CBCS SCHEME

USN

15MATDIP31

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: *Answer FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Express : $\dfrac{(1+i)(2+i)}{3+i}$ in the form a + ib. (05 Marks)

b. Express : $\dfrac{1+2i}{1-3i}$ in the polar form and their modulus and amplitude. (05 Marks)

c. Find the values of $(1+i)^{1/3}$. (06 Marks)

OR

2 a. If $\vec{a} = 4i + j + k$, $\vec{b} = 2i + j + 2k$, $\vec{c} = 3i + 4j + 5k$ find $(\vec{a}+\vec{b})\cdot(\vec{b}+\vec{c})$. (05 Marks)

b. Find the angle between the vectors $\vec{a} = 2i + 6j + 3k$ and $\vec{b} = 12i - 4j + 3k$. (05 Marks)

c. Find the constant λ such that the vectors $\vec{a} = 2i - j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = 3i + \lambda j + 5k$ are coplanar. (06 Marks)

Module-2

3 a. Find the n^{th} derivative of y = sin(ax + b). (05 Marks)

b. With usual notation, prove that $\tan\phi = r\cdot\dfrac{d\theta}{dr}$. (05 Marks)

c. State Euler's theorem on homogeneous function. If $u = \dfrac{x^3 + y^3}{\sqrt{x} + \sqrt{y}}$ than prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{5}{2}u$. (06 Marks)

OR

4 a. Find the pedal equation of the curve $r^n = a^n \cos n\theta$. (06 Marks)

b. Obtain the Maclaurin's expansion of the function $f(x) = \sin x + \cos x$ up to the terms containing fourth degree. (05 Marks)

c. If $Z = xy^2 + x^2y$ where x = at, y = 2at find $\dfrac{du}{dt}$ in terms of 't'. (05 Marks)

Module-3

5 a. Evaluate : $\displaystyle\int_0^1 \frac{x^9}{\sqrt{1-x^2}}\,dx$ by using Reduction formula. (05 Marks)

b. Evaluate $\displaystyle\int_0^\infty \frac{x^2\,dx}{(1+x^6)^{7/2}}$ by using reduction formula. (05 Marks)

c. Evaluate : $\displaystyle\int_0^1\int_x^{\sqrt{x}} xy\,dy\,dx$. (06 Marks)

OR

6 a. Evaluate : $\int_0^{\pi} x \sin^8 x \, dx$. **(05 Marks)**

b. Evaluate : $\int_0^2 \frac{x^4}{\sqrt{4-x^2}} dx$ by using reduction formula. **(05 Marks)**

c. Evaluate : $\int_0^3 \int_0^2 \int_0^1 (x+y+z) \, dz \, dx \, dy$. **(06 Marks)**

Module-4

7 a. A particle moves along the curve $C : x = t^3 - 4t, \; y = t^2 + 4t, z = 8t^2 - 3t^3$. Determine the velocity and acceleration. **(05 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at $(2, -1, 2)$. **(05 Marks)**

c. If $\vec{F} = (x+y+1)i + j - (x+y)k$. Show that $\vec{F} \cdot \text{curl} \, \vec{F} = 0$. **(06 Marks)**

OR

8 a. Find the angle between the tangents to the curve,
$\vec{r} = \left(t - \frac{t^3}{3}\right)i + t^2 j + \left(t + \frac{t^3}{3}\right)k$ at $t = \pm 3$. **(05 Marks)**

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of the vector $\bar{a} = 2i - j - 2k$. **(05 Marks)**

c. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar point function ϕ such that $\vec{F} = \nabla\phi$. **(06 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$. **(05 Marks)**

b. Solve : $\frac{dy}{dx} - 2\frac{y}{x} = x + x^2$. **(06 Marks)**

c. Solve : $(2x + y + 1) \, dx + (x + 2y + 1) \, dy = 0$. **(05 Marks)**

OR

10 a. Solve : $\frac{dy}{dx} = \frac{y}{x - \sqrt{xy}}$. **(05 Marks)**

b. Solve : $\frac{dy}{dx} + \frac{y}{x} = xy^2$. **(05 Marks)**

c. Solve $(x^2 + y^2 + x) \, dx + xy \, dy = 0$. **(06 Marks)**

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