

USN

BMATE101



First Semester B.E./B.Tech. Degree Examination, Jan./Feb. 2023  
**Mathematics - I for Electrical and Electronics Engineering Stream**

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
 2. VTU Formula Hand Book is permitted.  
 3. M : Marks, L: Bloom's level, C: Course outcomes.

Module - 1				M	L	C
Q.1	a.	If $\phi$ is the angle between the radius vector and tangent to the polar curve $r = r(\theta)$ , prove that $\tan \phi = r \frac{d\theta}{dr}$ .	6	L2	CO1	
	b.	Find the angle between the curves $r = \sin \theta + \cos \theta$ and $r = 2 \sin \theta$ .	7	L2	CO1	
	c.	Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $(\frac{3a}{2}, \frac{3a}{2})$ .	7	L2	CO1	
OR						
Q.2	a.	Prove that the polar curves $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$ intersect orthogonally.	6	L2	CO1	
	b.	Find the radius of curvature of the curve $y = 4 \sin x - \sin 2x$ at $x = \frac{\pi}{2}$ .	7	L2	CO1	
	c.	Using Modern mathematical tool, write a program / code to plot the curve $r = 2  \cos 2 \theta $ .	7	L3	CO5	
Module - 2						
Q.3	a.	Expand $f(x) = \cos x + \sin x$ in a Maclaurin series upto the term involving $x^2$ .	6	L2	CO2	
	b.	If $U = \log(\tan x + \tan y + \tan z)$ , show that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$ .	7	L2	CO2	
	c.	If $u = x^2 + y^2 + z^2$ , $v = xy + yz + zx$ , $w = x + y + z$ , find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ .	7	L2	CO2	
OR						
Q.4	a.	Evaluate $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$ .	6	L1	CO2	
	b.	If $V = f(x-y, y-z, z-x)$ , show that $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0$ .	7	L2	CO2	
	c.	Using modern mathematics tool, write a program / code to show that $U_{xx} + U_{yy} = 0$ . Given $U_3 = e^x (x \cos y - y \sin y)$ .	7	L3	CO2	

Module - 3						
Q.5	a.	Solve $(5x^4 + 3x^2y^2 - 2xy^3) dx + (2x^3y - 3x^2y^2 - 5y^4) dy = 0$ .	6	L2	CO3	
	b.	Find the Orthogonal trajectory of the family of confocal ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ , where $\lambda$ is a parameter.	7	L3	CO3	
	c.	Solve $xyp^2 - (x^2 + y^2)p + xy = 0$ .	7	L2	CO3	
OR						
Q.6	a.	Solve $\frac{dy}{dx} + y \tan x = y^2 \sec x$ .	6	L2	CO3	
	b.	Find the current $i$ at any time $t$ , if initially there is no current in the circuit governed by the differential equation $L \left( \frac{di}{dt} \right) + Ri = 200 \sin 300t$ , when $L = 0.05$ and $R = 100$ .	7	L3	CO3	
	c.	Find the general solution of the equation $(px - y)(py + x) = a^2p$ by reducing into Clairaut's form, taking the substitution $X = x^2, Y = y^2$ .	7	L2	CO3	
Module - 4						
Q.7	a.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy \, dy \, dx$ , by changing the order of integration.	6	L2	CO4	
	b.	Evaluate $\int_{-p}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) \, dy \, dx \, dz$ .	7	L2	CO4	
	c.	Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ .	7	L1	CO4	
OR						
Q.8	a.	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) \, dx \, dy$ by changing to polar coordinates.	6	L2	CO4	
	b.	Using double integration, find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ .	7	L3	CO4	
	c.	Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} \, d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$ by using Beta and Gamma functions.	7	L2	CO4	
Module - 5						
Q.9	a.	Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ .	6	L2	CO5	

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	<p><b>b.</b> Solve the system of linear equations by Gauss – Jordan method</p> $\begin{aligned} x + y + z &= 11 \\ 3x - y + 2z &= 12 \\ 2x + y - z &= 3. \end{aligned}$	7	L2	CO5
	<p><b>c.</b> Determine the largest eigen value and the corresponding eigen vector of the matrix.</p> $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ <p>by using Power method with initial eigen vector <math>X_0 = [1 \ 0 \ 0]^T</math>. Carry out six iterations.</p>	7	L2	CO5
<b>OR</b>				
<b>Q.10</b>	<p><b>a.</b> For what values of <math>\lambda</math> and <math>\mu</math> the system of equations <math>x + y + z = 6</math>, <math>x + 2y + 3z = 10</math> and <math>x + 2y + \lambda z = \mu</math> may have</p> <p>i) Unique solution ii) Infinite number of solution iii) No solution.</p>	6	L2	CO5
	<p><b>b.</b> Solve by using Gauss elimination method the equations <math>3x + y - z = 3</math>; <math>2x - 8y + z = -5</math>; <math>x - 2y + 9z = 8</math>.</p>	7	L2	CO5
	<p><b>c.</b> Using modern mathematical tool, write a program / code to find the largest eigen value of</p> $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ <p>by power method.</p>	7	L3	CO5

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