

CBCS SCHEME

18MATDIP31



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Third Semester B.E. Degree Examination, Jan./Feb. 2023

Additional Mathematics – I

Max. Marks: 100

Time: 3 hrs.

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- Express the complex number $\frac{5+5i}{3-4i}$ in the form $x + iy$. (06 Marks)
 - Find the amplitude and modulus of the complex number $\frac{4+2i}{2-3i}$. (07 Marks)
 - Prove that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1} \cdot \cos^n \frac{\theta}{2} \cos\left(\frac{n\theta}{2}\right)$. (07 Marks)

OR

- Show that the points A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5) and D(-3, 2, 1) are coplanar. (06 Marks)
 - Find the cube roots of $1 - i$. (07 Marks)
 - Find the sine of the angle between $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$. (07 Marks)

Module-2

- Prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \dots$ by using Maclaurin's series. (06 Marks)
 - If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} + z \cdot \frac{\partial u}{\partial z} = 0$. (07 Marks)
 - If $u = \tan^{-1}\left(\frac{x^3 y^3}{x^3 + y^3}\right)$ prove that $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = \frac{3}{2} \sin 2u$. (07 Marks)

OR

- Obtain the Maclaurin's series expansion of e^x . (06 Marks)
 - If $u = e^{x^3+y^3}$ prove that $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 3u \log u$. (07 Marks)
 - If $u = x - y$ and $v = \frac{1}{x - y}$, find $\frac{\partial(u, v)}{\partial(x, y)}$. (07 Marks)

Module-3

- Find the directional derivative of $x^2 y z^3$ at $(1, 1, 1)$ in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. (06 Marks)
 - A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2 \sin 3t$, where t is the time. Determine the component of velocity and acceleration at $t = 0$ in the direction of $\hat{i} + \hat{j} + \hat{k}$. (07 Marks)
 - Find the angle between the tangents to the curve $x = t^2$, $y = t^3$, $z = t^4$ at $t = 2$ and $t = 3$. (07 Marks)

OR

- 6 a. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \nabla(xy + yz + zx)$ (06 Marks)
 b. Find the constants a, b, c such that the vector $\vec{F} = (x + y + az)\hat{i} + (x + cy + 2z)\hat{k} + (bx + 2y - z)\hat{j}$ is irrotational. (07 Marks)
 c. If $\vec{F} = 2x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$ and $\phi = 2z - x^3y$, find $\vec{F} \cdot (\nabla\phi)$ and $\vec{F} \times (\nabla\phi)$ at $(1, -1, 1)$. (07 Marks)

Module-4

- 7 a. Find the reduction formula for $\int \cos^n x \, dx : n > 0$ (06 Marks)
 b. Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2 - x^2}} \, dx$ (07 Marks)
 c. Evaluate $\int_0^1 \int_{x^2}^x (x^2 + 3y + 2) \, dy \, dx$ (07 Marks)

OR

- 8 a. Find the reduction formula for $\int \sin^n x \, dx : n > 0$ (06 Marks)
 b. Evaluate $\int_0^\pi x \cdot \cos^6 x \, dx$ (07 Marks)
 c. Evaluate $\int_0^a \int_0^a \int_0^a e^{x+y+z} \, dx \, dy \, dz$ (07 Marks)

Module-5

- 9 a. Solve: $\frac{dy}{dx} = -\frac{y}{x} + y^2x$ (06 Marks)
 b. Solve $y \sin 2x \, dx - (1 + y + \cos^2 x) \, dy = 0$ (07 Marks)
 c. Solve $x \cdot \frac{dy}{dx} + y = x^3y^6$ (07 Marks)

OR

- 10 a. Solve $\frac{dy}{dx} = \frac{x^2 - 2xy}{x^2 - \sin y}$ (06 Marks)
 b. Solve $dx + x \, dy = e^{-y} \cdot \sec y \, dy$ (07 Marks)
 c. Solve $3x(x + y^2) \, dy + (x^3 - 3xy - 2y^3) \, dx = 0$ (07 Marks)
