| USN | | | | | | | | | | |
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Internal Assessment Test 3 – JAN. 2023-Scheme and solutions

| Sub: | Analysis of Indeterminate structures | | | | | Sub Code: | 18CV52 | Branch: | Civil Engg |
|---|---|---|---|---|---|---|---|---|---|
| Date: | 19.01.2023 | Duration: | 90 min's | Max Marks: | 50 | Sem / Sec: | 4 A | | OBE |

Question number 1 is mandatory; answer any 2 full questions from Q2 to Q4.

| | | MARKS | CO | RBT |
|---|---|---|---|---|
| 1 (a) | Figure 1 shows a continuous beam ABCD, calculate the fixed end moments, if the support D rotates by 0.002 radians in anticlockwise direction and supports B sinks by 8mm. Take E=210 GPa and I=0.1 Gmm4 | [10] | CO1 | L2 |

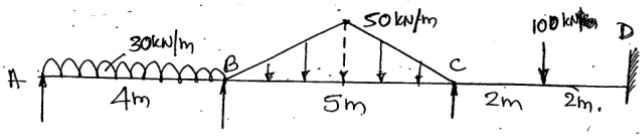
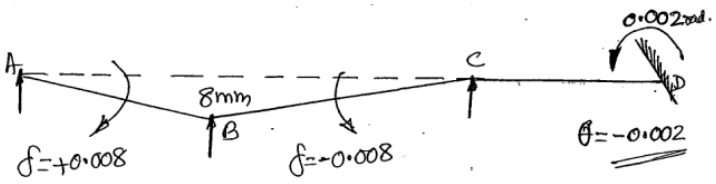



Fig.1.a




$E = 210 \times 10^9 \times 10^{-6} = 210 \times 10^3\ N/mm^2$

$I = 0.1 \times 10^9\ mm^4$

$EI = \dfrac{(210 \times 10^3)(0.1)10^9}{(10^3)(10^3)^2}\ N\text{-}mm^2 = 21000\ kN\text{-}m^2$

(a) FEM :

Additional moment $= -\dfrac{6EI\delta}{l^2}$ (Sinking)

$= \dfrac{4EI\theta}{l}$ → Near end Rotation

$= \frac{2EI\theta}{l}$ → Far end rotation

$M_{FAB} = -\frac{\omega l^2}{12} - \frac{6EI\delta}{l^2} = -\frac{30\times4^2}{12} - \frac{6(21000)(0.008)}{4^2}$

$= -103.0$ kN-m

$M_{FBA} = +\frac{\omega l^2}{12} - \frac{6EI\delta}{l^2} = -23$ kN-m.

$M_{FBC} = -\frac{5\omega l^2}{96} - \frac{6EI\delta}{l^2} = -\frac{5(50)5^2}{96} - \frac{6(2100)(-0.008)}{5^2}$

$= -24.78$

$M_{FCB} = +\frac{5}{96}\omega l^2 - \frac{6EI\delta}{l^2} = 105.42$ kN-m.

$M_{FCD} = -\frac{Wl}{8} + \frac{2EI\theta}{l}$ (Far end) $= -\frac{100\times4}{8} + \frac{2(21000)(-0.00}{4}$

$= -71$ kN-m

$M^F_{DC} = +\frac{Wl}{8} + \frac{4EI\theta}{l}$ (Near) $= \frac{100\times4}{8} + \frac{4(21000)(-0.002)}{4}$

$= +8$

| | | | | |
|---|---|---|---|---|
| 2 (a) | Analyse the continuous beam shown in Fig.2.a by Kani's method and draw BMD, SFD and EC. | [20] | CO3 | L3 |

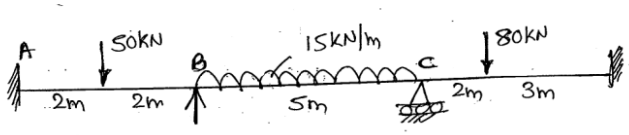



Fig.2.a

Sol: (a) FEM

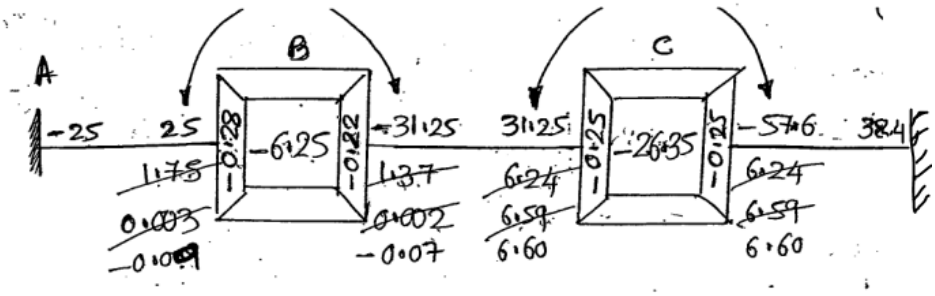
$M_{FAB} = -25$ kN-m, $M_{FBA} = +25$

$M_{FBC} = -31.25$, $M_{FCB} = +31.25$

$M_{FCD} = -57.6$, $M_{FDC} = 38.4$.

## (b) Rotation Factor (For Intermediate)

| | | k | $\Sigma k$ | $U=\left(-\frac{1}{2}\right)\frac{k}{\Sigma k}$ |
|---|---|---|---|---|
| B | BA | $I/4 = 0.25I$ | $0.45I$ | $-0.28$ |
| | BC | $I/5 = 0.20I$ | | $-0.22$ |
| C | CB | $I/5 = 0.20I$ | $0.4I$ | $-0.25$ |
| | CD | $I/5 = 0.20I$ | | $-0.25$ |




Rotation Moment $m'_{AB} = U\left[\Sigma M_F + \Sigma \text{Far end Rotation moment}\right]$

### Trial (1)

$m'_{BA} = -0.28(-6.25 + 0) = 1.75$

$m'_{BC} = -0.22(-6.25 + 0) = 1.37$

$m'_{CB} = -0.25(-26.35 + 1.37) = 6.24$

$m'_{CD} = -0.25(-26.35 + 1.37) = 6.24$

Trial (2)

$m'_{BA} = -0.28(-6.25 + 6.24) = 0.003$

$m'_{BC} = -0.22(-6.25 + 6.25) = 0.002$

$m'_{CB} = -0.25(-26.35 + 0.002) = 6.59$

$m'_{CD} = -0.25(-26.35 + 0.002) = 6.59$

Trial (3)

$m'_{BA} = -0.28(-6.25 + 6.59) = -0.09$

$m'_{BC} = -0.22(-6.25 + 6.59) = -0.07$

$m'_{CB} = -0.25(-26.35 - 0.07) = 6.60$

$m'_{CD} = -0.25(-26.35 - 0.07) = 6.60$

Final Moment

$M = FEM + 2(\text{Near End Rotation moment}) + 1(\text{Far end Rotation moment})$

$M_{AB} = -25 + 2(0) - 0.09 = -25.09$ kN-m ↺

$M_{BA} = +25 + 2(-0.09) + 0 = 24.82$ kN-m ↻

$M_{BC} = -31.25 + 2(-0.07) + 6.60 = -24.79$ kN-m ↺

$M_{CB} = +31.25 + 2(6.60) - 0.07 = 44.38$ " ↻

$M_{CD} = -57.6 + 2(6.60) - 0 = -44.40$ " ↺

$M_{DC} = 38.4 + 2(0) + 6.60 = 45$ kN-m ↻

| | | | | |
|---|---|---|---|---|
| 3 (a) | Analyse the continuous beam shown in Fig.3.a by Stiffness matrix method (system approach) and draw BMD, SFD and EC. | [20] | CO5 | L3 |

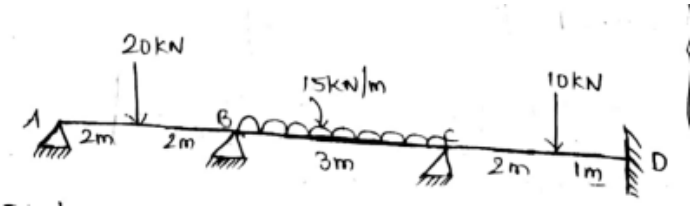



Fig.3.a

Step 1:- FEM's :-

$$M_{FAB} = \frac{-wl}{8} = \frac{-20\times 4}{8} = -10\,kNm$$

$$M_{FBA} = \frac{wl}{8} = \frac{20\times 4}{8} = 10\,kNm$$


$$M_{FBC} = \frac{-wl^2}{12} = \frac{-15\times 3^2}{12} = -11.25\,kNm$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{15\times 3^2}{12} = 11.25\,kNm$$

$$M_{FCD} = \frac{-wab^2}{l^2} = \frac{-10\times 2\times 1^2}{3^2} = -2.222\,kNm$$

$$M_{FDC} = \frac{wa^2b}{l^2} = \frac{10\times 4\times 1}{9} = 4.444\,kNm$$

Step 2:- $[\Delta]$, $[P]$, $[P_L]$

$$\therefore [P] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad [P_L] = \begin{bmatrix} -10 \\ -1.25 \\ 9.03 \end{bmatrix}, \quad [\Delta] = \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \end{bmatrix}$$

Step 3:- Stiffness matrix

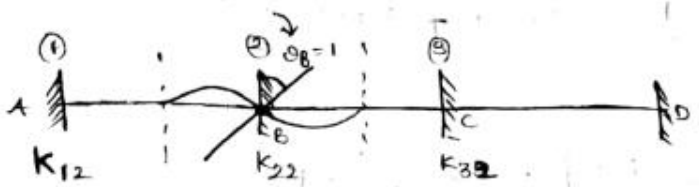
i) Applying unit rotation along co-ordinate direction ①


$$k_{11} = \left(\frac{4EI}{l}\right)_{AB} = \frac{4EI}{4} = EI$$

$$k_{21} = \left(\frac{2EI}{l}\right)_{BA} = \frac{2EI}{4} = 0.5EI$$

$$k_{31} = 0$$

ii) Applying unit rotation along Co-ordinate direction (2)

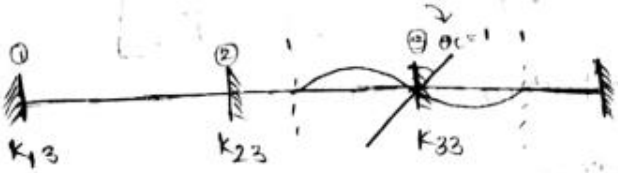



$$K_{12} = \left(\frac{2EI}{l}\right)_{AB} = \frac{2EI}{4} = 0.5EI$$

$$K_{22} = \left(\frac{4EI}{l}\right)_{BA} + \left(\frac{4EI}{l}\right)_{BC} = EI + 1.33EI = 2.333EI$$

$$K_{32} = \left(\frac{2EI}{l}\right)_{CB} = \frac{2}{3}EI = 0.667EI$$

iii) Applying unit rotation along Co-ordinate direction (3)




$$K_{13} = 0$$

$$K_{23} = \left(\frac{2EI}{l}\right)_{CB} = 0.667EI$$

$$K_{33} = \left(\frac{4EI}{l}\right)_{CB} + \left(\frac{4EI}{l}\right)_{CD} = \frac{4}{3}EI \times 2 = 2.667EI$$

$$\therefore [k] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} = EI \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2.333 & 0.667 \\ 0 & 0.667 & 2.667 \end{bmatrix}$$

$$\therefore [\Delta] = [k]^{-1} \times [P - P_L]$$

$$\begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 2.333 & 0.667 \\ 0 & 0.667 & 2.666 \end{bmatrix}^{-1} \times \begin{bmatrix} 10 \\ 1.25 \\ -9.03 \end{bmatrix}$$

$$\therefore \theta_A = \frac{10.389}{EI}, \quad \theta_B = \frac{-0.778}{EI}, \quad \theta_C = \frac{-3.192}{EI}$$

Step 4 :- Slope - deflection equations :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= -10 + \frac{2EI}{4}\left[\frac{2 \times 10.389}{EI} + \frac{-0.778}{EI} - \frac{3 \times 0}{4}\right]$$

$$= -10 + \frac{1}{2} \times (2 \times 10.389) + \frac{1}{2}(-0.778)$$

$$M_{AB} = 0$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$= 10 + \frac{2EI}{4} \times \frac{1}{2}\left[2 \times \left(\frac{-0.778}{EI}\right) + \frac{10.389}{EI}\right]$$

$$= 10 + (-0.778) + 5.1945$$

$$M_{BA} = 14.4165 \text{ kNm}$$

$$M_{BC} = -11.25 + \frac{2}{3}EI\left[2\left(\frac{-0.778}{EI}\right) + \frac{-3.192}{EI}\right]$$

$$= -11.25 - 1.0373 - 2.128$$

$$M_{BC} = -14.431 \text{ kNm}$$

$$M_{CB} = 11.25 + \frac{2}{3}EI\left[2\left(\frac{-3.192}{EI}\right) + \left(\frac{-0.778}{EI}\right)\right]$$

$$= 11.25 - 4.256 - 0.5186$$

$$M_{CB} = 6.4754 \text{ kNm}$$

$$M_{CD} = -2.222 + \frac{2EI}{3}\left[2\times\left(\frac{-3.192}{EI}\right) + (0)\right]$$

$$= -2.222 - 4.256$$

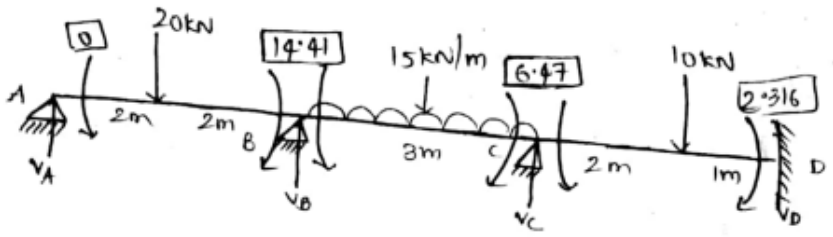
$$M_{CD} = -6.478 \text{ kNm}$$

$$M_{DC} = 4.444 + \frac{2}{3}EI\left[0 + \left(\frac{-3.192}{EI}\right)\right]$$

$$= 4.444 - 2.128$$

$$M_{DC} = 2.316 \text{ kNm}$$

Step 5:- SFD & BMD:-




$\therefore \Sigma F_y = 0$

$\Rightarrow V_A + V_B + V_C + V_D = 20 + 45 + 10$

$\Rightarrow \Sigma M_B = 0$ (LHS)

$\Rightarrow V_A \times 4 - 20 \times 2 - 14.41 = 0$

$\boxed{V_A = 13.6\,kN}$

$\Sigma M_C = 0$ (RHS)

$\Rightarrow -V_D \times 3 + 10 \times 2 + 2.316 - 6.47 = 0$

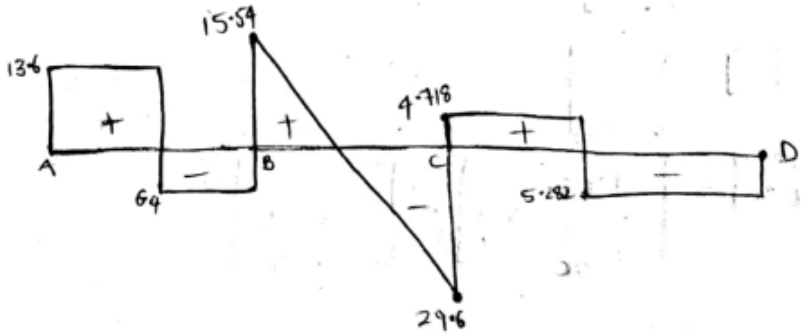
$\boxed{V_D = 5.282\,kN}$

$\Sigma M_C = 0$ (LHS)

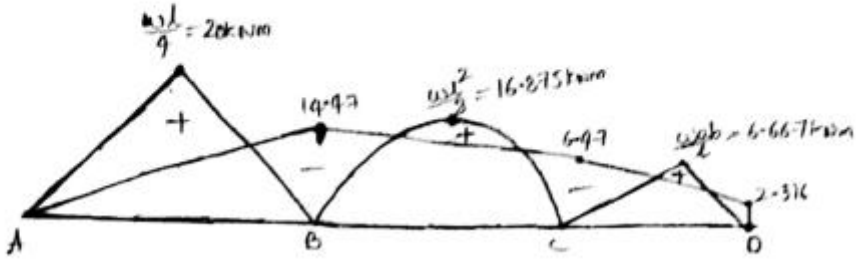
$\Rightarrow V_A \times 7 + V_B \times 3 + 6.47 - 20 \times 5 - 15 \times 3 \times \frac{3}{2} = 0$

$\Rightarrow \boxed{V_B = 21.94\,kN}$

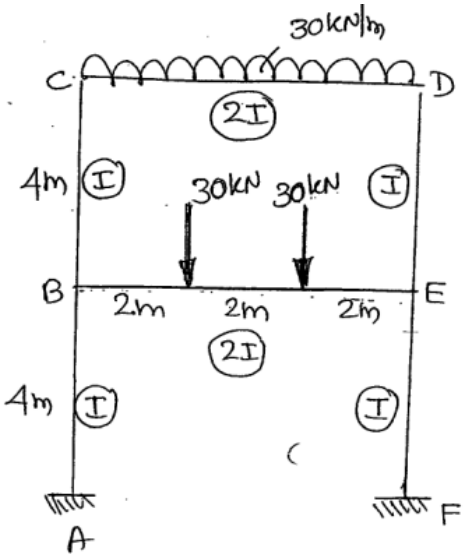
$\Rightarrow \boxed{V_C = 34.178\,kN}$

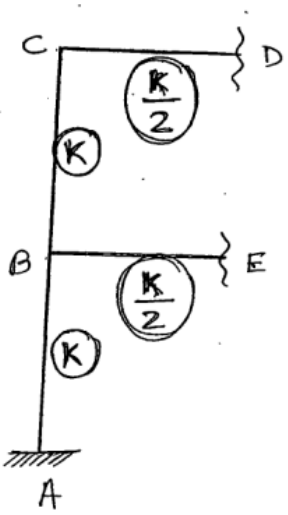


BMD:-




| | | | | |
|---|---|---|---|---|
| 4 (a) | Analyse the frame shown in Fig.4.a by Kani's method and draw BMD. | [20] | CO3 | L3 |



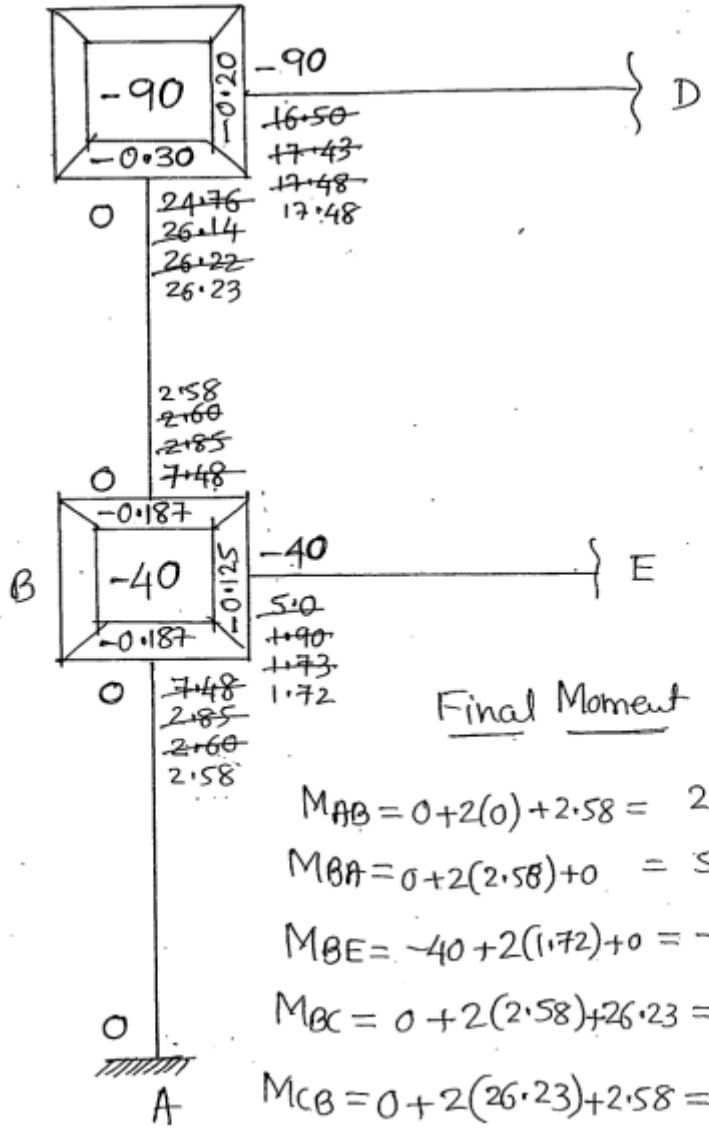

(a) FEM

$$M_{FBE} = \frac{-Wab^2}{l^2} = -\left[\frac{30\times2\times4^2}{6^2} + \frac{30\times4\times2^2}{6^2}\right] = -40$$

$$M_{FCD} = \frac{-wl^2}{12} = -90$$

(b) R.F (only For "B" & "C")

| | | K | ΣK | U |
|---|---|---|---|---|
| B | BA | $K = I/l = I/4 = 0.25I$ | 0.667I | -0.187 |
| | BC | $K = I/l = I/4 = 0.25I$ | | -0.187 |
| | BE | $\frac{K}{2} = \frac{1}{2}\left(\frac{I}{l}\right) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$ | | -0.125 |
| C | CB | $K = I/l = I/4 = 0.25I$ | 0.417I | -0.30 |
| | CD | $\frac{K}{2} = \frac{1}{2}\left(\frac{I}{l}\right) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$ | | -0.20 |




Final Moment

$M_{AB} = 0 + 2(0) + 2.58 = 2.58$ kN-m

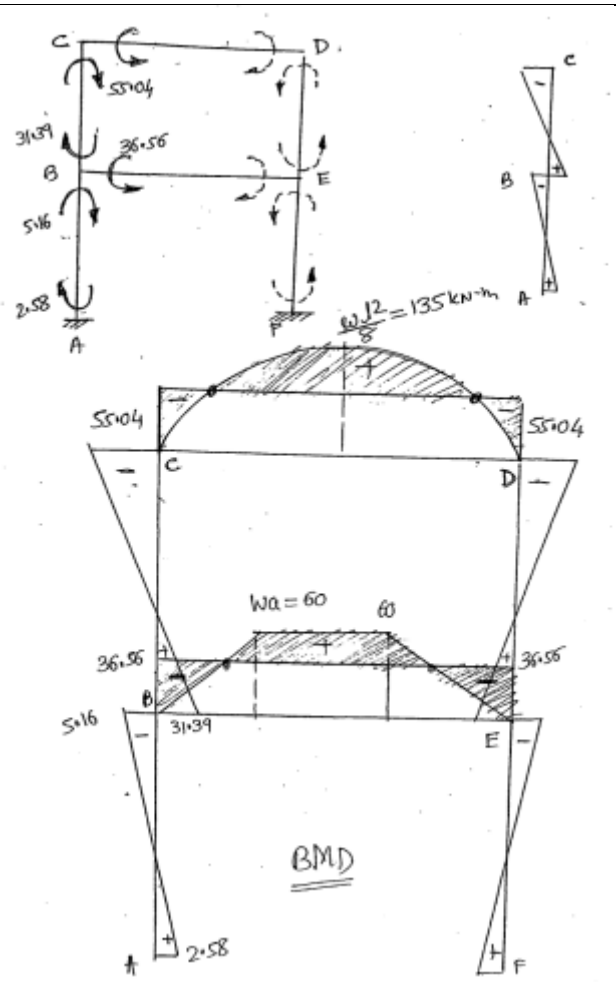
$M_{BA} = 0 + 2(2.58) + 0 = 5.16$ kN-m

$M_{BE} = -40 + 2(1.72) + 0 = -36.56$ "

$M_{BC} = 0 + 2(2.58) + 26.23 = 31.39$ "

$M_{CB} = 0 + 2(26.23) + 2.58 = 55.04$ "

$M_{CD} = -90 + 2(17.48) + 0 = -55.04$ "


C
D
55·04
31·39
36·56
B
E
5·16
2·58
A
F
C
B
A
$\frac{wl^2}{8} = 135\,kN\cdot m$
55·04
55·04
C
D
Wa = 60
60
36·56
36·56
B
5·16
31·39
E
BMD
2·58
F