CBCS SCHEME

USN

17MAT11

First Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Obtain the n^{th} derivative of $$\frac{x}{(1+x)(1+2x)}$$ **(06 Marks)**

b. Prove that the curves $r = a\sec^2\theta/2$ and $r = a\,\mathrm{cosec}^2\theta/2$ cut orthogonally. **(07 Marks)**

c. Find the radius of curvature at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$ on the curve $x^3 + y^3 = 3xy$. **(07 Marks)**

OR

2 a. If $y = e^{a\sin^{-1}x}$, then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. **(06 Marks)**

b. Prove that with usual notation, $\tan\phi = r\frac{d\theta}{dr}$ **(07 Marks)**

c. Find the pedal equation of the curve $\frac{2a}{r} = (1 - \cos\theta)$ **(07 Marks)**

Module-2

3 a. If $u = \sin^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that $xu_x + yu_y = 2\tan u$ **(06 Marks)**

b. Obtain Taylor's series expansion of log(cos x) about the point $x = \pi/3$ upto the fourth degree term. **(07 Marks)**

c. If $u = x + 3y^2 - z^3$; $v = 4x^2yz$; $w = 2z^2 - xy$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at (1, –1, 0). **(07 Marks)**

OR

4 a. Evaluate $\underset{x\to 0}{\mathrm{Lt}}\left[\frac{\sin^2 x - x^2}{x^2\sin^2 x}\right]$ **(06 Marks)**

b. Expand log(1 + sinx) in power of x by Maclaurin's expansion upto the term containing x^3. **(07 Marks)**

c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ **(07 Marks)**

Module-3

5 a. A particle moves along the curve whose parametric equation are $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the components of velocity and acceleration at t = 1 in the direction of $\hat{i} + \hat{j} + 3\hat{k}$. **(10 Marks)**

b. A vector field is given by $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$. Show that the field is irrotational and find its scalar potential such that $\vec{F} = \nabla\phi$. **(10 Marks)**

OR

6 a. If $\vec{F} = (x + y + 1)i + j - (x + y)k$, show that $\vec{F} \cdot \text{curl}\,\vec{F} = 0$ **(06 Marks)**

b. Show that $\vec{F} = \dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

c. Show that $\text{div}(\text{curl}\,\vec{F}) = 0$ **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \cos^n x\,dx$ where n is a positive integer hence evaluate $\int_0^{\pi/2} \cos^n x\,dx$ **(06 Marks)**

b. Solve $y(2x - y + 1) + x(3x - 4y + 3)dy = 0$ **(07 Marks)**

c. Find the orthogonal trajectories of the family of curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2 + \lambda} = 1$ where λ is the parameter. **(07 Marks)**

OR

8 a. Evaluate $\int_0^a x\sqrt{ax - xz}\,dx$. **(06 Marks)**

b. Solve $x^3\dfrac{dy}{dx} - x^2 y = -y^4 \cos x$. **(07 Marks)**

c. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes. Find the temperature of the body after 24 minutes. **(07 Marks)**

Module-5

9 a. Find the rank of a matrix

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

using elementary row operating. **(06 Marks)**

b. Solve the system of equation $2x + 5y + 7z = 52$, $2x + y - z = 0$, $x + y + z = 9$ by using Gauss-Jordan method. **(07 Marks)**

c. Diagnolise the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. **(07 Marks)**

OR

10 a. Show that the transformation $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular, find the inverse transformation. **(06 Marks)**

b. Using power method, find the dominant eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ taking the initial vector as $[1, 0, 0]^T$. Carry out five iterations. **(07 Marks)**

c. Reduce the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into canonical form, by using orthogonal transformation. **(07 Marks)**