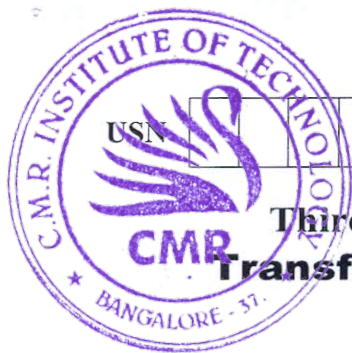


Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.



# CBCS SCHEME

--	--	--	--	--	--	--	--	--	--

21MAT31

## Third Semester B.E. Degree Examination, Jan./Feb. 2023 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Find the Laplace transform of  $te^{2t} - \frac{2 \sin 3t}{t}$ . (06 Marks)
- b. Given that  $f(t) = \begin{cases} E, & 0 < t < a/2 \\ -E, & a/2 < t < a \end{cases}$   
 where  $f(t+a) = f(t)$  show that  $L\{f(t)\} = \frac{E}{s} \tan h\left(\frac{as}{4}\right)$ . (07 Marks)
- c. Using convolution theorem obtain the inverse Laplace transform of the following function :  
 $\frac{1}{(s-1)(s^2+1)}$ . (07 Marks)

OR

- 2 a. Find the inverse Laplace transform of :  
 $\frac{s+5}{s^2-6s+13}$ . (06 Marks)
- b. Express the following function in terms of unit step function and hence find their Laplace transform.  
 $f(t) = \begin{cases} 1, & 0 < t < 1 \\ t, & 1 < t \leq 2 \\ t^2, & t > 2. \end{cases}$  (07 Marks)
- c. Solve the following initial value problem by using Laplace transform :  
 $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}, y(0) = 0, y'(0) = 0$ . (07 Marks)

### Module-2

- 3 a. Obtain Fourier series of  $f(x) = \frac{\pi-x}{2}$  in  $0 < x < 2\pi$ . Hence deduce that  
 $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$ . (06 Marks)
- b. Find a cosine Fourier series for  $f(x) = (x-1)^2, 0 \leq x \leq 1$ . (07 Marks)
- c. Obtain the Fourier series of  $y$  upto the First harmonic for the following values.

$x^\circ$	45	90	135	180	225	270	315	360
$y$	4.0	3.8	2.4	2.0	-1.5	0	2.8	3.4

(07 Marks)

OR

- 4 a. Obtain Fourier series for

$$f(x) = \begin{cases} \pi x & \text{in } 0 \leq x \leq 1 \\ \pi(2-x) & \text{in } 1 \leq x \leq 2 \end{cases}$$

(06 Marks)

- b. Obtain the sine half range series for the function :

$$f(x) = 1 - \left(\frac{x}{\pi}\right) \text{ in } 0 \leq x \leq \pi.$$

(07 Marks)

- c. The following values of y and x are given. Find Fourier series of upto first harmonics.

x	0	2	4	6	8	10	12
y	9.0	18.2	24.4	27.8	27.5	22.0	9.0

(07 Marks)

**Module-3**

- 5 a. If
- $f(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$
- . Find Fourier transform of f(x) and hence find the value of

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx.$$

(06 Marks)

- b. Find the Fourier sine transform of
- $f(x) = e^{-|x|}$
- and hence evaluate

$$\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx, \quad m > 0.$$

(07 Marks)

- c. Solve by using Z-Transforms
- $U_{n+2} + 2U_{n+1} + U_n = n$
- with
- $U_0 = 0 = U_1$
- .

(07 Marks)

OR

- 6 a. Obtain the Fourier cosine transform of the function :

$$f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4-x, & 1 < x \leq 4 \\ 0, & x > 4. \end{cases}$$

(06 Marks)

- b. Obtain the Z-transform of
- $\cos n\theta$
- and
- $\sin n\theta$

(07 Marks)

- c. Compute the inverse Z-transform of
- $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$
- .

(07 Marks)

**Module-4**

- 7 a. Classify the following partial differential equations :

i)  $x^2 u_{xx} + (1-y^2) u_{yy} = 0, \quad -\infty < x < \infty, -1 < y < 1$

ii)  $(1+x^2) u_{xx} + (5+2x^2) u_{xt} + (4+x^2) u_{tt} = 0$

iii)  $(x+1) u_{xx} - 2(x+2) u_{xy} + (x+3) u_{yy} = 0.$

(10 Marks)

- b. Solve
- $u_t = u_{xx}$
- subject to the conditions
- $u(0, t) = 0 = u(1, t)$
- and
- $u(x, 0) = \sin(\pi x)$
- by taking
- $h = 0.2$
- for 5 levels. Further write down the following values from the table

i)  $u(0.2, 0.04)$

ii)  $u(0.4, 0.08)$

iii)  $u(0.6, 0.06).$

(10 Marks)



OR

- 8 a. Solve the elliptic equation  $u_{xx} + u_{yy} = 0$  for the following square Mesh with boundary values as shown. Find the iterative values of  $u_i$  (1 to 9) to the nearest integer.

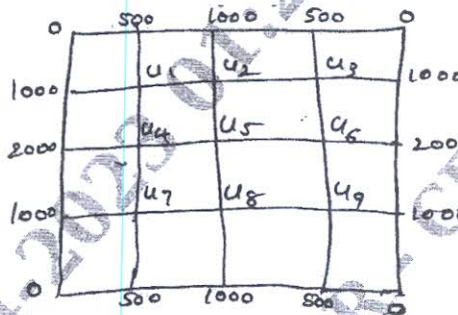


Fig.Q8(a)

- b. Solve  $25u_{xx} = u_{tt}$  at the pivotal points given  $u(0, t) = 0 = u(5, t)$ ,  $u_t(x, 0) = 0$  and  $u(x, 0) = \begin{cases} 20x, & 0 \leq x \leq 1 \\ 5(5-x), & 1 \leq x \leq 5 \end{cases}$  by taking  $h = 1$  compute  $u(x, t)$  for  $0 \leq t \leq 1$ .

**Module-5**

- 9 a. Given  $y'' - xy' - y = 0$  with the initial conditions  $y(0) = 1, y'(0) = 0$  compute  $y(0.2)$  using fourth order Runge - Kutta method. (06 Marks)  
 b. Derive the Euler's equation. (07 Marks)  
 c. Find the extremal of the functional. (07 Marks)  

$$\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx .$$

OR

- 10 a. Obtain the solution of the equation  $2 \frac{d^2 y}{dx^2} = 4x + \frac{dy}{dx}$  by computing the value of  $y(1.4)$  by applying Milne's method using following data :

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

- b. Find the curve on which the functional  $\int_0^1 [(y')^2 + 12xy] dx$  with  $y(0) = 0$  and  $y(1) = 1$  can be determined. (07 Marks)  
 c. Prove that the shortest distance between two points in a plane is straight line. (07 Marks)

\*\*\*\*\*