USN

18EC44

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Statistics & Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define cdf, pdf and pmf with example. **(06 Marks)**

b. The following is the pdf for random variable U,

$$f_U(u) = \begin{cases} C\exp\left(-\dfrac{u}{2}\right), & 0 \le u < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value that C must have and evaluate $F_U(0.5)$. **(06 Marks)**

c. Given the data in the following table :

k	x_k	$P(x_k)$
1	2.1	0.21
2	3.2	0.18
3	4.8	0.20
4	5.4	0.22
5	6.9	0.19

(i) Plot pdf and cdf of the discrete random variable X.

(ii) Write expression for $f_X(x)$ and $F_X(x)$ using unit delta functions and unit step function. **(08 Marks)**

OR

2 a. Define Expectation, Variance and characteristic functions. **(04 Marks)**

b. Explain the probability models for Gaussian and exponential random variables. **(08 Marks)**

c. The random variable X is uniformly distributed between 0 and 4. The random variable Y is obtained from X using $y = (x-2)^2$. Evaluate CDF and PDF for Y. **(08 Marks)**

Module-2

3 a. Obtain the expressions for different bivariate expectations. **(06 Marks)**

b. It is given that $E[X] = 2.0$ and that $E[X^2] = 6$. Find the standard deviation of X. Also if $Y = 6X^2+2X-13$, find μ_Y. **(07 Marks)**

c. The mean and variance of random variable X are -2 and 3 ; the mean and variance of Y are 3 & 5. The covariance $COV[XY] = -0.8$. Find correlation co-efficient ρ_{XY} and correlation E[XY]. **(07 Marks)**

OR

4 a. The joint pdf of a bivariate random variable X and Y is given by,

$$F_{XY}(x,y) = \begin{cases} k(x+y), & 0 < x, y < z \\ 0, & \text{otherwise} \end{cases}$$ where k is constant.

(i) Find the value of k.

(ii) Find the marginal pdf's of X and Y.

(iii) Are X and Y independent? **(06 Marks)**

b. The random variable U has a mean of 0.3 and a variance of 1.5

(i) Find the mean and variance of Y if $Y = \frac{1}{53}\sum_{i=1}^{53} u_i$

(ii) Find the mean and variance of Z if $Z = \sum_{i=1}^{53} u_i$

In these two sums, the u_i's are IID. **(04 Marks)**

c. Explain briefly Chi square random variable. **(10 Marks)**

Module-3

5 a. Explain Random process, stationarity and wide sense stationarity random process. **(06 Marks)**

b. X(t) and Y(t) are independent, jointly wide sense stationarily random processes given by $X(t) = A\cos(\omega_1 t + \theta_1)$ and $Y(t) = B\cos(\omega_2 t + \theta_2)$. If W(t) = X(t).Y(t), find Auto Correlation function $R_W(Z)$. **(06 Marks)**

c. Define Auto Correlation Function (ACF) of a random process. List and prove the properties of Auto Correlation. **(08 Marks)**

OR

6 a. Explain Wiener-Kenchin relations. **(06 Marks)**

b. A PSD is shown in Fig. Q6 (b) where constants are a = 55, b = 5, $\omega_0 = 1000$, $\omega_1 = 100$. Solve the values for $E[X^2(t)]$, σ_X^2 and μ_X.

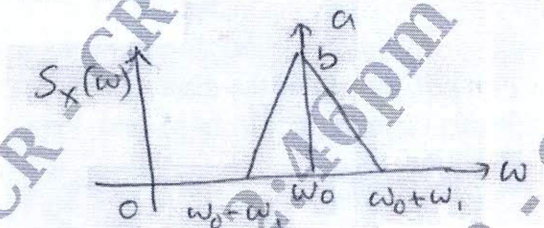

Fig. Q6 (b) **(06 Marks)**

c. Assume that the following table is obtained from a windowed sample function obtained from a random Ergodic process. Solve the ACF for Z = 0, 2 and 4 ms.

x(t)	1.5	2.1	1.0	2.2	−1.6	−2.0	−2.5	2.5	1.6	1.8
k	0	1	2	3	4	5	6	7	8	9

(08 Marks)

Module-4

7 a. Define vector space and axioms of vector spaces. **(06 Marks)**

b. Let W be the subspace of R^5 spanned by,

$x_1 = (1 \quad 2 \quad -1 \quad 3 \quad 4)$, $x_2 = (2 \quad 4 \quad -2 \quad 6 \quad 8)$, $x_3 = (1 \quad 3 \quad 2 \quad 2 \quad 6)$

$x_4 = (1 \quad 4 \quad 5 \quad 1 \quad 8)$, $x_5 = (2 \quad 7 \quad 3 \quad 3 \quad 9)$

Find the basis and dimension of W. **(06 Marks)**

c. If vectors $u = \begin{bmatrix}1\\2\\2\end{bmatrix}$, $v = \begin{bmatrix}2\\-2\\1\end{bmatrix}$, $w = \begin{bmatrix}2\\1\\-2\end{bmatrix}$

Then show that the vectors U, V and W form orthogonal pairs. Also find the length of vectors U, V and W. **(08 Marks)**

OR

8 a. Determine whether the vectors $(1 \quad 4 \quad 9)$, $(3 \quad 1 \quad 9)$ and $(9 \quad 3 \quad 12)$ are linearly dependent or independent. **(06 Marks)**

b. List and explain four fundamental subspaces. **(06 Marks)**

c. Apply Gram-Schmidt process to vectors to obtain an orthonormal basis for $v_3(R)$ with the standard inner product. $v_1 = (2 \quad 2 \quad 1)$, $v_2 = (1 \quad 3 \quad 1)$, $v_3 = (1 \quad 2 \quad 2)$ **(08 Marks)**

Module-5

9 a. Reduce the matrix A to U. Find det(A). $A = \begin{bmatrix} 2 & 5 & 3 \\ 1 & 2 & 4 \\ -1 & 3 & 6 \end{bmatrix}$. **(04 Marks)**

b. Find Eigen values and Eigen vectors of matrix, $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$. **(10 Marks)**

c. What is positive definite matrix? Mention the methods of testing positive definiteness. Check the following matrix for positive definiteness.

$S_1 = \begin{bmatrix} 5 & 6 \\ 6 & 7 \end{bmatrix}$. **(06 Marks)**

OR

10 a. Compute A^TA and AA^T. Find eigen values and unit Eigen vectors for $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$. Multiply the three matrices. $U\sum V^T$ to recover A. **(12 Marks)**

b. Expand the determinant $A = \begin{bmatrix} 3 & 1 & 4 & 2 \\ 1 & 5 & 2 & 6 \\ 2 & 3 & 7 & 1 \\ 4 & 1 & 2 & 3 \end{bmatrix}$ **(08 Marks)**

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