



# CBCS SCHEME

18MAT21

## Second Semester B.E. Degree Examination, Jan./Feb. 2023 Advanced Calculus and Numerical Methods

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- Find the Directional derivative of  $\phi = xy^2 + yz^3$  at  $(2, -1, 1)$  along  $i + 2j + 2k$ . (06 Marks)
  - Find  $\text{div } \vec{A}$ ,  $\text{Curl } \vec{A}$ ,  $\text{div}(\text{Curl } \vec{A})$ , where  $\vec{A} = x^2yi + y^2zj + z^2yk$ . (07 Marks)
  - If  $u = x^2yz$ ,  $v = xy - 3z^2$ , then find  $\nabla \cdot (\nabla u \times \nabla v)$ . (07 Marks)

### OR

- Find the work done in moving a particle in the force field  $\vec{F} = 3x^2i + (2xz - y)j + zk$  along the straight line from  $(0, 0, 0)$  to  $(2, 1, 3)$ . (06 Marks)
  - Using Divergence theorem, evaluate  $\iiint_V \text{div } \vec{F} \, dV$ , where  $V$  is the region bounded by the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$  and  $2x + 2y + z = 4$ . (07 Marks)
  - Using Stoke's theorem, evaluate  $\int_C (x + y)dx + (2x - y)dy + (y + z)dz$ , where  $C$  is the boundary of the triangle with vertices  $(2, 0, 0)$ ,  $(0, 3, 0)$  and  $(0, 0, 6)$ . (07 Marks)

### Module-2

- Solve  $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$ . (06 Marks)
  - Solve  $(D^2 - 4D + 3)y = e^{3x} + 2^x + 7$ . (07 Marks)
  - Using the method of variation of parameter, solve  $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$ . (07 Marks)

### OR

- Solve the differential equation  $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = \sin 2x + x$ . (06 Marks)
  - Solve  $(1 - 2x)^2 y'' + 6(1 - 2x)y' + 16y = 4(1 - 2x)^2$ . (07 Marks)
  - The current  $i$  and the charge  $q$  in a series circuit containing an inductance  $L$ , capacitance  $C$ , e.m.f  $E$  satisfy the differential equation  $L \frac{di}{dt} + \frac{q}{C} = E$ ,  $i = \frac{dq}{dt}$ . Express  $q$  in terms of  $t$ , given that  $L, C, E$  are constants and the value of  $i, q$  are both zero initially. (07 Marks)

### Module-3

- Find the partial differential equation by eliminating the function from  $Z = f(x^2 + y^2) + x + y$ . (06 Marks)
  - Solve  $\frac{\partial^3 z}{\partial x^2 \partial y} = 18xy^2 + \sin(2x - y)$ . (07 Marks)
  - Find all possible solution of  $u_{tt} = c^2 u_{xx}$  one dimensional wave equation by Variable Separable method. (07 Marks)

OR

- 6 a. Solve  $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} - 3z = 0$ ,  $z = 1$ ,  $\frac{\partial z}{\partial x} = 1$ , when  $x = 0$ . (06 Marks)
- b. Find the general solution of  $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ . (07 Marks)
- c. Derive one dimensional heat equation. (07 Marks)

**Module-4**

- 7 a. Test for converges for  $\frac{2}{3} + \frac{2.3}{3.5} + \frac{2.3.4}{3.5.7} + \dots$  (06 Marks)
- b. With usual notation prove that  $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ . (07 Marks)
- c. Express  $f(x) = x^4 - 3x^3 - x^2 + 5x - 2$  in terms of Legendre polynomial. (07 Marks)

OR

- 8 a. Discuss the nature of the series  $\frac{2}{3} + \left(\frac{3}{5}\right)^2 + \left(\frac{4}{7}\right)^3 + \dots \infty$ . (06 Marks)
- b. Obtain the series solution of Legendre differential equation in terms of  $P_n(x)$ .  
 $(1 - x^2)y'' - 2xy' + n(n + 1)y = 0$ . (07 Marks)
- c. Show that i)  $P_2(\cos \theta) = \frac{1}{4}(1 + 3 \cos 2\theta)$  ii)  $P_3(\cos \theta) = \frac{1}{8}[3 \cos \theta + 5 \cos 3\theta]$ . (07 Marks)

**Module-5**

- 9 a. Find the population of a town for the year 1974. Given that

Year	1939	1949	1959	1969	1979	1989
Population in thousands	12	15	20	27	39	52

- (06 Marks)
- b. Using Newton's general interpolation formula, find the polynomial and hence find  $f(3)$ .

x	0	1	2	4	5	6
y	22	48	50	30	32	58

- (07 Marks)
- c. Using Newton Raphson method, find correct to 4(four) decimal places, the smallest root of  $\log x = \cos x$ . (07 Marks)

OR

- 10 a. Using Regula Falsi method, determine a solution of  $2x = \cos x + 3$  correct to four decimal places. (06 Marks)
- b. Find the polynomial  $f(x)$  using Lagrange's Interpolation formula for

x	1	3	4	6
y	0	12	33	135

Hence find  $f(2)$ . (07 Marks)

- c. Use Weddle's rule to find  $\int_0^{0.6} e^{-x^2} dx$ , by taking seven ordinates. (07 Marks)

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