CBCS SCHEME

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17MAT41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Apply modified Euler's method to solve $\frac{dy}{dx} = x + y$, $y(0) = 1$. Compute $y(0.2)$ taking $h = 0.1$. **(07 Marks)**

b. Using fourth order Runge-Kutta method, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$ taking $h = 0.2$. **(06 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.0679$, $y(0.3) = 2.090$, find $y(0.4)$ correct to four decimal places by using Milne's predictor and corrector method. **(07 Marks)**

OR

2 a. Use Taylor's series method to find $y(4.1)$ given that $\frac{dy}{dx} = \frac{1}{x^2 + y}$ and $y(4) = 4$. **(06 Marks)**

b. Solve $(y^2 - x^2)dx = (y^2 + x^2)dy$ in the range $0 \le x \le 0.4$ given that $y = 1$ at $x = 0$ initially by applying R-K method of fourth order. **(07 Marks)**

c. Apply Milne's method to compute $y(1.4)$ correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$ **(07 Marks)**

Module-2

3 a. Given $y'' - xy' - y = 0$ with the initial condition $y(0) = 1$, $y'(0) = 0$. Compute $y(0.2)$ and $y'(0.2)$ using fourth order R-K method. **(07 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that

$$\int_0^1 x J_n(\alpha x) J_n(\beta x)\,dx = 0 \text{ if } \alpha \ne \beta.$$ **(07 Marks)**

c. Express the polynomial $4x^3 - 2x^2 - 3x + 8$ interms of Legendre polynomials. **(06 Marks)**

OR

4 a. Applying Milne's predictor and corrector formulae to compute $y(0.8)$ given that y satisfies the equation $y'' = 2yy'$ using the following data :
$y(0) = 0$, $y(0.2) = 0.2027$, $y(0.4) = 0.4228$, $y(0.6) = 0.6841$, $y'(0) = 1$, $y'(0.2) = 1.041$, $y'(0.4) = 1.179$, $y'(0.6) = 1.468$. **(07 Marks)**

b. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

c. Derive Rodrigue's formula, $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$. (07 Marks)

Module-3

5 a. Derive Cauchy-Riemann equation in the polar form. (07 Marks)

b. State and prove Cauchy's-Integral formula, $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a} dz$. (06 Marks)

c. Find the bilinear transformation which map the points z = 1, i −1 into w = i, 0, −i. (07 Marks)

OR

6 a. Find the analytic function f(z) whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. (06 Marks)

b. Using Cauchy's residue theorem, evaluate $\int_C \frac{z\cos z}{\left(z - \frac{\pi}{2}\right)^3} dz$ where C : $|z-1| = 1$. (07 Marks)

c. Discuss the transformation $\omega = e^z$. (07 Marks)

Module-4

7 a. If the mean and standard deviation of the number of correctly answered questions in a test given to 4096 students 2.5 and $\sqrt{1.875}$. Find an estimate of the number of candidates answering correctly,
   (i) 8 or more questions
   (ii) 2 or less
   (iii) 5 questions (07 Marks)

b. Derive mean and standard deviations of binomial distributions. (06 Marks)

c. The joint probability distribution for two random variables X and Y is as follows :

X \ Y	−3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine : (i) Marginal distribution of X and Y (ii) COV (X, Y)
(iii) Correlations of X and Y. (07 Marks)

OR

8 a. Derive mean and standard deviations of Exponential distribution. (06 Marks)

b. X and Y are independent random variables. X takes values 2, 5, 7 with probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ respectively. Y take values 3, 4, 5 with the probability $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$.
   (i) Find the Joint probability distribution of X and Y.
   (ii) Show that the covariance of X and Y.
   (iii) Find the probability distribution of Z = X + Y (07 Marks)

c. In 800 families with 5 childrens each how many families would be expected to have, (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) atmost 2 girls by assuming probabilities for boys and girls to be equal. (07 Marks)

Module-5

9 a. A survey was conducted in a slum locality of 2000 families by selecting a sample size 800. It was revealed that 180 families were illiterates. Find the probable limits of the illiterate families in the population of 2000. **(07 Marks)**

b. Ten individuals are choosen at random from a population and their heights in inches are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the suggestion that the mean height of the population is 65 inches given that $t_{0.05} = 2.262$ for 9 d.f. **(07 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}.$$

(06 Marks)

OR

10 a. Four coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit ($x^2_{0.05} = 9.49$ for 4 d.f) **(07 Marks)**

No. of heads	0	1	2	3	4
Frequency	5	29	36	25	5

b. The transition probability matrix of a Markov chain is given by,

$$P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

and the initial probability distribution is $P^{(0)} = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$. Find $P_{13}^{(2)}$, $P_{23}^{(2)}$, $P^{(2)}$ and $P_1^{(2)}$.

(06 Marks)

c. A man's smoking habits are as follows. If he smokes filters cigarettes one week, he switches to non filter cigarettes the next week with probability 0.2. On the other hand if he smokes nonfilter cigarettes one week there is a probability of 0.7 that he will smoke non filter cigarettes the next week as well. In the long run how often does he smoke filter cigarettes? **(07 Marks)**

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