BC.

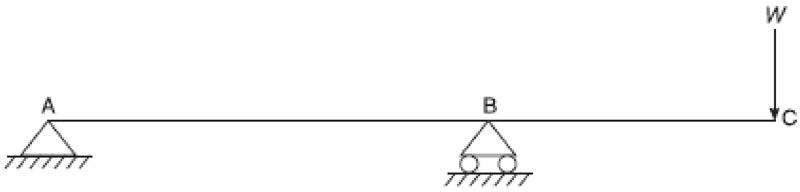


Figure 6.9 Overhanging beam.

Types of Loads

Concentrated load

A load which is concentrated at a point in a beam is known as concentrated load (Figure 6.10).

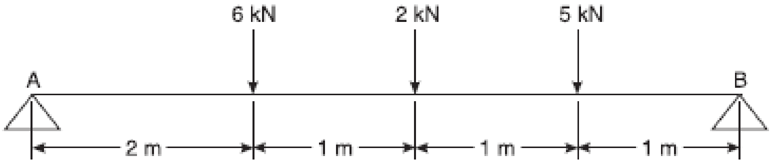


Figure 6.10 Concentrated loads.

Uniformly distributed load

A load which is distributed uniformly along the entire length of the beam is known as uniformly distributed load (such as the load 20 kN per metre (UDL), length shown in Figure 6.11.

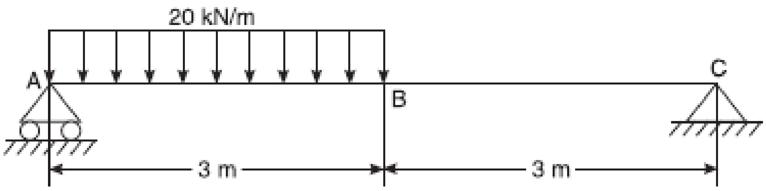


Figure 6.11 Uniformly distributed load.

To convert the 20 kN/m UDL into a point load which is acting at the centre of a particular span (i.e. 3 m), we proceed as follows:

$$\text{Magnitude of point load} = 20 \text{ kN/m} \times 3 \text{ m} = 60 \text{ kN}$$

Uniformly varying load

A load which varies with the length of the beam is known as uniformly varying load (Figure 6.12). The magnitude of the point load corresponding to a uniformly varying load such as that shown in Figure 6.12, is calculated as follows:

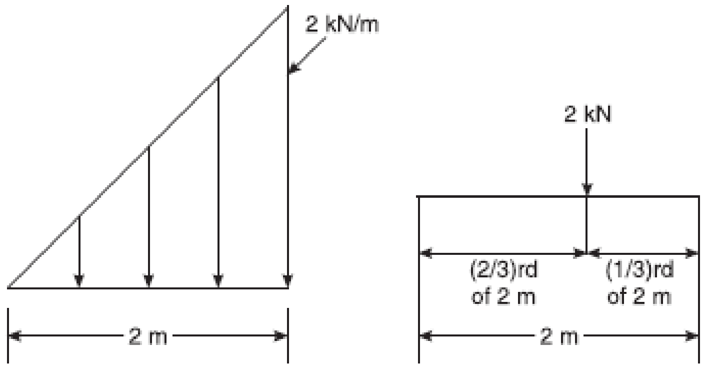


Figure 6.12 Uniformly varying load.

$$\text{Magnitude of point load} = \text{Area of the triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2 \times 2 = 2 \text{ kN}$$

The point load acts at the centre of gravity (CG) of the triangle.

6b) A simply supported beam of span 6 m is subjected to loading as shown in the figure. Determine the reactions at A and B.

Example 6.2 A simply supported beam of span 6 m is subjected to loading as shown in Figure 6.15. Determine the reactions at A and B.

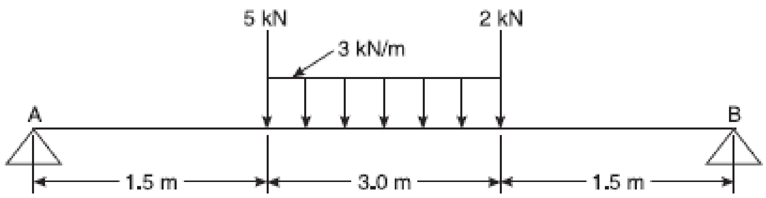


Figure 6.15 Example 6.2.

Solution Converting the UDL of 3 kN/m over a span of 3 m into a point load, and applying the laws of equilibrium (Figure 6.16), we get

$$F_y = 0$$

or $$R_A + R_B - 5 - 9 - 2 = 0$$

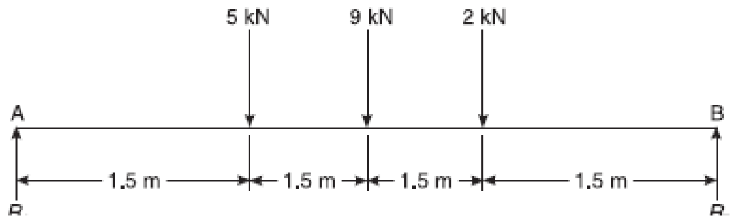
or $$R_A + R_B = 16$$

$$\Sigma M_B = 0$$

or $$-2 \times 1.5 - 9 \times 3 - 5 \times 4.5 + R_A \times 6 = 0$$

∴ $$R_A = \frac{52.5}{6} = 8.75 \text{ kN}$$ **Ans.**

or $$R_B = 7.25 \text{ kN}$$ **Ans.**



7) Find the support reactions of the beam loaded as shown in Figure.

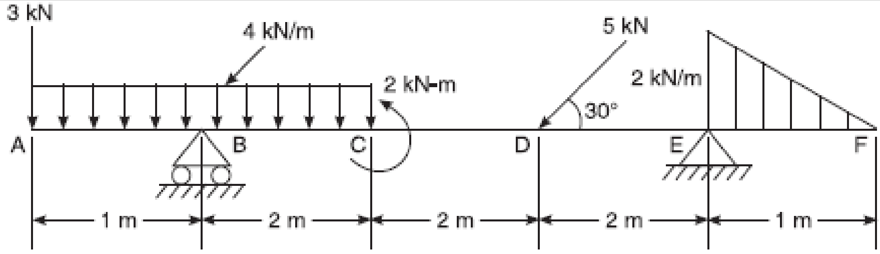


Figure 6.31 Example 6.10.

Solution Using the conditions of equilibrium shown in Figure 6.32, we have

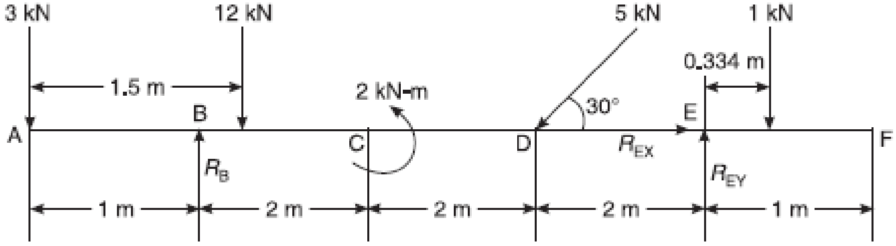


Figure 6.32 Example 6.10.

$$\Sigma F_x = 0$$

or $$R_{EX} - 5 \cos 30° = 0$$

∴ $$R_{EX} = 4.33 \text{ kN}$$

Also, $$\Sigma F_y = 0$$

or $$R_B + R_{EY} - 3 - 12 - 5 \sin 30° - 1 = 0$$

or $$R_B + R_{EY} = 18.5 \qquad \text{(i)}$$

Also, $$\Sigma M_B = 0$$

or $$-R_{EY} \times 6 + 12 \times 0.5 - 3 \times 1 - 2 + 5 \sin 30° \times 4 + 1 \times 6.334 = 0$$

∴ $$R_{EY} = 2.889 \text{ kN}$$

Substituting the value of R_{EY} in (i), we get

$$R_B = 15.611 \text{ kN} \qquad \textbf{Ans.}$$

$$R_E = \sqrt{(4.33)^2 + (2.889)^2} = 5.205 \text{ kN} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{R_{EY}}{R_{EX}}\right)$$

∴ $$\theta = \tan^{-1}\left(\frac{2.889}{4.33}\right) = 33.71° \qquad \textbf{Ans.}$$