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Internal Assessment Test 1-QP and solutions – Nov. 2022

Sub:	Analysis of Indeterminate structures					Sub Code:	18CV52	Branch:	Civil Engg
Date:	04.11.2022	Duration:	90 min’s	Max Marks:	50	Sem / Sec:	4 A		OBE

	Question number 1 is mandatory, answer any 2 full question from Q2 to Q4.	MARKS	CO	RBT
1 (a)	Determine the fixed end moments for the beam shown in Fig.1.a. Consider that EI is constant through the span of continuous beam	[10]	CO1	L2

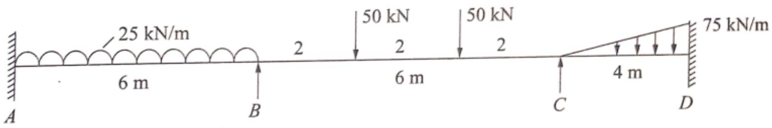
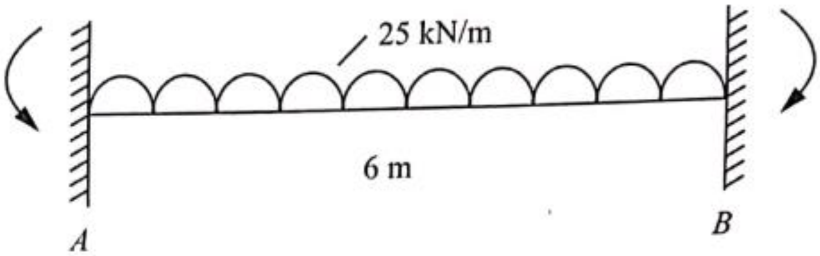


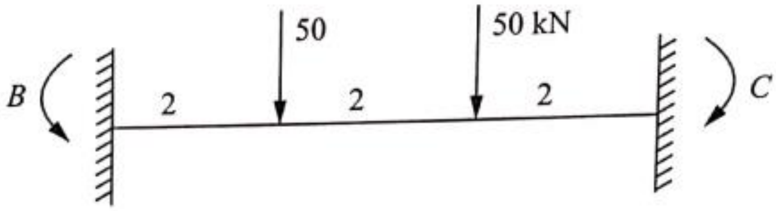
Fig.1.a.

Soln :

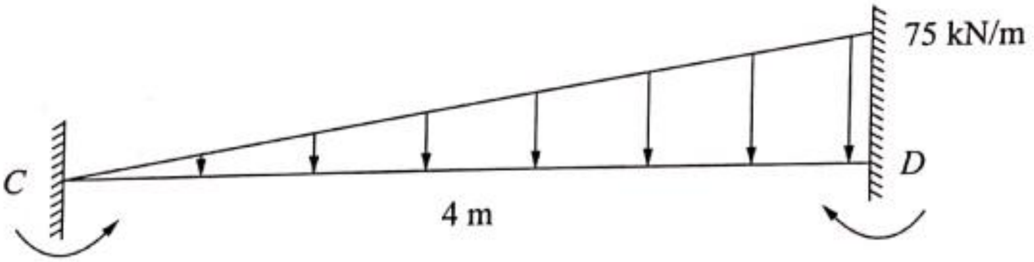
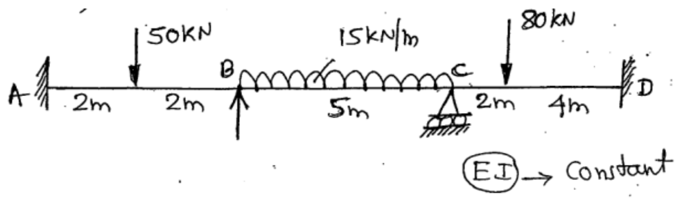
3. Fixed Moments



$$\frac{-25 \times 6^2}{12} = -75 \text{ kNm} \qquad \frac{+25 \times 6^2}{12} = +75 \text{ kNm}$$



$$\frac{-2 \times 50 \times 4}{6} = -66.67 \text{ kNm} \qquad \frac{+2 \times 50 \times 4}{6} = +66.67 \text{ kNm}$$

	 **FIG. 1.15** $\frac{-75 \times 4^2}{30} = -40$ kNm $\frac{+75 \times 4^2}{20} = +60$ kNm			
2 (a)	Analyse the continuous beam shown in Fig.2.a by slope deflection method and draw BMD, SFD and EC.  Fig.2.a **Soln:**	[20]	CO1	L3

Sol

(a) FEM

$$M_{FAB} = -\frac{Wl}{8} = \frac{-50\times4}{8} = -25\,kN\text{-}m$$

$$M_{FBA} = +\frac{Wl}{8} = +25\,kN\text{-}m$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{15\times5^2}{12} = -31.25$$

$$M_{FCB} = +\frac{wl^2}{12} = +31.25$$

$$M_{FCD} = -\frac{Wab^2}{l^2} = -\frac{80\times2\times4^2}{6^2} = -71.11\,kN\text{-}m$$

$$M_{FDC} = +\frac{Wa^2b}{l^2} = \frac{80\times2^2\times4}{6^2} = +35.56\,kN\text{-}m$$

(b) S.D. Equation :

$\theta_A = \theta_D = 0$ ($\because$ Fixed Support)

$\delta = 0$ ($\because$ No sinking)

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}$$

$$M_{AB} = \frac{2EI}{4}\left[0+\theta_B-0\right]-25 = 0.5EI\theta_B - 25 \quad \text{—(}$$

$$M_{BA} = \frac{2EI}{4}\left[2\theta_B+0-0\right]+25 = EI\theta_B + 25 \quad \text{—(II)}$$

$$M_{BC} = \frac{2EI}{5}\left[2\theta_B+\theta_C-0\right]-31.25$$

$$= 0.8EI\theta_B + 0.4EI\theta_C - 31.25 \quad \text{—(II}$$

$$M_{CB} = \frac{2EI}{5}\left[2\theta_C+\theta_B-0\right]+31.25$$

$$= 0.8EI\theta_C + 0.4EI\theta_B + 31.25 \quad \text{—(IV}$$

$$M_{CD} = \frac{2EI}{6}\left[2\theta_C+0-0\right]-71.11 = 0.667EI\theta_C - 71.11 \quad -$$

$$M_{DC} = \frac{2EI}{6}\left[0+\theta_C-0\right]+35.56 = 0.333EI\theta_C + 35. \quad \text{—(}$$

(c) Equillibrium Condition :—

(1) At "B" $\boxed{M_{BA} + M_{BC} = 0}$ → Intermediate support

$$\left[EI\theta_B+25\right] + \left[0.8EI\theta_B + 0.4EI\theta_C - 31.25\right] = 0$$

$1.8EI\theta_B + 0.4EI\theta_C = 6.25 \rightarrow$ (I)

(2) At "c" $\boxed{M_{CB} + M_{CD} = 0}$

$$\left[0.8EI\theta_C + 0.4EI\theta_B + 31.25\right] + \left[0.667EI\theta_C - 71.11\right] = 0$$

$0.4EI\theta_B + 1.467EI\theta_C = 39.86 \rightarrow$ (II)

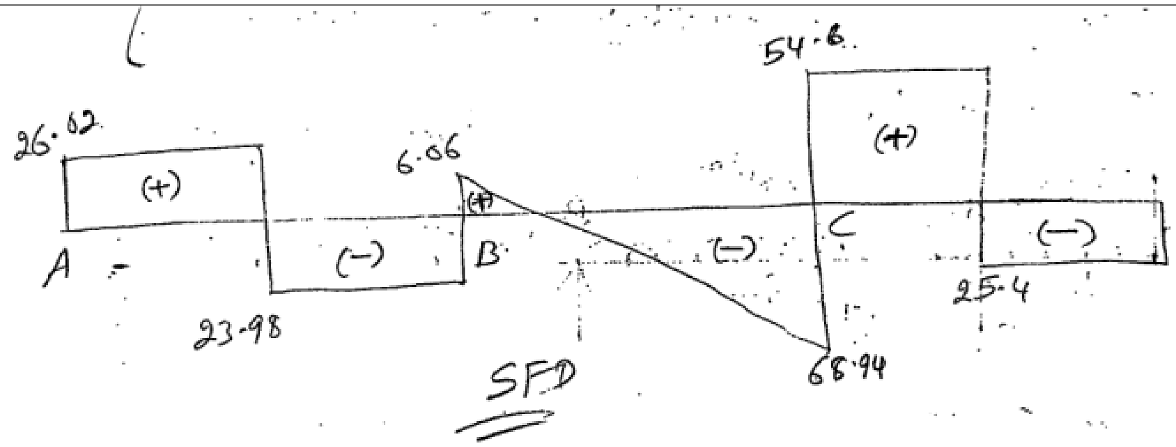
Solving
$$\theta_B = -2.73/EI$$
$$\theta_C = +27.91/EI$$

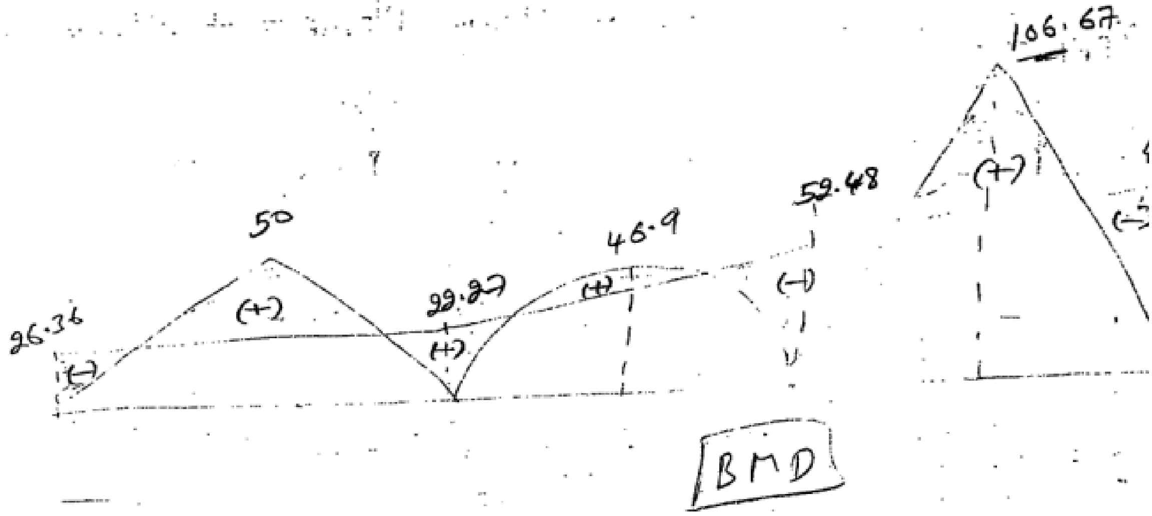
(d) Final Moments [substitute θ value in eq^n (i) to (vi)]

$M_{AB} = -26.36$ kN-m ↺ $\qquad M_{CB} = 52.48$ kN-m ↻

$M_{BA} = 22.27$ kN-m ↻ $\qquad M_{CD} = -52.49$ kN-m ↺

$M_{BC} = -22.27$ kN-m ↺ $\qquad M_{DC} = 44.85$ kN-m ↻





3 (a)	Analyse the continuous beam shown in Fig.3.a by slope deflection method and draw BMD, SFD and EC.	[20]	CO1	L3

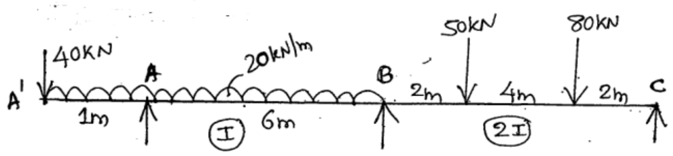
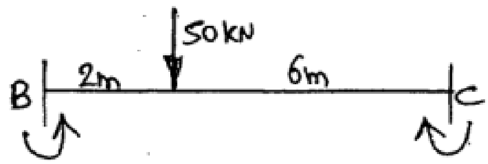
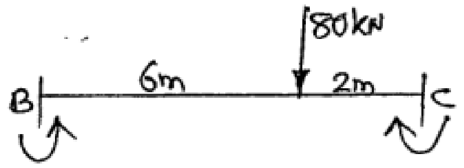


Fig.3.a

Sol^n^ (a) FEM

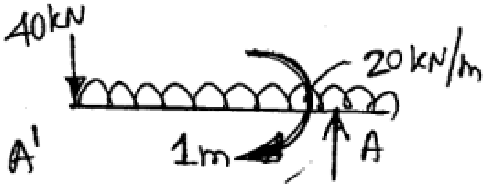
$$M_{FAB} = -\frac{wl^2}{12} = -\frac{20\times6^2}{12} = -60\,kN\text{-}m, \quad M_{FBA} = +60\,kN\text{-}m$$





$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times2\times6^2}{8^2} + \frac{80\times6\times2^2}{8^2}\right] = -86.25\,kN\text{-}m$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +\left[\frac{50\times2^2\times6}{8^2} + \frac{80\times6^2\times2}{8^2}\right] = +108.75\,kN\text{-}m$$



$$M_{AA'} = +\left[40\times1 + 20\times1\times\tfrac{1}{2}\right]$$

$$= +50\,kN\text{-}m$$

+ve sign for clockwise resisting moment.

(b) S.D. Equation

$\delta = 0$ (No sinking)

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}$$

$$M_{AB} = \frac{2(1\times EI)}{6}\left[2\theta_A + \theta_B - 0\right] - 60$$

$$= (0.667EI)\theta_A + (0.333EI)\theta_B - 60 \quad \text{— (i)}$$

$$M_{BA} = \frac{2(1\times EI)}{6}\left[2\theta_B + \theta_A - 0\right] + 60$$

$$= (0.667EI)\theta_B + (0.333EI)\theta_A + 60 \quad \text{— (ii)}$$

$$M_{BC} = \frac{2(2EI)}{8}\left[2\theta_B + \theta_C - 0\right] - 86.25$$

$$= EI\theta_B + 0.5EI\theta_C - 86.25 \quad \text{— (iii)}$$

$$M_{CB} = \frac{2(2EI)}{8}\left[2\theta_C + \theta_B - 0\right] + 108.75$$

$$= EI\theta_C + 0.5EI\theta_B + 108.75 \quad \text{— (iv)}$$

★ **Note:-** There is **NO** S.D. Equation for "overhang"

(c) Equillibrium condition :-

(i) At "A" $\boxed{M_{AA'} + M_{AB} = 0}$

$$[50] + [0.667EI\theta_A + 0.333EI\theta_B - 60] = 0$$

$$(0.667EI)\theta_A + (0.333EI)\theta_B = 10 \rightarrow \text{(I)}$$

(ii) At "B" $M_{BA} + M_{BC} = 0$ (19

$(0.333\,EI)\theta_A + (1.667\,EI)\theta_B + (0.5\,EI)\theta_C = 26.25 \rightarrow$ (II)

(iii) At "C" $M_{CB} = 0$ ($\because$ last Simple or Hinge or Roller support)

$(0.5\,EI)\theta_B + (1\,EI)\theta_C = -108.75 \longrightarrow$ (III)

Solving

$\theta_A = \frac{-15.197}{EI}$

$\theta_B = \frac{60.47}{EI}$

$\theta_C = \frac{-138.98}{EI}$

(d) Final Moment:

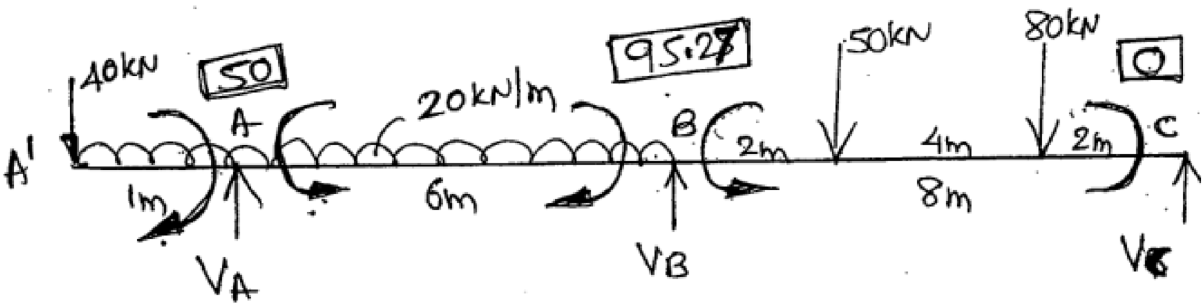
$M_{AB} = -50$ kN-m ↻

$M_{BA} = 95.27$ kN-m ↺

$M_{BC} = -95.27$ " ↻

$M_{CB} = 0$

$M_{AA'} = +50$ kN-m ↺



$\Sigma V = 0, \; V_A + V_B + V_C = 40 + 50 + 80 + 20 \times 7 = 310$ — (i)

$\Sigma M_B = 0$ (RHS) $- V_C \times 8 + 50 \times 2 + 80 \times 6 - 95.27 = 0$

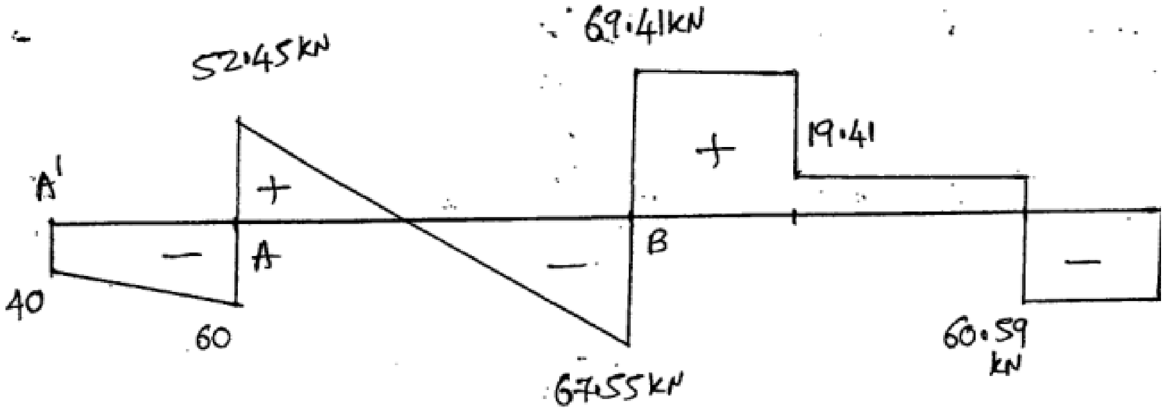
$V_C = 60.59$ kN

$\Sigma M_B = 0$ (LHS)

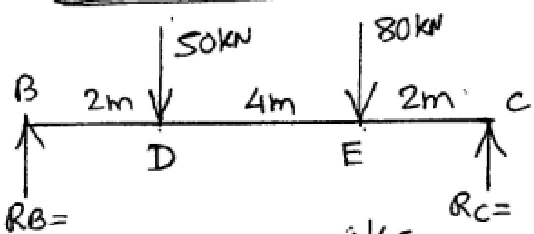
$V_A \times 6 - 40 \times 7 - 20 \times 7 \times 7/2 + 50 - 50 + 95.27 = 0$

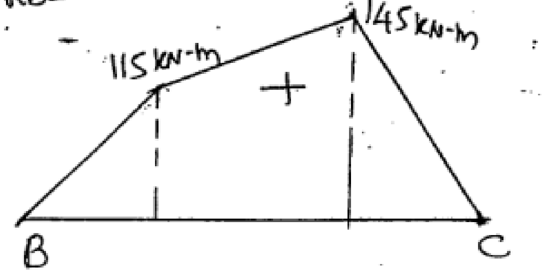
$V_A = 112.45$ kN

From (i) $112.45 + V_B + 60.59 = 310$ $\therefore V_B = 136.96$



Free BMD for "BC"





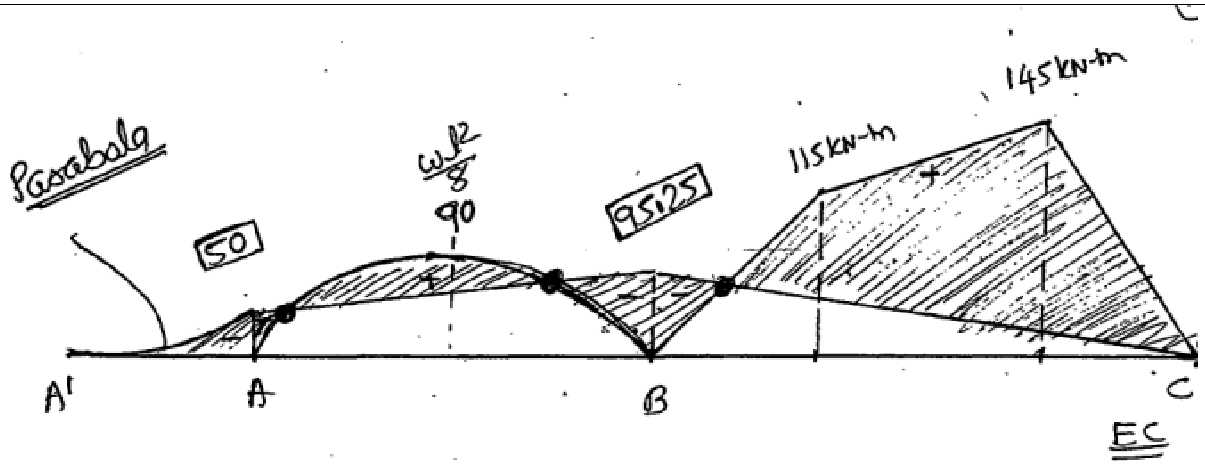
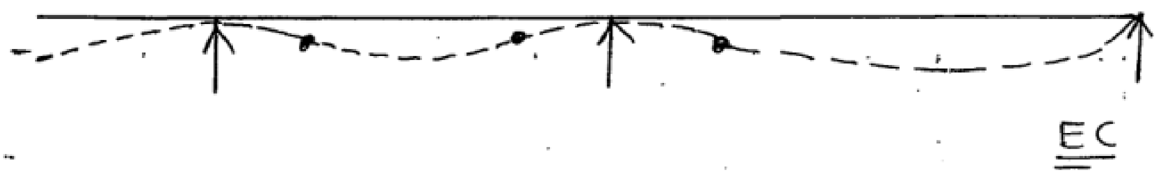
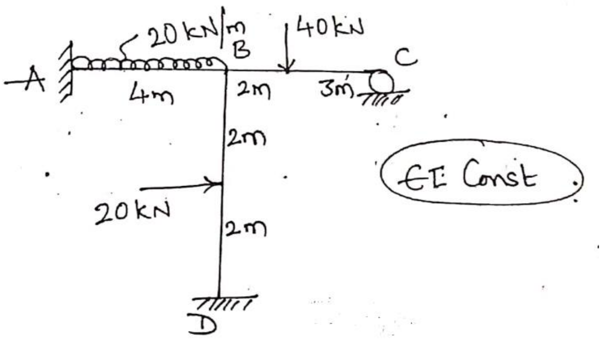
Reaction

$R_C = 72.5$ kN

$R_B = 57.5$ kN

$\therefore M_D = R_B \times 2 = 115$ kN-m

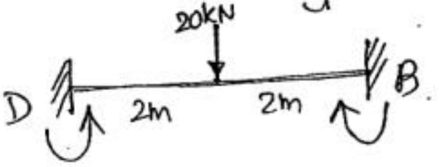
$M_E = R_C \times 2 = 145$ kN-m

	 			
4 (a)	Analyse the rigid frame shown in Fig.4.a by slope deflection method and draw BMD and EC.  Fig.4.a	[20]	CO1	L3

Sol[12] (a) **FEM**

$M_{FAB} = -\frac{\omega l^2}{12} = -26.67$, $M_{FBA} = +26.67$

$M_{FBC} = -\frac{Wab^2}{l^2} = -28.8$, $M_{FCB} = +\frac{Wa^2b}{l^2} = +19.2$



$M_{FDB} = -\frac{Wl}{8} = -10$

$M_{FBD} = +10$

(b) **S.D. Equation** :

$\theta_A = \theta_D = 0$ (Fixed)

$\delta = 0$ (Non-Sway)

$M_{AB} = \frac{2EI}{4}\left[\theta_B\right] - 26.67 = 0.5EI\theta_B - 26.67$ —(i)

$M_{BA} = \frac{2EI}{4}\left[2\theta_B\right] + 26.67 = EI\theta_B + 26.67$ —(ii)

$M_{BC} = \frac{2EI}{5}\left[2\theta_B + \theta_C\right] - 28.8$ (39)

$= 0.8EI(\theta_B) + 0.4EI\theta_C - 28.8$ —(iii)

$M_{CB} = \frac{2EI}{5}\left[2\theta_C + \theta_B\right] + 19.2$

$= 0.8EI\theta_C + 0.4EI\theta_B + 19.2$ —(iv)

$M_{BD} = \frac{2EI}{4}\left[2\theta_B\right] + 10 = EI(\theta_B) + 10$ —(v)

$M_{DB} = \frac{2EI}{4}\left[\theta_B\right] - 10 = 0.5EI(\theta_B) - 10$ —(vi)

(c) Equillibrium Condition :-

at "B" $M_{BA} + M_{BC} + M_{BD} = 0$

$$2.8EI(\theta_B) + 0.4EI(\theta_C) = -7.87 \rightarrow (I)$$

At "C" $M_{CB} = 0$

$$0.4EI(\theta_B) + 0.8EI(\theta_C) = -19.2 \rightarrow (II)$$

solving

$$\theta_B = \frac{0.67}{EI}$$

$$\theta_C = -\frac{24.33}{EI}$$

(d) Final Moment

$M_{AB} = -26.33$ kN-m

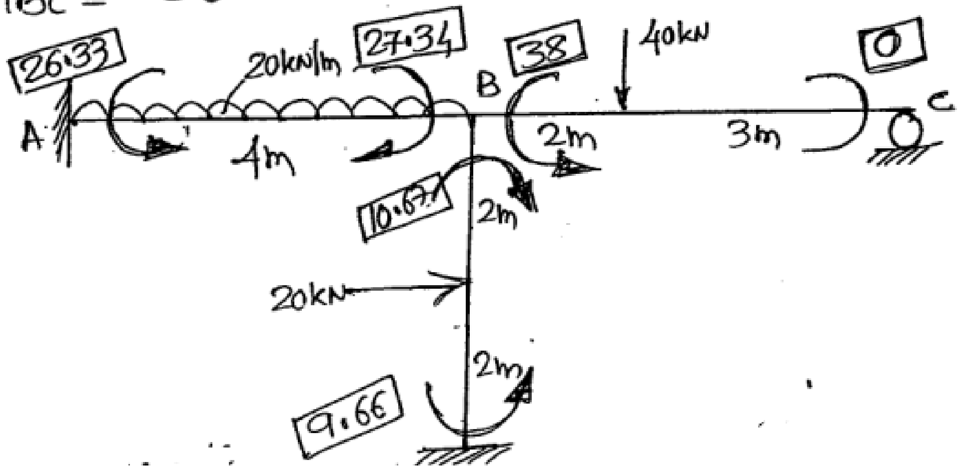
$M_{CB} = 0$

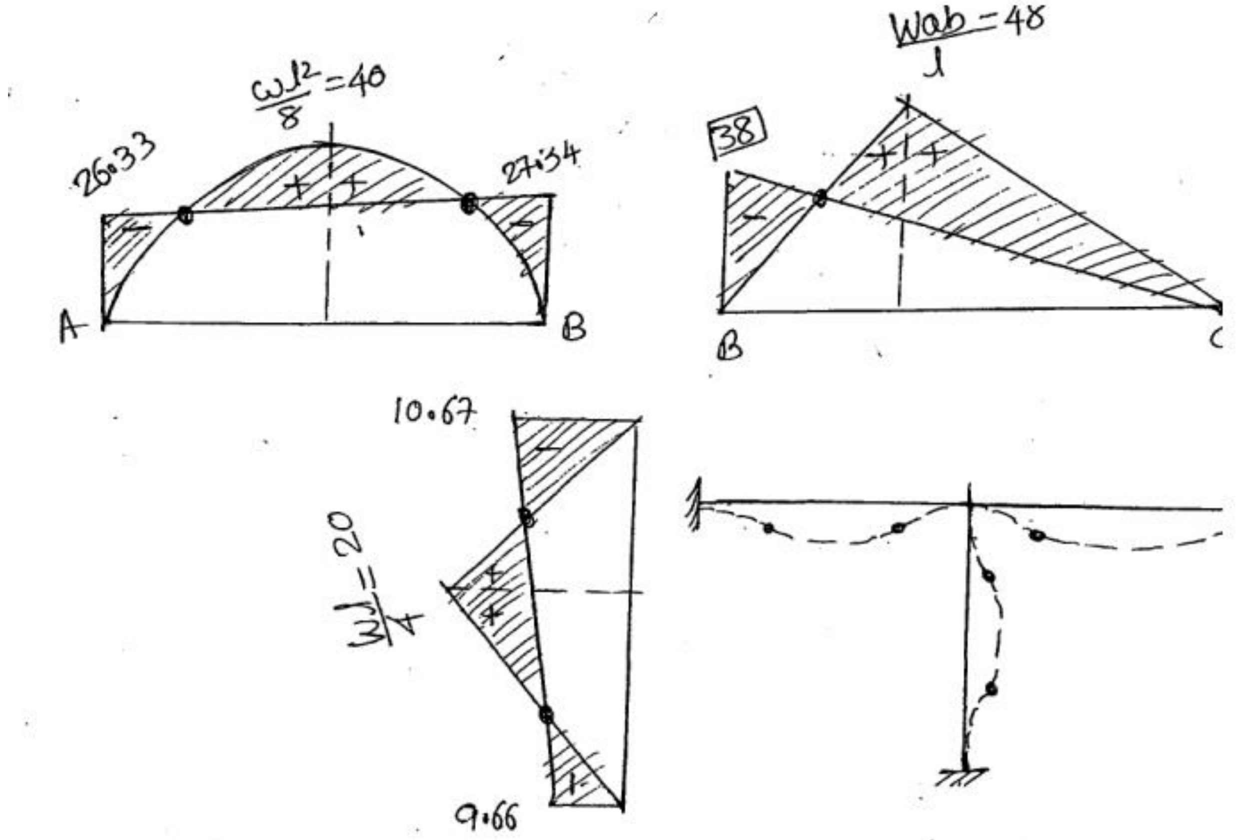
$M_{BA} = 27.34$ kN-m

$M_{BD} = 10.67$ kN-m

$M_{BC} = -38.00$ kN-m

$M_{DB} = -9.66$ kN-m





$\frac{wl^2}{8} = 40$
26.33
27.34
A
B
$\frac{Wab}{l} = 48$
38
B
C
10.67
$\frac{Wl}{4} = 20$
9.66