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Internal Assessment Test - II

| Sub: | SIGNALS AND SYSTEMS | | | | | | | Code: | 18EE54 |
|---|---|---|---|---|---|---|---|---|---|
| Date: | 3/12/22 | Duration: | 90 mins | Max Marks: | 50 | Sem: | 5th | Branch: | EEE |

Answer Any FIVE FULL Questions

| | | Marks | OBE | |
|---|---|---|---|---|
| | | | CO | RBT |
| 1 | Compute the convolution sum of the 2 sequences, $x_1$[n] and $x_2$[n], given below<br>$x_1[n] =$ ( 3,2,1,2) , $x_2[n] =$ (1,2,1,2).Also verify the results with tabular method. | 10 | CO2 | L3 |
| 2 | Find the step response of an LTI system, if the impulse response are<br>(i)$h(t)=t^2u(t)$ ii)$h(n)=(\frac{1}{2})^n u(n)$ | 10 | CO2 | L3 |
| 3 | Find the natural response, forced response and complete response of the given differential equation.$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+8y(t)=\frac{dx(t)}{dt}+2x(t)$with y(0)= 2, $\frac{dy(t)}{dt}\|_{t=0}=3$ and $x(t)=e^{-t}u(t)$ | 10 | CO2 | L3 |

**P.T.O**

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**P.T.O**

| | | | | |
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| 4 | Obtain direct form-1 and direct form 2 representations for the following<br>.$\frac{d^2y(t)}{dt^2}$+3 $\frac{dy(t)}{dt}$ +2 y(t)= $\frac{d^2x(t)}{dt^2}$+ $\frac{dx(t)}{dt}$ | 10 | CO2 | L3 |
| 5 | Explain the following properties of impulse response (a)Distributive and causal<br>(b)Associative | 10 | CO2 | L2 |
| 6 | If $h(t) = u(t) - u(t-2)$ and $x(t) = u(t) - u(t-3)$, Determine the output $y(t)$ given $h(t)$ is the impulse response and $x(t)$ is the input for the LTI system. | 10 | CO2 | L3 |

**CI** **CCI** **HOD**


| | | | | |
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**CI** **CCI** **HOD**

1.

1) $x_1[n] = (3, \underset{\uparrow}{2}, 1, 2)$

$x_2[n] = (1, 2, \underset{\uparrow}{1}, 2)$

$x_1 \rightarrow 0$ to $3$

$x_2 \rightarrow 0$ to $3$

$y[n] = \sum_{-\infty}^{\infty} x[n]\, h[n-K]$ $\quad n \rightarrow 0$ to $6$

K limit is same as n

$y[n] = \sum x[n]h[n] + x[n]h[n-1] + x[n]h[n-2] + x[n]h[n-3]$
$+ x[n]h[n-4] + x[n]h[n-5] + x[n]h[n-6]$

$\therefore y[0] = x[0]h[0] + x[0]h[-1] + x[0]h[-2] + x[0]h[-3] + x[6]h[-4]$
$+ x[5]h[-5] + x[6]h[-6]$

$= 3\times1 + 2\times0 + 3\times0 + 3\times0 + 3\times0 + 3\times0$

$\boxed{y[0] = 3}$

$y[1] = x[1]h[1] + x[1]h[0] + x[1]h[0] + x[1]h[-2] + x[1]h[-3]$
$+ x[1]h[-4] + x[1]h[-5]$

$= 2\times2 + 3\times1 + 00$

$\boxed{y[1] = 8}$

$y[3] = x[3]h[3] + x[3]h[2] + x[3]h[1] + x[3]h[0] + x[3]h[-1]$
$+ x[3]h[-2] + x[3]h[-3]$

$= 2\times2 + 2\times1 + 2\times2 + 2\times1$

$\boxed{y[3] = 12}$

$y[4] = x[4]h[4] + x[4]h[3] + x[4]h[2] + x[4]h[1] + x[4]h[0]$

$= 0 + 0 + 3\times3$

$\boxed{y[4] = 9}$

$y[5] = x[5]h[6] + x[5]h[4] + x[5]h[3] + x[5]h[2] + x[5]h[1]$

$= 2\times1 + 1\times2$

$\boxed{y[5] = 4}$

$y[6] = x[6]h[6] + x[6]h[5] + x[6]h[4] + x[6]h[3] + x[6]h[2]$
$x[6]h[1] + x[6]h[0]$

$= 0 + 0 + 1\times2 + 2 + 0$ $\quad \boxed{y[6] = 4}$

| $x_2[n]$ \ $x_1[n]$ | 3 | 2 | 1 | 2 |
|---|---|---|---|---|
| 1 | 3 | 2 | 1 | 2 |
| 2 | 6 | 4 | 2 | 4 |
| 1 | 3 | 2 | 1 | 2 |
| 2 | 6 | 4 | 2 | 4 |

$y[n] = \{3, 8, 8, \underset{\uparrow}{12}, 9, 4, 4\}$

2.

$$h(t) = t^2 u(t)$$

We know that the step response is given by.

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$

$$= \int_{0}^{t} \tau^2 d\tau$$

$$= \frac{\tau^3}{2} \Bigg|_{0}^{t}$$

$$= \frac{t^3}{2} \qquad ; t \geq 0$$

$\therefore$ The step response $s(t) = \frac{t^3}{2} u(t)$

$$s(n) = \sum_{k=-\infty}^{n} h(k)$$

For $n<0$ : $s(n) = 0$

For $n \geq 0$ $\therefore$ $s(n) = \sum_{k=0}^{n} (½)^k$

$$s(n) = \frac{1 - (½)^{n+1}}{1 - ½}$$

$$= 2[1-(½)^{n+1}] \qquad : n \geq 0$$

$$\therefore s(n) = 2[1-(½)^{n+1}]\, u(n)$$

**3.**

$$r^2+6r+8=0$$
$$r^2+4r+2r+8=0$$
$$r(r+4)+2(r+4)=0 \quad (r+4)(r+2)=0$$
$$y^h(t) = c_1 e^{-4t} + c_2 e^{-2t}$$
$$y(0)=2 \qquad 2 = c_1 + c_2$$
$$\frac{dy(t)}{dt} = -4c_1 e^{-4t} - 2c_2 e^{-2t}$$
$$3 = -4c_1 - 2c_2$$
$$c_1 = -\frac{7}{2} \quad c_2 = \frac{11}{2} \qquad 4\times c_1 +$$
$$\boxed{y^h(t) = -\frac{7}{2} e^{-4t} + \frac{11}{2} e^{-2t}}$$
$$x(t) = e^{-t}u(t) \quad y^p(t) = Ke^{-t}$$
$$\frac{d^2}{dt^2}(Ke^{-t}) + 6\frac{d}{dt}(Ke^{-t}) + 8Ke^{-t} = \frac{d}{dt}(e^{-t}) + 2e^{-t}$$
$$+Ke^{-t} - 6Ke^{-t} + 8Ke^{-t} = -e^{-t} - 2e^{-t}$$
$$3Ke^{-t} = -3e^{-t} \quad \boxed{K=1}$$
$$y^f(t) = c_3 e^{-4t} + c_4 e^{-2t} + e^{-t}$$

$y(0) = 0 \qquad \left.\frac{dy(t)}{dt}\right|_{t=0} = 0$

$0 = C_3 + C_4 + 1$

$\frac{dy(t)}{dt} = -4C_3 e^{-4t} - 2C_4 e^{-2t} - e^{-t}$

$0 = -4C_3 - 2C_4 - 1$

$C_3 + C_4 = -1$

$-4C_3 - 2C_4 = 1$

$C_3 = 1/2 \qquad C_4 = -3/2$

$y^f(t) = \frac{1}{2} e^{-4t} - \frac{3}{2} e^{-2t} + e^{-t}$

$y(t) = y^n(t) + y^f(t)$

$= -\frac{7}{2} e^{-4t} + \frac{11}{2} e^{-2t} + \frac{1}{2} e^{-4t} - \frac{3}{2} e^{-2t} + e^{-t}$

$-3e^{-4t} + 2e^{-2t} + e^{-t}$

$\boxed{y(t) = -3e^{-4t} + 2e^{-2t} + e^{-t}}$

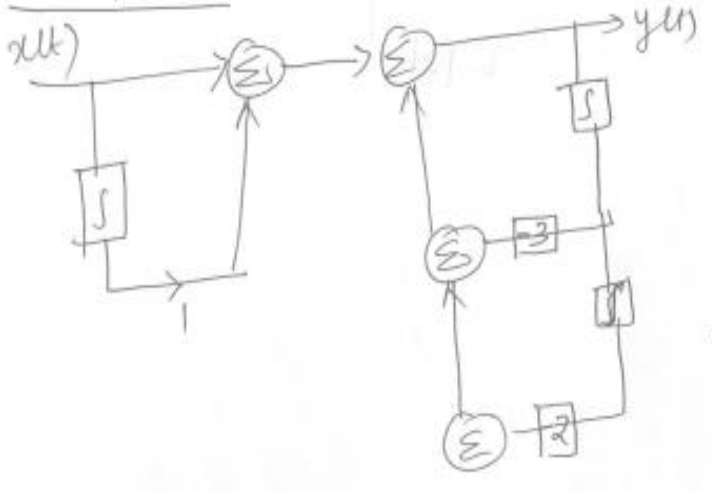
**4.**

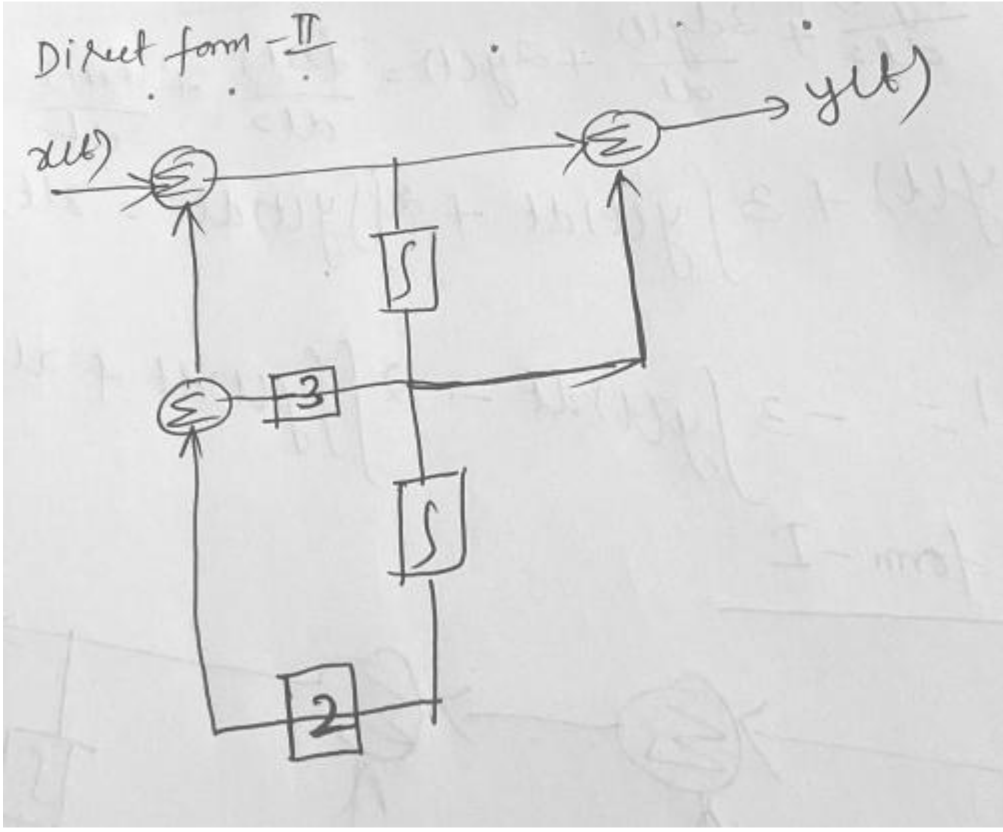
(4) $\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt}$

$y(t) + 3\int y(t)dt + 2\iint y(t)dt = x(t) + \int x(t)dt$

$y(t) = -3\int y(t)dt - 2\iint y(t)dt + x(t) + \int x(t)dt$

Direct form-I






**5.**

### 2.3.2 The Distributive Property

Another basic property of convolution is 'distributive property'.

i.e. In discrete-time

$$x(n) * \{h_1(n) + h_2(n)\} = x(n) * h_1(n) + x(n) * h_2(n)$$

and in continuous-time,

$$x(t) * \{h_1(t) + h_2(t)\} = x(t) * h_1(t) + x(t) * h_2(t)$$

Consider an interconnection of discrete-time LTI system as shown in Fig. 2.8

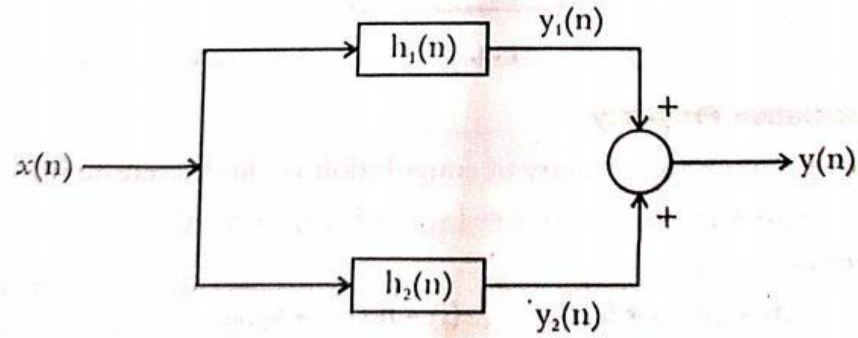



**Fig. 2.8**

### 2.3.3 The Associative Property

This is another important property of convolution i.e. in discrete-time,

$$x(n) * \{h_1(n) * h_2(n)\} = \{x(n) * h_1(n)\} * h_2(n)$$

and in continuous-time,

$$x(t) * \{h_1(t) * h_2(t)\} = \{x(t) * h_1(t)\} * h_2(t)$$



Consider a cascade connection of continuous-time LTI system as shown in Fig. 2.12

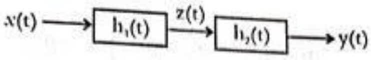



**Fig. 2.12**

From Fig. 2.12, we have,

$$y(t) = z(t) * h_2(t)$$

$$= \int_{-\infty}^{\infty} z(\tau)\, h_2(t-\tau)\, d\tau \qquad \text{.............................} \qquad (2.16)$$

where z(t) is the output of the first system

∴ We have,

$$z(\tau) = x(t) * h_1(\tau)$$

$$= \int_{-\infty}^{\infty} x(\eta)\, h_1(\tau-\eta)\, d\eta \qquad \text{.............................} \qquad (2.17)$$

Substituting eqn. 2.17 in 2.16, we get,

$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\eta)\, h_1(\tau-\eta)\, h_2(t-\tau)\, d\eta\, d\tau$$

$\tau = (m+\eta)$

Substituting $m = \tau-\eta$ and interchanging the order of integration, we get,

$$= \int_{-\infty}^{\infty} x(\eta) \left[ \int_{-\infty}^{\infty} h_1(m)\, h_2(t-\eta-m)\, dm \right] d\eta$$

$$= \int_{-\infty}^{\infty} x(\eta)\, [h(t-\eta)]\, d\eta$$

where $h(t-\eta) = h_1(t-\eta) * h_2(t-\eta)$

$\therefore h(t) = h_1(t) * h_2(t)$

$\therefore y(t) = x(t) * h(t)$ ............................. (2.18)

$\therefore y(t) = x(t) * [h_1(t) * h_2(t)]$ ............................. (2.19)

Observing eqn. 2.18 and 2.19, we can write the system shown in Fig. 2.12 as shown in Fig. 2.13 below.

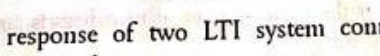



**Fig. 2.13**

Hence the impulse response of two LTI system connected in cascade is the convolution of the individual impulse responses.

Also we know that convolution is commutative. Therefore we can write the system in Fig. 2.13 as below.

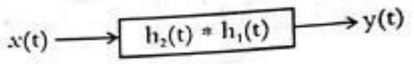



Fig. 2.14

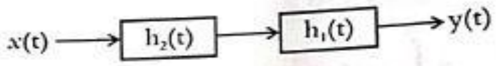



Fig. 2.15

From the above discussion we conclude that the output of a cascade combination of LTI systems is independent of the order in which the systems are connected.

Similarly, for discrete-time we have,

$$x(n) * \{h_1(n) * h_2(n)\} = x(n) * \{h_2(n) * h_1(n)\}$$

### 2.3.5 Causal System

We know that the output of a causal system depends only on the present and/or past values of the input to the system. i.e. for a discrete-time LTI system to be causal, its $h(n) = 0$ for $n<0$. Similarly, for a continuous-time LTI system to be causal, its $h(t) = 0$ for $t<0$.

**6.**

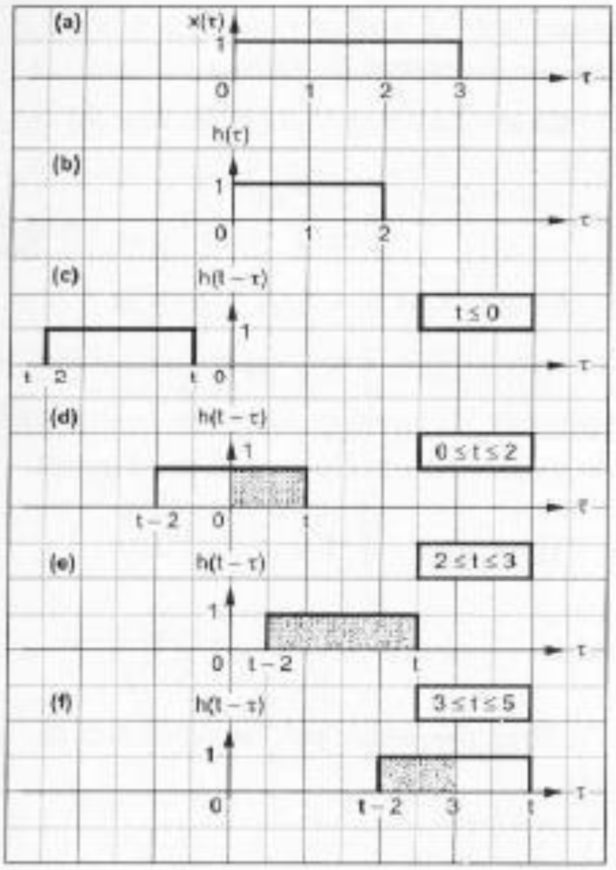
Here $\quad x(\tau) = u(\tau) - u(\tau-3)$

$$= \begin{cases} 1 & for \;\; 0 \le \tau \le 3 \\ 0 & elsewhere \end{cases}$$

and $\quad h(\tau) = u(\tau) - u(\tau-2)$

$$= \begin{cases} 1 & for \;\; 0 \le \tau \le 2 \\ 0 & elsewhere \end{cases}$$

Fig. 2.2.34 shows $x(\tau)$ and $h(\tau)$. It also shows $h(t-\tau)$ for various values of t. Output is given as $y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$. The overlap of $x(\tau)$ and $h(t-\tau)$ is shown for various values of t.




**Case-I :** $t \leq 0$ *and* $t \geq 5$

For this case there is no overlap betwe $x(\tau)$ and $h(t-\tau)$. Hence $y(t)=0$.

**Case-II :** $0 \leq t \leq 2$ Refer Fig. (d)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{0}^{t} 1\, d\tau = t$$

**Case-III :** $2 \leq t \leq 3$ Refer Fig. (e)

$$y(t) = \int_{t-2}^{t} 1\, d\tau = 2$$

**Case-IV :** $3 \leq t \leq 5$ Refer Fig. (f)

$$y(t) = \int_{t-2}^{3} 1\, d\tau = 5-t$$

Thus the result will be,

$$y(t) = \begin{cases} t & \text{for } 0 \leq t \leq 2 \\ 2 & \text{for } 2 \leq t \leq 3 \\ 5-t & \text{for } 3 \leq t \leq 5 \end{cases}$$