CBCS SCHEME

USN

18CS36

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Test the validity of the following :

(i)
$$\begin{array}{l} p \wedge q \\ q \to r \\ \underline{r \to s} \\ \therefore s \end{array}$$

(ii)
$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \underline{\neg r \to s} \\ \therefore q \end{array}$$

(06 Marks)

b. State the converse inverse and contrapositive of the following conditions:
"If a triangle is not isosceles, then it is not equilateral". **(06 Marks)**

c. Prove that the following argument is valid

$$\begin{array}{l} \forall x\, [p(x) \to q(x)] \\ \underline{\forall x,\, [q(x) \to r(x)]} \\ \therefore \forall x\, [p(x) \to r(x)] \end{array}$$

(04 Marks)

d. Prove that for all integers m and n, if m and n are both odd, then m + n is even and mn is odd. **(04 Marks)**

OR

2 a. Simplify the following compound proposition using the laws of logic
i) $(p \vee q) \wedge [\neg\{(\neg p) \wedge q\}]$ ii) $(p \vee q) \wedge \{(\neg p \wedge \neg p) \wedge q\} \Leftrightarrow \neg p \wedge q$ **(06 Marks)**

b. Prove that for any proposition p, q, r the compound proposition
$[(p \vee q) \wedge \{(p \to q) \wedge (q \to r)\}]$ is a tautology. **(05 Marks)**

c. Determine the truth value of each of the following quantified statement, the universe being the set of all non zero integers.
(i) $\exists x, \exists y\, [xy = 1]$ (ii) $\exists x\ \forall y [xy = 1]$ (iii) $\forall x\ \exists y [xy = 1]$
(iv) $\exists x\ \exists y\, [(2x+y=5) \wedge (x - 3y = 8)]$ **(04 Marks)**

d. Simplify the following switch network.

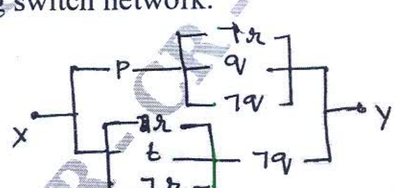

Fig.Q2(d) **(05 Marks)**

Module-2

3 a. Show that $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ **(06 Marks)**

b. $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$ prove that $a_n \leq 3^n$. **(07 Marks)**

c. How many ways 10 roses, 14 sunflowers, 15 daffodils can be distributed among 3 girls? **(07 Marks)**

OR

4 a. Prove that $3 + 3^2 + 3^3 + \ldots + 3^n = \dfrac{3(3^n - 1)}{2}$. **(07 Marks)**

b. Find the number of signals that can be generated using six different colored flags when any number of them may be hoisted at any time. **(07 Marks)**

c. Determine the co-efficient xyz^2 in the expansion of $(2x - y - z)^4$. **(06 Marks)**

Module-3

5 a. For any non-empty set A, B, C prove that $(A \cap B) \times C = (A \times C) \cap (B \times C)$. **(05 Marks)**

b. Let A = {1, 2, 3, 4, 5, 6, 7} and B = {w, x, y, z}. Find the number of onto functions from A to B. **(07 Marks)**

c. Let f, g, h be functions from z to z defined by f(x) = x − 1, g(x) = 3x, $h(x) = \begin{cases} 0 \text{ if } x \text{ is even} \\ 1 \text{ if } x \text{ is odd} \end{cases}$, determine f o (g o h) (x) = (f o g) o h(x) and verify that f o (g o h)(x) = (fog) o h(x). **(08 Marks)**

OR

6 a. How many persons must be chosen in order that at least seven of them will have birthday in the same calendar month? **(04 Marks)**

b. For a given set A = {1, 2, 3, 4} and let R be a relation on A. R = {(1, 2) (1, 3) (1, 4) (2, 3) (2, 4) (3, 4) (2, 1) (3, 1) (4, 1)}.

i) Draw the diagraph of R.

ii) Determine the indegree and outdegree of the vertices in the diagraph. **(06 Marks)**

c. Find the number of edges used in Hasse diagram, for the Poset [{2, 3, 6, 12, 15, 48, 120, 240}, where x divides y]. Also determine maximal and minimal elements in Poset upper bound and lower bound LUB and GLB for the set B = {12, 15}. **(10 Marks)**

Module-4

7 a. How many solutions are there to $x_1 + x_2 + x_3 = 17$ where $x_i \leq 7$ for $1 \leq i \leq 3$. **(07 Marks)**

b. A Girl student has sarees of 5 different colors blue, green, red, white and yellow. On Monday she does not wear green, on Tuesdays blue or red, on Wednesday blue or green, on Thursday red or yellow, on Friday red. In how many ways can she dress without repeating a color during a week? **(07 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this number increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

OR

8 a. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns FUN, CASE, FLOW occur. **(07 Marks)**

b. In how many ways can you put 7 fruits into their respective fruit box such that exactly 3 go into the right fruit boxes? **(06 Marks)**

c. Determine rook polynomial for the following :

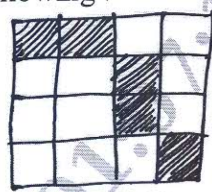

Fig.Q8(c) **(07 Marks)**

Module-5

9 a. For the graph mentioned below :

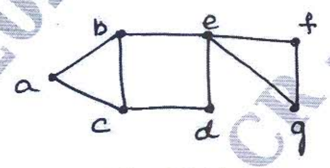

Fig.Q9(a)

What vertices would you use to
- i) Determine a walk from b to d that is not a trail.
- ii) Determine b to d trial is not a path.
- iii) A path from b to d.
- iv) A closed walk from b to b that is not a cycle.
- v) A cycle from b to b. **(10 Marks)**

b. Prove that every tree T = < V, E > = | E | + 1 = | V |. **(05 Marks)**

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z, that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(05 Marks)**

OR

10 a. Define the following with an example :
i) Induced graph ii) Complete graph iii) Isomorphic graph. **(10 Marks)**

b. Prove that for every tree T = < V, E >, if | V | ≥ 2, then T has at least two pendant vertices. **(05 Marks)**

c. Illustrate with an example Eulerian Graph. **(05 Marks)**

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