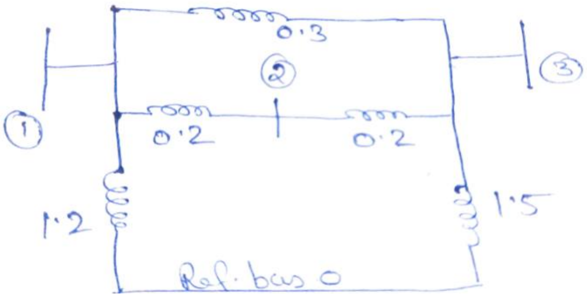
CMR
INSTITUTE OF
TECHNOLOGY

USN

Internal Assesment Test - III

Sub:	Power System Analysis II							Code:	18EE71
Date:	26/12/2022	Duration:	90 mins	Max Marks:	50	Sem:	7	Branch:	EEE
Answer Any FIVE FULL Questions									

<table>
<tr><th rowspan="2"></th><th rowspan="2"></th><th rowspan="2">Marks</th><th colspan="2">OBE</th></tr>
<tr><th>CO</th><th>RBT</th></tr>
<tr><td>1</td><td>With a usual notation derive generalized transmission loss formula and B –coefficients</td><td>10</td><td>CO4</td><td>L3</td></tr>
<tr><td>2</td><td>Explain the method of equal incremental cost for the economic operation of generators with transmission loss considered with the help of flow chart</td><td>10</td><td>CO4</td><td>L3</td></tr>
<tr><td>3</td><td>The fuel inputs per hour of plants 1 and 2 are given as
$F_1 = 0.2 P_1^2 + 40 P_1 + 120$ Rs per hr.
$F_2 = 0.25 P_2^2 + 30 P_2 + 150$ Rs per hr.
Determine the economic operating schedule and the corresponding cost of generation if the maximum and minimum loading on each unit is 100 MW and 25 MW, the demand is 180 MW and transmission losses are neglected. If the load is equally shared by both the units, determine the saving obtained by loading the units as per the incremental production cost.</td><td>10</td><td>CO4</td><td>L4</td></tr>
<tr><td>4</td><td>Derive the algorithm for the formation of bus impedance matrix Z_{bus} for a single phase system when a link element is added to the partial network</td><td>10</td><td>CO4</td><td>L4</td></tr>
<tr><td>5</td><td>For the positive sequence network data shown in table below, obtain Z bus by building procedure.

 </td><td>10</td><td>CO5</td><td>L4</td></tr>
<tr><td>6</td><td>Calculate the loss coefficient in pu and MW^{-1} on a base of 50 MVA for the network as shown in figure. Corresponding data is given below.
<table><tr><td>I_a=1.2 - j0.4 pu</td><td>Z_a = 0.02+j 0.08 pu</td></tr><tr><td>I_b=0.4 - j0.2 pu</td><td>Z_b = 0.08+j 0.32 pu</td></tr><tr><td>I_c=0.8 - j0.1 pu</td><td>Z_c = 0.02+j 0.08 pu</td></tr><tr><td>I_d=0.8 - j0.2 pu</td><td>Z_d = 0.03+j 0.12 pu</td></tr><tr><td>I_e=1.2 - j0.3 pu</td><td>Z_e = 0.03+j 0.12 pu</td></tr></table></td><td>10</td><td>CO4</td><td>L5</td></tr>
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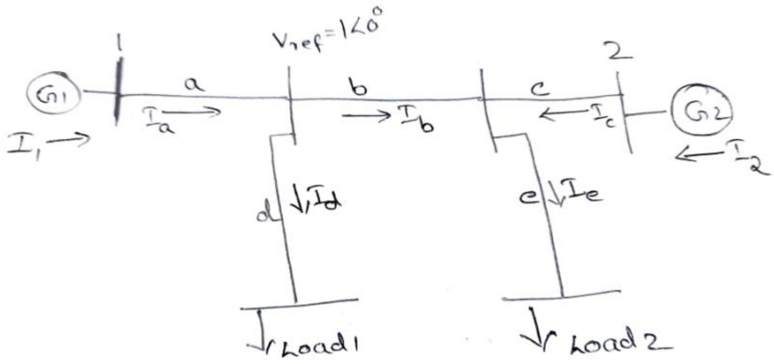
				

Solutions

1)

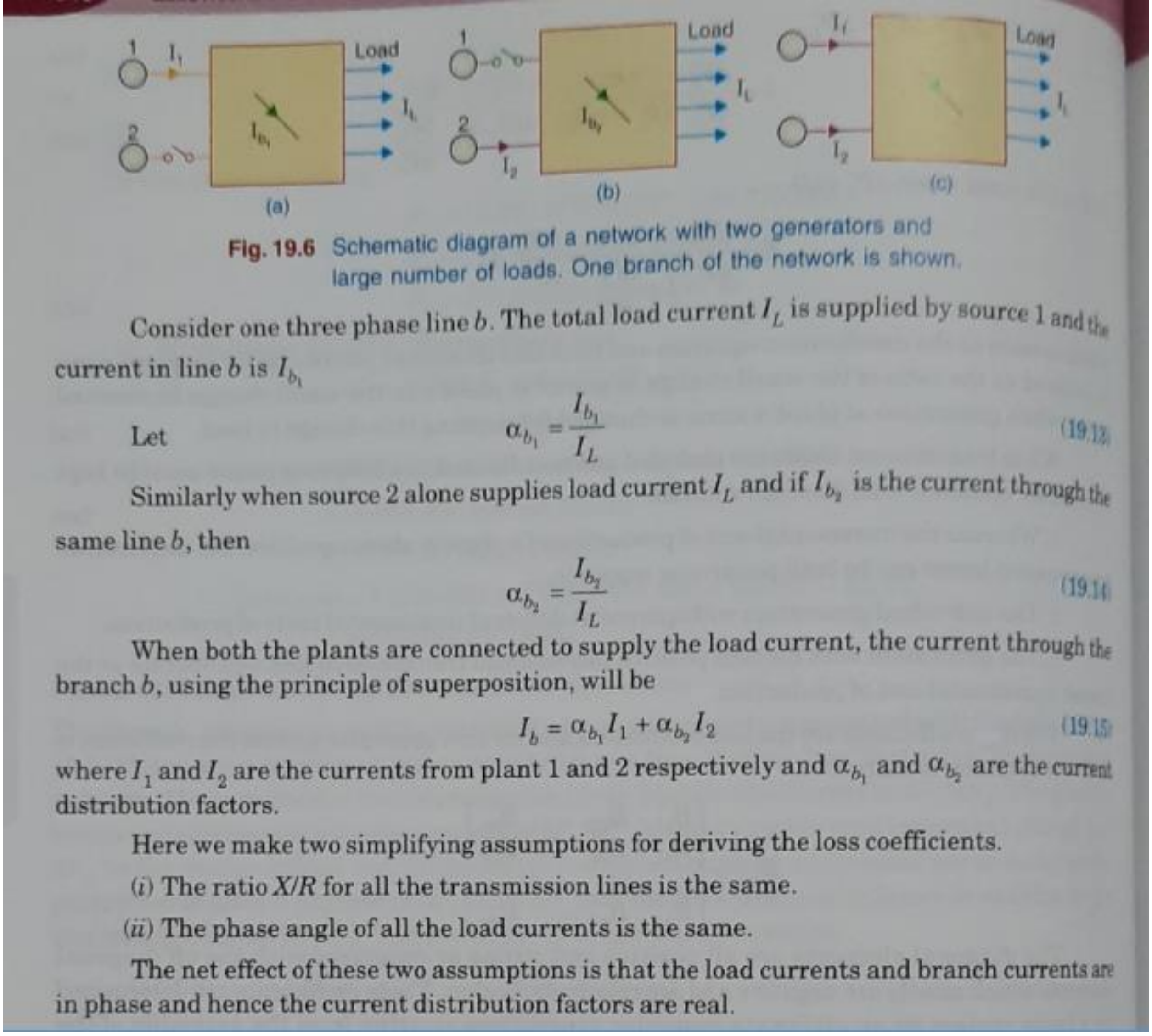


Fig. 19.6 Schematic diagram of a network with two generators and large number of loads. One branch of the network is shown.

Consider one three phase line b. The total load current I_L is supplied by source 1 and the current in line b is I_{b_1}

Let
$$\alpha_{b_1} = \frac{I_{b_1}}{I_L} \qquad (19.13)$$

Similarly when source 2 alone supplies load current I_L and if I_{b_2} is the current through the same line b, then

$$\alpha_{b_2} = \frac{I_{b_2}}{I_L} \qquad (19.14)$$

When both the plants are connected to supply the load current, the current through the branch b, using the principle of superposition, will be

$$I_b = \alpha_{b_1} I_1 + \alpha_{b_2} I_2 \qquad (19.15)$$

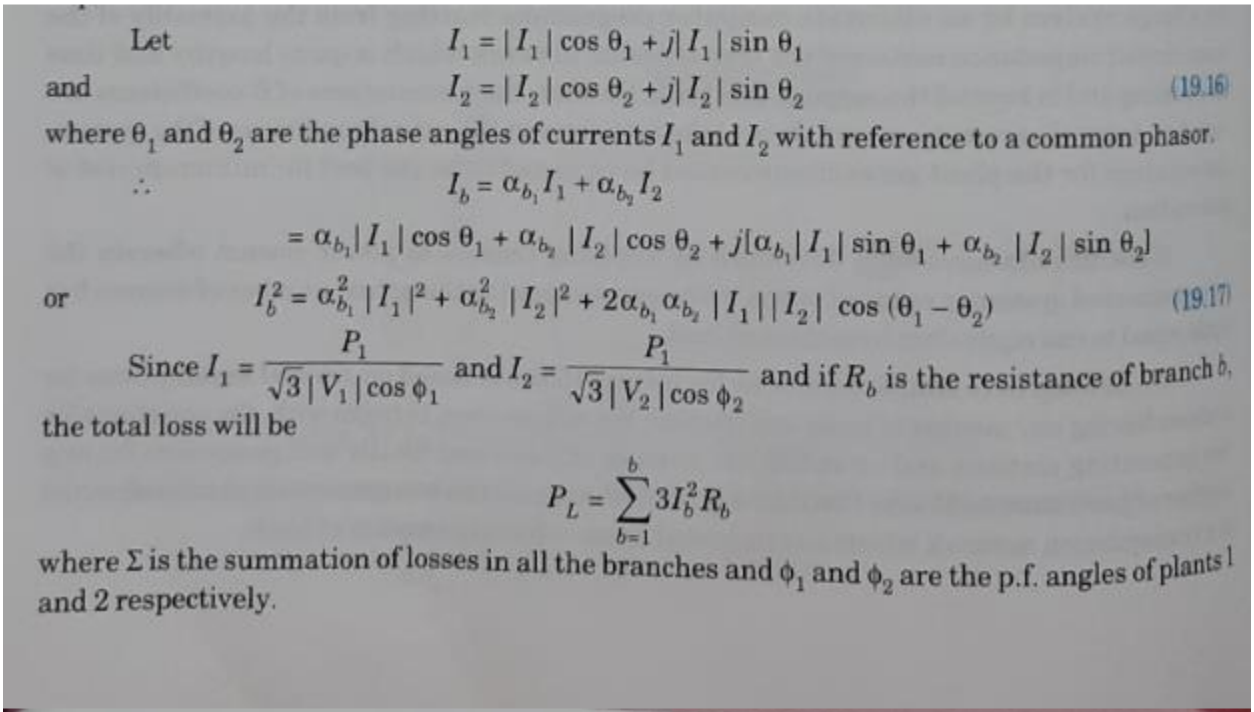
where I_1 and I_2 are the currents from plant 1 and 2 respectively and α_{b_1} and α_{b_2} are the current distribution factors.

Here we make two simplifying assumptions for deriving the loss coefficients.

(*i*) The ratio X/R for all the transmission lines is the same.

(*ii*) The phase angle of all the load currents is the same.

The net effect of these two assumptions is that the load currents and branch currents are in phase and hence the current distribution factors are real.



Let $I_1 = |I_1|\cos\theta_1 + j|I_1|\sin\theta_1$

and $I_2 = |I_2|\cos\theta_2 + j|I_2|\sin\theta_2$ (19.16)

where θ_1 and θ_2 are the phase angles of currents I_1 and I_2 with reference to a common phasor.

$$\therefore \quad I_b = \alpha_{b_1} I_1 + \alpha_{b_2} I_2$$

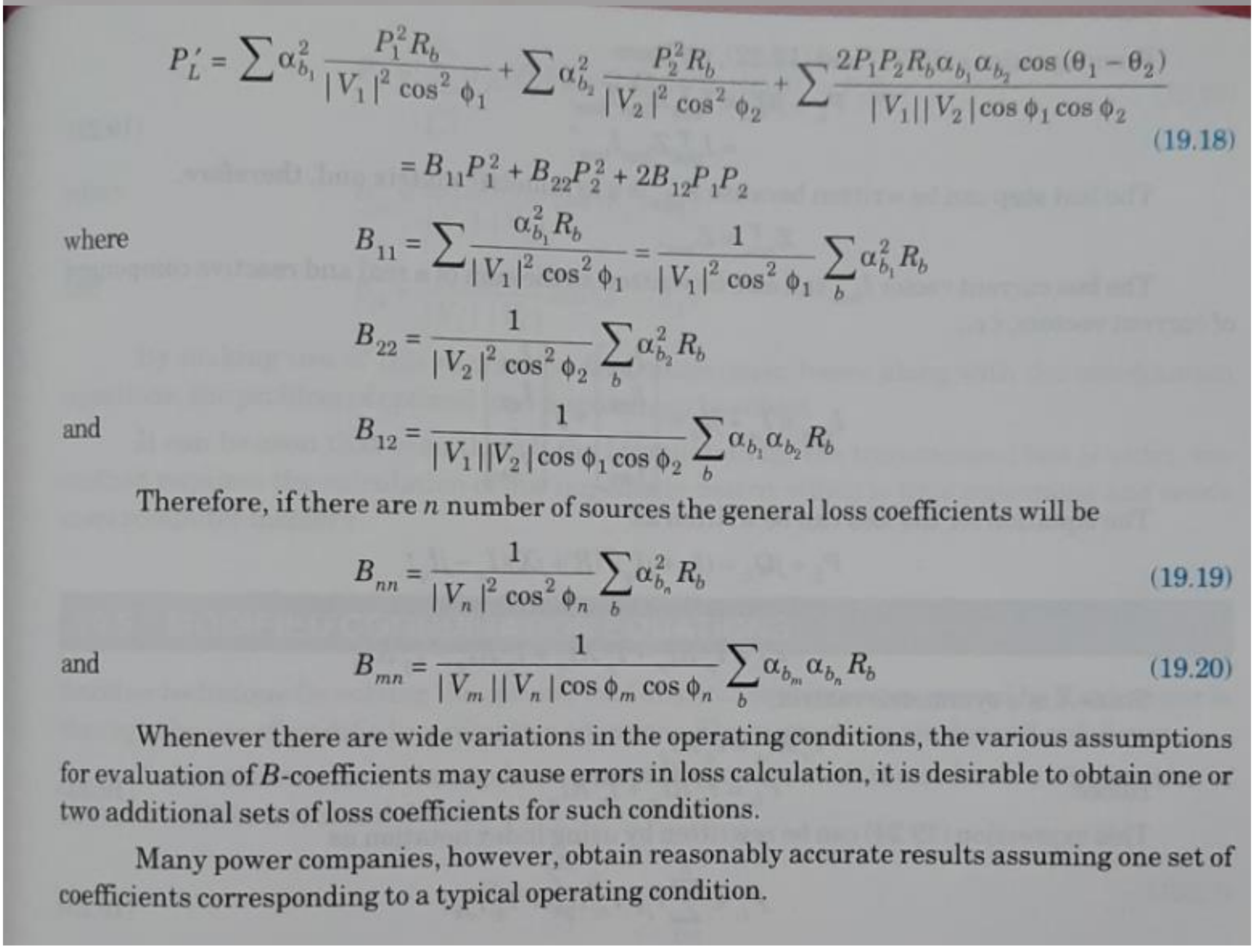
$$= \alpha_{b_1}|I_1|\cos\theta_1 + \alpha_{b_2}|I_2|\cos\theta_2 + j[\alpha_{b_1}|I_1|\sin\theta_1 + \alpha_{b_2}|I_2|\sin\theta_2]$$

or
$$I_b^2 = \alpha_{b_1}^2|I_1|^2 + \alpha_{b_2}^2|I_2|^2 + 2\alpha_{b_1}\alpha_{b_2}|I_1||I_2|\cos(\theta_1 - \theta_2) \qquad (19.17)$$

Since $I_1 = \frac{P_1}{\sqrt{3}|V_1|\cos\phi_1}$ and $I_2 = \frac{P_1}{\sqrt{3}|V_2|\cos\phi_2}$ and if R_b is the resistance of branch b, the total loss will be

$$P_L = \sum_{b=1}^{b} 3I_b^2 R_b$$

where Σ is the summation of losses in all the branches and ϕ_1 and ϕ_2 are the p.f. angles of plants 1 and 2 respectively.



$$P_L' = \sum \alpha_{b_1}^2 \frac{P_1^2 R_b}{|V_1|^2\cos^2\phi_1} + \sum \alpha_{b_2}^2 \frac{P_2^2 R_b}{|V_2|^2\cos^2\phi_2} + \sum \frac{2P_1P_2R_b\alpha_{b_1}\alpha_{b_2}\cos(\theta_1-\theta_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}$$

$$= B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2 \qquad (19.18)$$

where
$$B_{11} = \sum \frac{\alpha_{b_1}^2 R_b}{|V_1|^2\cos^2\phi_1} = \frac{1}{|V_1|^2\cos^2\phi_1}\sum_b \alpha_{b_1}^2 R_b$$

$$B_{22} = \frac{1}{|V_2|^2\cos^2\phi_2}\sum_b \alpha_{b_2}^2 R_b$$

and
$$B_{12} = \frac{1}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_b \alpha_{b_1}\alpha_{b_2} R_b$$

Therefore, if there are n number of sources the general loss coefficients will be

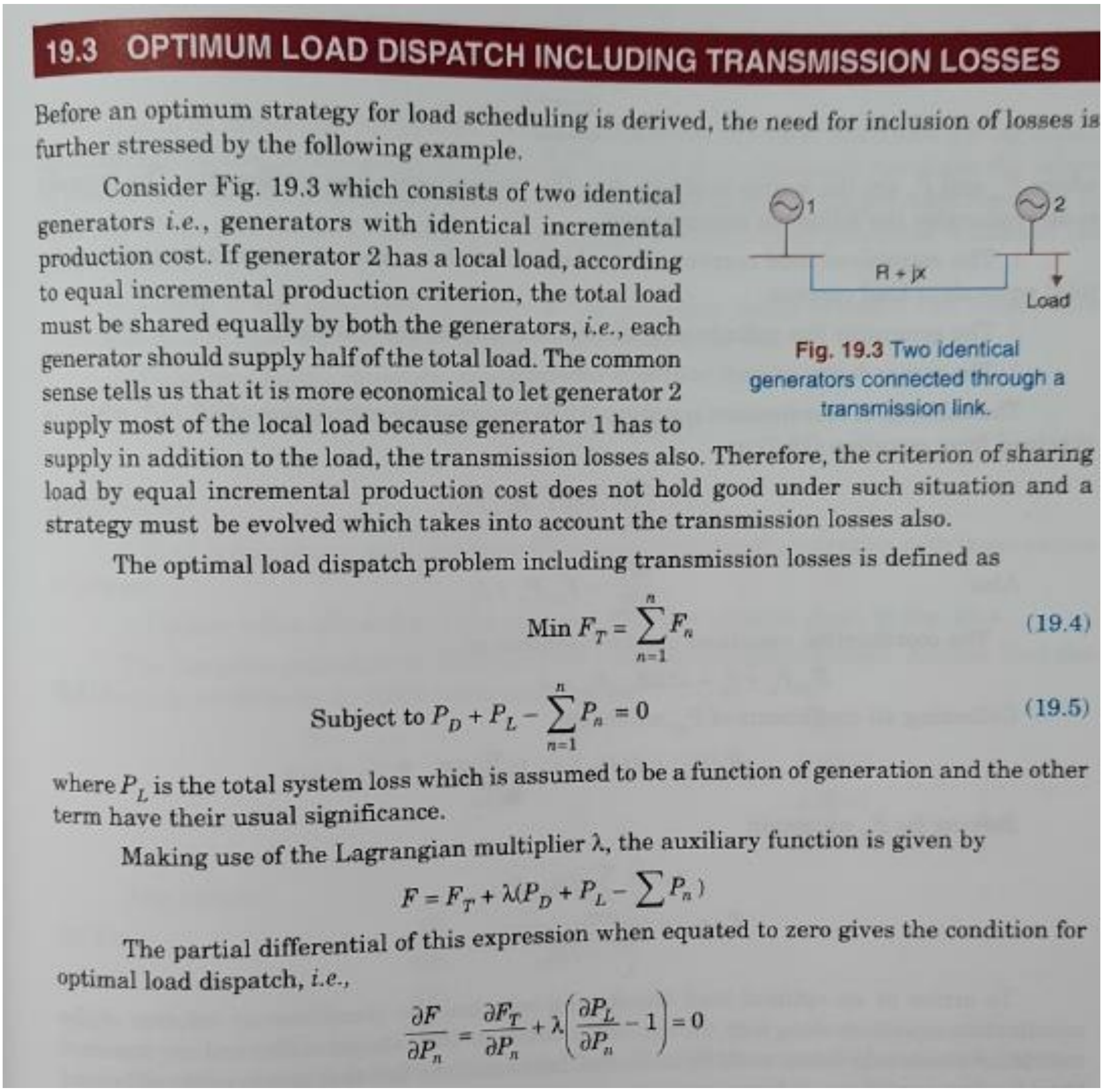
$$B_{nn} = \frac{1}{|V_n|^2\cos^2\phi_n}\sum_b \alpha_{b_n}^2 R_b \qquad (19.19)$$

and
$$B_{mn} = \frac{1}{|V_m||V_n|\cos\phi_m\cos\phi_n}\sum_b \alpha_{b_m}\alpha_{b_n} R_b \qquad (19.20)$$

Whenever there are wide variations in the operating conditions, the various assumptions for evaluation of B-coefficients may cause errors in loss calculation, it is desirable to obtain one or two additional sets of loss coefficients for such conditions.

Many power companies, however, obtain reasonably accurate results assuming one set of coefficients corresponding to a typical operating condition.

2)



19.3 OPTIMUM LOAD DISPATCH INCLUDING TRANSMISSION LOSSES

Before an optimum strategy for load scheduling is derived, the need for inclusion of losses is further stressed by the following example.

Consider Fig. 19.3 which consists of two identical generators *i.e.*, generators with identical incremental production cost. If generator 2 has a local load, according to equal incremental production criterion, the total load must be shared equally by both the generators, *i.e.*, each generator should supply half of the total load. The common sense tells us that it is more economical to let generator 2 supply most of the local load because generator 1 has to supply in addition to the load, the transmission losses also. Therefore, the criterion of sharing load by equal incremental production cost does not hold good under such situation and a strategy must be evolved which takes into account the transmission losses also.

Fig. 19.3 Two identical generators connected through a transmission link.

The optimal load dispatch problem including transmission losses is defined as

$$\text{Min } F_T = \sum_{n=1}^{n} F_n \tag{19.4}$$

$$\text{Subject to } P_D + P_L - \sum_{n=1}^{n} P_n = 0 \tag{19.5}$$

where P_L is the total system loss which is assumed to be a function of generation and the other term have their usual significance.

Making use of the Lagrangian multiplier λ, the auxiliary function is given by

$$F = F_T + \lambda(P_D + P_L - \sum P_n)$$

The partial differential of this expression when equated to zero gives the condition for optimal load dispatch, *i.e.*,

$$\frac{\partial F}{\partial P_n} = \frac{\partial F_T}{\partial P_n} + \lambda\left(\frac{\partial P_L}{\partial P_n} - 1\right) = 0$$

or
$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda \tag{19.6}$$

Here the term $\frac{\partial P_L}{\partial P_n}$ is known as the incremental transmission loss at plant n and λ is known as the incremental cost of received power in ₹ per MWhr.

The equation (19.6) is a set of n equations with $(n + 1)$ unknowns. Here n generations are unknown and λ is also unknown. These equations are known as coordination equations because they coordinate the incremental transmission losses with the incremental cost of production.

To solve these equations the loss formula equation (19.7) is expressed in terms of generations and is approximately expressed as

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \tag{19.7}$$

where P_m and P_n are the source loadings, B_{mn} the transmission loss coefficients. The formula is derived under the following assumptions:

1. The equivalent load current at any bus remains a constant complex fraction of the total equivalent load current.
2. The generator bus voltage magnitudes and angles are constant.
3. The power factor of each source is constant.

The solution of coordination equation (19.6) requires the calculation of $\partial P_L/\partial P_n$ which is obtained from equation (19.7) as

$$\frac{\partial P_L}{\partial P_n} = 2\sum_m B_{mn} P_m \tag{19.8}$$

Also
$$\frac{dF_n}{dP_n} = F_{nn} P_n + f_n$$

∴ The coordination equations can be rewritten as

$$F_{nn} P_n + f_n + \lambda \Sigma 2 B_{mn} P_m = \lambda \tag{19.9}$$

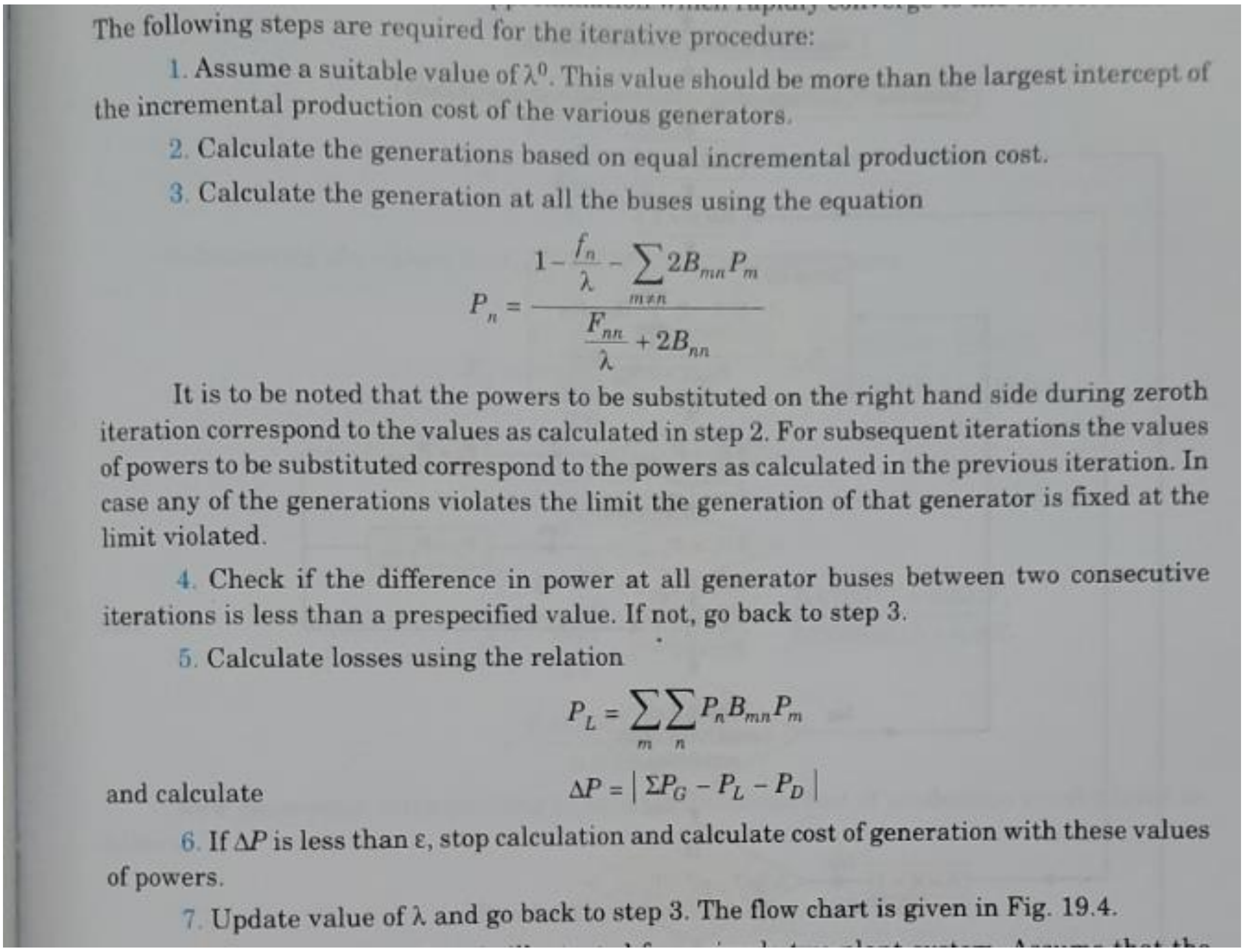
Collecting all coefficients of P_n, we obtain

$$P_n (F_{nn} + 2\lambda B_{nn}) = -\lambda (\sum_{m \neq n} 2 B_{mn} P_m) - f_n + \lambda$$

Solving for P_n we obtain

$$P_n = \frac{1 - \frac{f_n}{\lambda} \sum_{m \neq n} 2 B_{mn} P_m}{\frac{F_{nn}}{\lambda} + 2 B_{nn}} \tag{19.10}$$

To arrive at an optimal load dispatching solution, the simultaneous solution of the coordination equations along with the equality constraint (19.5) should suffice and any standard matrix inversion subroutine could be used. But, because of the fact that plants might go beyond their loading conditions, it becomes necessary to solve a new set of equations and thus by the



The following steps are required for the iterative procedure:

1. Assume a suitable value of λ^0. This value should be more than the largest intercept of the incremental production cost of the various generators.

2. Calculate the generations based on equal incremental production cost.

3. Calculate the generation at all the buses using the equation

$$P_n = \frac{1 - \frac{f_n}{\lambda} - \sum_{m \neq n} 2B_{mn} P_m}{\frac{F_{nn}}{\lambda} + 2B_{nn}}$$

It is to be noted that the powers to be substituted on the right hand side during zeroth iteration correspond to the values as calculated in step 2. For subsequent iterations the values of powers to be substituted correspond to the powers as calculated in the previous iteration. In case any of the generations violates the limit the generation of that generator is fixed at the limit violated.

4. Check if the difference in power at all generator buses between two consecutive iterations is less than a prespecified value. If not, go back to step 3.

5. Calculate losses using the relation

$$P_L = \sum_m \sum_n P_n B_{mn} P_m$$

and calculate

$$\Delta P = | \Sigma P_G - P_L - P_D |$$

6. If ΔP is less than ε, stop calculation and calculate cost of generation with these values of powers.

7. Update value of λ and go back to step 3. The flow chart is given in Fig. 19.4.

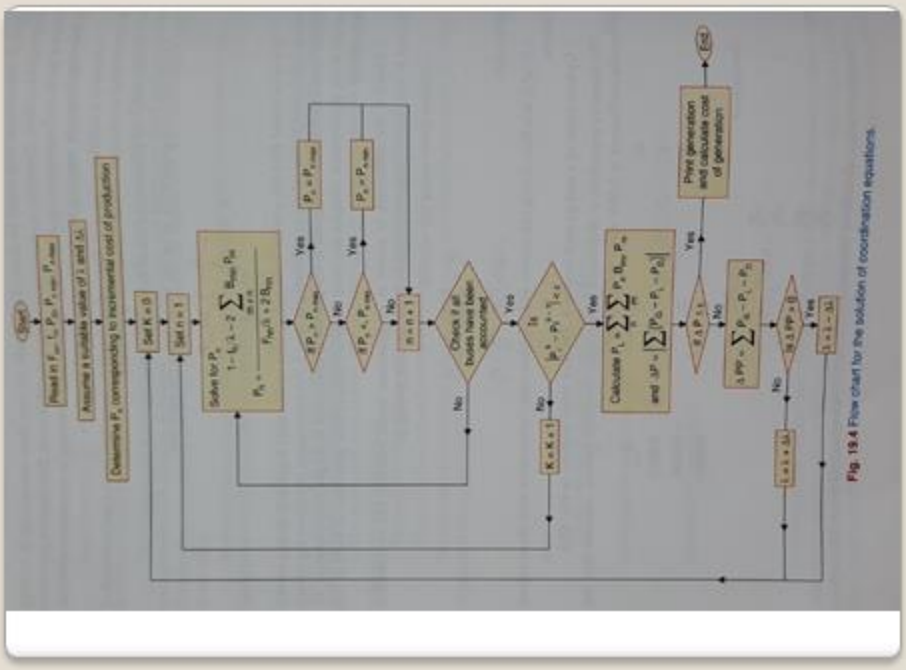
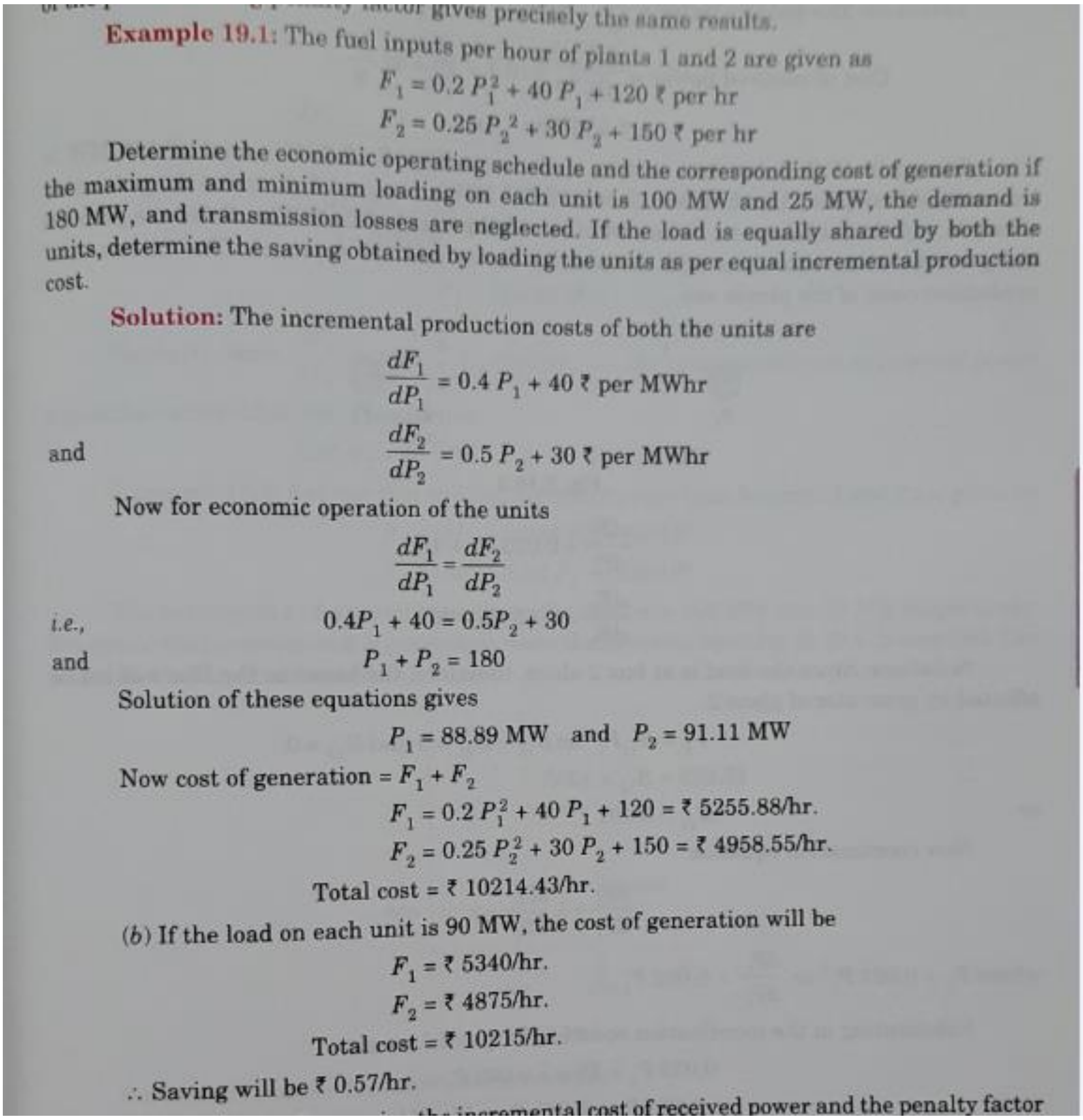


Fig. 19.4 Flow chart for the solution of coordination equations.

3)



gives precisely the same results.

Example 19.1: The fuel inputs per hour of plants 1 and 2 are given as

$$F_1 = 0.2\,P_1^2 + 40\,P_1 + 120 \text{ ₹ per hr}$$

$$F_2 = 0.25\,P_2^{\,2} + 30\,P_2 + 150 \text{ ₹ per hr}$$

Determine the economic operating schedule and the corresponding cost of generation if the maximum and minimum loading on each unit is 100 MW and 25 MW, the demand is 180 MW, and transmission losses are neglected. If the load is equally shared by both the units, determine the saving obtained by loading the units as per equal incremental production cost.

Solution: The incremental production costs of both the units are

$$\frac{dF_1}{dP_1} = 0.4\,P_1 + 40 \text{ ₹ per MWhr}$$

and

$$\frac{dF_2}{dP_2} = 0.5\,P_2 + 30 \text{ ₹ per MWhr}$$

Now for economic operation of the units

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

i.e., $0.4P_1 + 40 = 0.5P_2 + 30$

and $P_1 + P_2 = 180$

Solution of these equations gives

$$P_1 = 88.89 \text{ MW} \quad \text{and} \quad P_2 = 91.11 \text{ MW}$$

Now cost of generation = $F_1 + F_2$

$$F_1 = 0.2\,P_1^2 + 40\,P_1 + 120 = \text{₹ } 5255.88\text{/hr.}$$

$$F_2 = 0.25\,P_2^2 + 30\,P_2 + 150 = \text{₹ } 4958.55\text{/hr.}$$

Total cost = ₹ 10214.43/hr.

(*b*) If the load on each unit is 90 MW, the cost of generation will be

$$F_1 = \text{₹ } 5340\text{/hr.}$$

$$F_2 = \text{₹ } 4875\text{/hr.}$$

Total cost = ₹ 10215/hr.

∴ Saving will be ₹ 0.57/hr.

incremental cost of received power and the penalty factor

4)

Addition of a link

If the added element p–q is a link, the procedure for recalculating the elements of the bus impedance matrix is to connect in series with the added element a voltage source e_l as shown in Fig. 4.4. This creates a

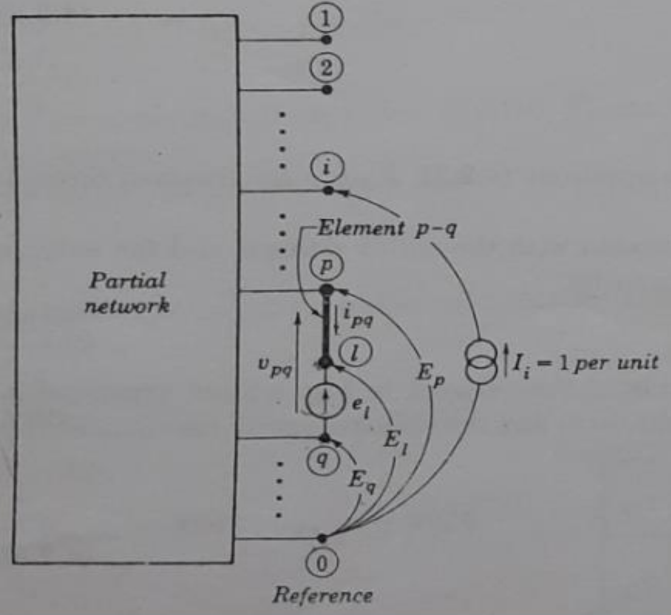
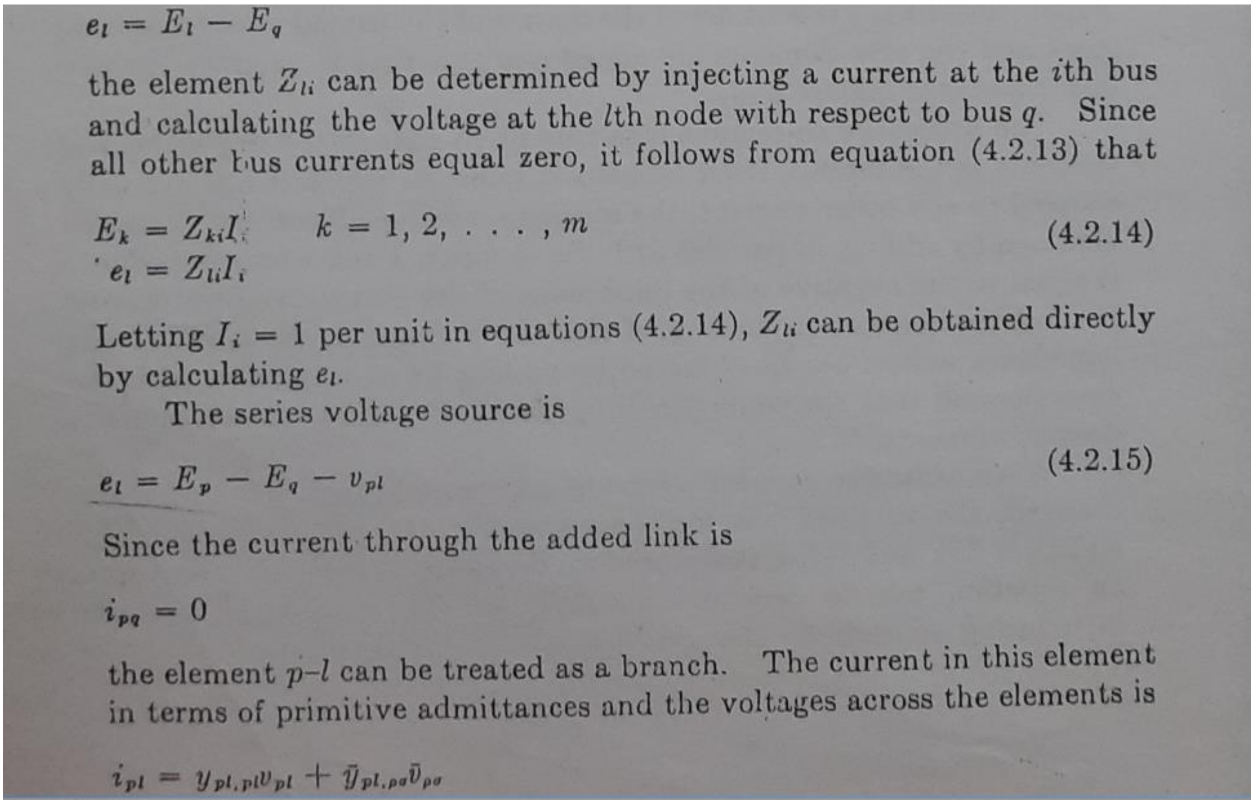


Fig. 4.4 Injected current voltage source in series with added link and bus voltages for calculation of Z_{li}.

through the added link is zero.

The performance equation for the partial network with the added element p–l and the series voltage source e_l is

$$\begin{array}{ccc|ccccccc|c}
 & & & 1 & & & p & & m & l & \\
E_1 & & 1 & Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1l} & I_1\\
E_2 & & & Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2l} & I_2\\
\cdots & & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\
E_p & = & p & Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pl} & I_p\\
\cdots & & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\
E_m & & m & Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{ml} & I_m\\
\hline
e_l & & l & Z_{l1} & Z_{l2} & \cdots & Z_{lp} & \cdots & Z_{lm} & Z_{ll} & I_l
\end{array} \qquad (4.2.13)$$



$e_l = E_l - E_q$

the element Z_{li} can be determined by injecting a current at the ith bus and calculating the voltage at the lth node with respect to bus q. Since all other bus currents equal zero, it follows from equation (4.2.13) that

$$E_k = Z_{ki} I_i \qquad k = 1, 2, \ldots, m \qquad (4.2.14)$$
$$e_l = Z_{li} I_i$$

Letting $I_i = 1$ per unit in equations (4.2.14), Z_{li} can be obtained directly by calculating e_l.

The series voltage source is

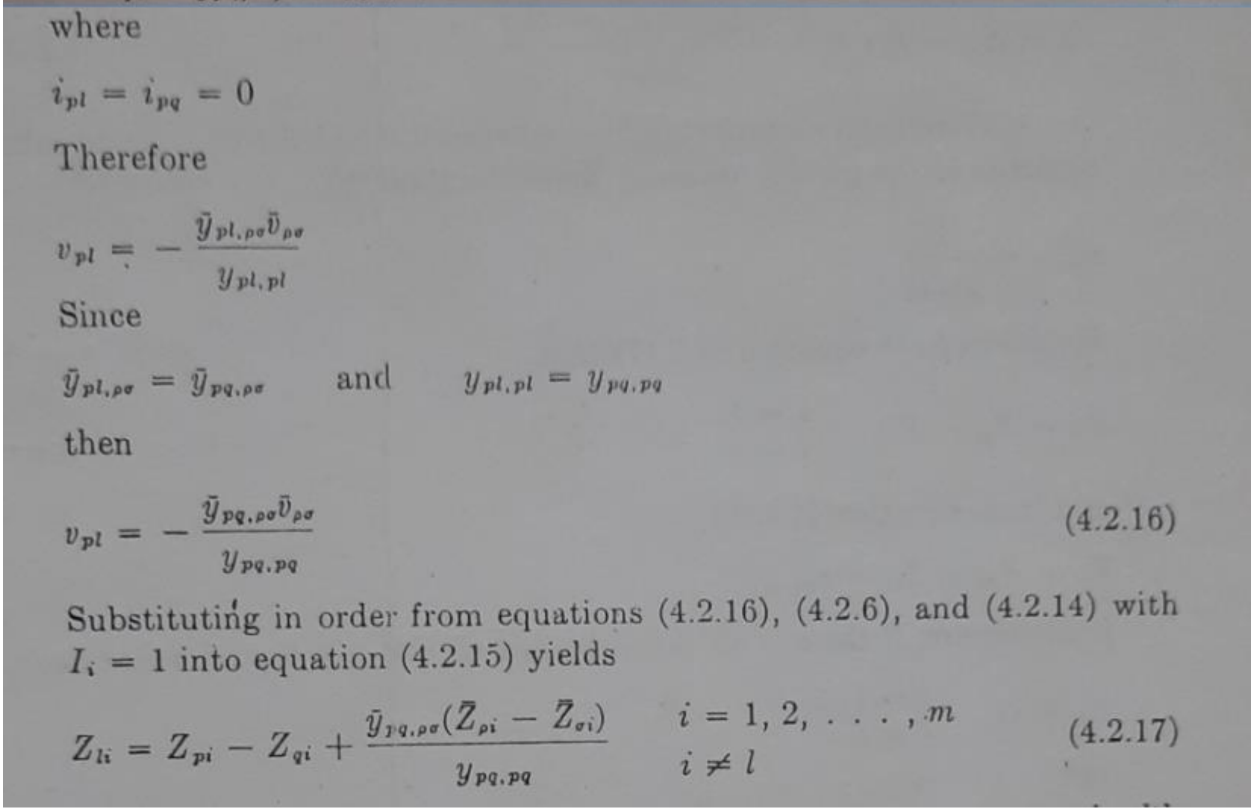
$$e_l = E_p - E_q - v_{pl} \qquad (4.2.15)$$

Since the current through the added link is

$i_{pq} = 0$

the element p–l can be treated as a branch. The current in this element in terms of primitive admittances and the voltages across the elements is

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,\rho\sigma} \bar{v}_{\rho\sigma}$$



where

$i_{pl} = i_{pq} = 0$

Therefore

$$v_{pl} = -\frac{\bar{y}_{pl,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pl,pl}}$$

Since

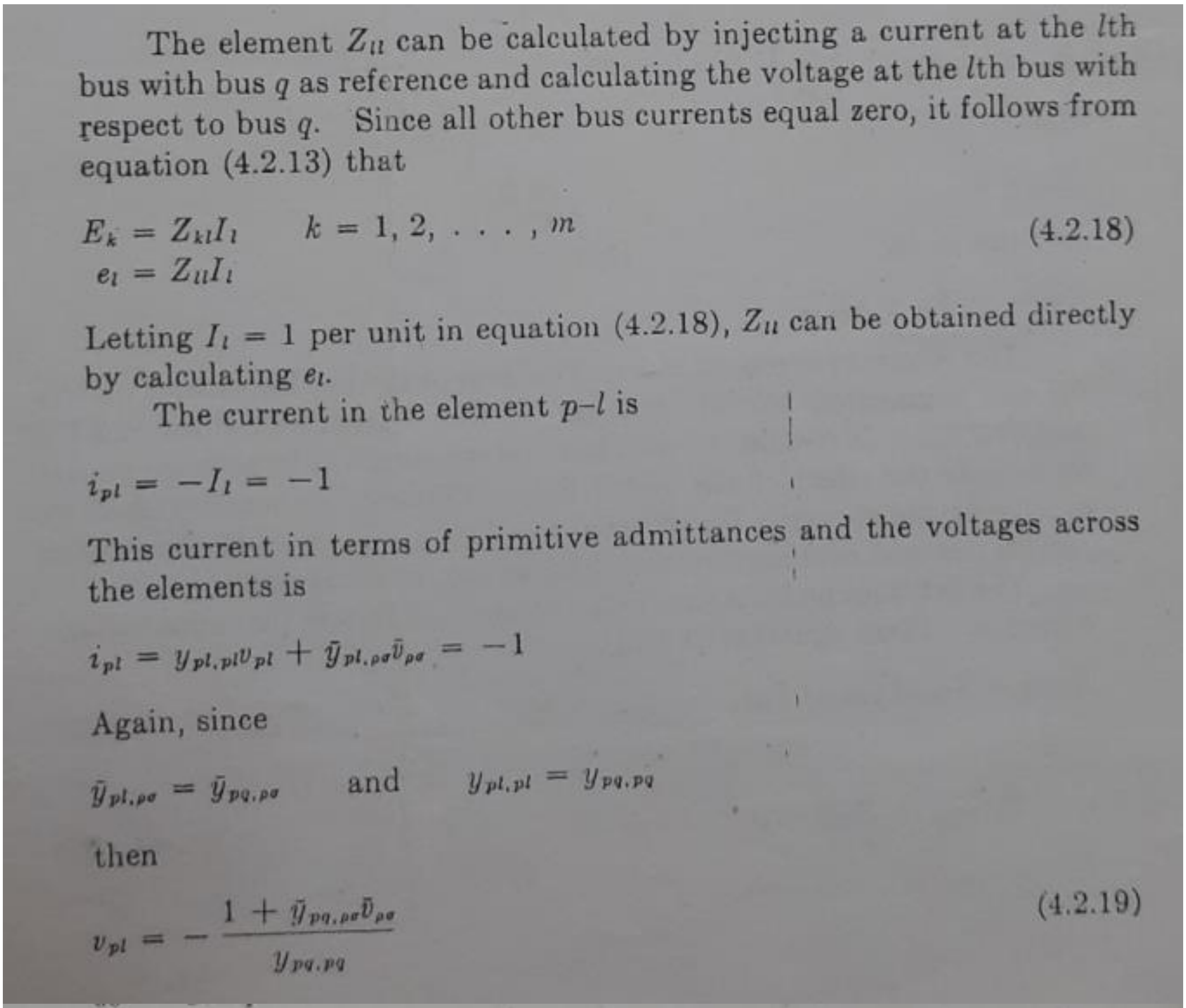
$\bar{y}_{pl,\rho\sigma} = \bar{y}_{pq,\rho\sigma}$ and $y_{pl,pl} = y_{pq,pq}$

then

$$v_{pl} = -\frac{\bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pq,pq}} \qquad (4.2.16)$$

Substituting in order from equations (4.2.16), (4.2.6), and (4.2.14) with $I_i = 1$ into equation (4.2.15) yields

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho i} - \bar{Z}_{\sigma i})}{y_{pq,pq}} \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ i \neq l \end{array} \qquad (4.2.17)$$



The element Z_{ll} can be calculated by injecting a current at the lth bus with bus q as reference and calculating the voltage at the lth bus with respect to bus q. Since all other bus currents equal zero, it follows from equation (4.2.13) that

$$E_k = Z_{kl} I_l \qquad k = 1, 2, \ . \ . \ . \ , m \tag{4.2.18}$$
$$e_l = Z_{ll} I_l$$

Letting $I_l = 1$ per unit in equation (4.2.18), Z_{ll} can be obtained directly by calculating e_l.

The current in the element p–l is

$$i_{pl} = -I_l = -1$$

This current in terms of primitive admittances and the voltages across the elements is

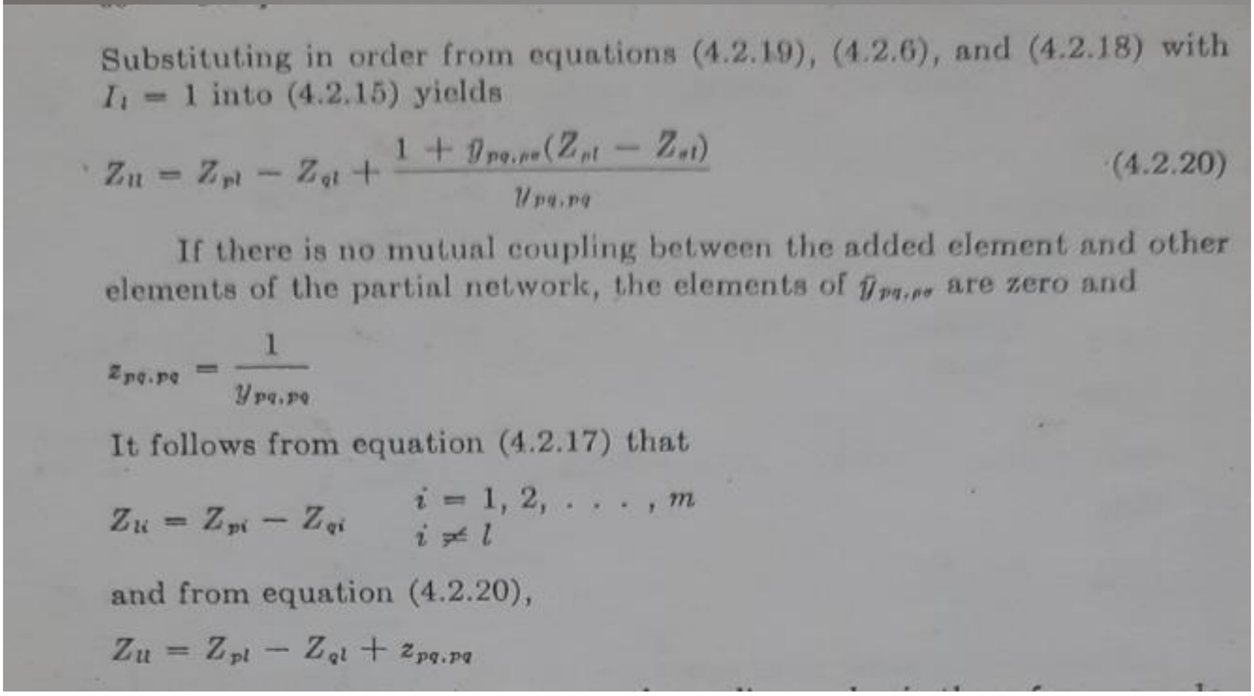
$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,\rho\sigma} \bar{v}_{\rho\sigma} = -1$$

Again, since

$$\bar{y}_{pl,\rho\sigma} = \bar{y}_{pq,\rho\sigma} \qquad \text{and} \qquad y_{pl,pl} = y_{pq,pq}$$

then

$$v_{pl} = -\frac{1 + \bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pq,pq}} \tag{4.2.19}$$



Substituting in order from equations (4.2.19), (4.2.6), and (4.2.18) with $I_l = 1$ into (4.2.15) yields

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho l} - \bar{Z}_{\sigma l})}{y_{pq,pq}} \tag{4.2.20}$$

If there is no mutual coupling between the added element and other elements of the partial network, the elements of $\bar{y}_{pq,\rho\sigma}$ are zero and

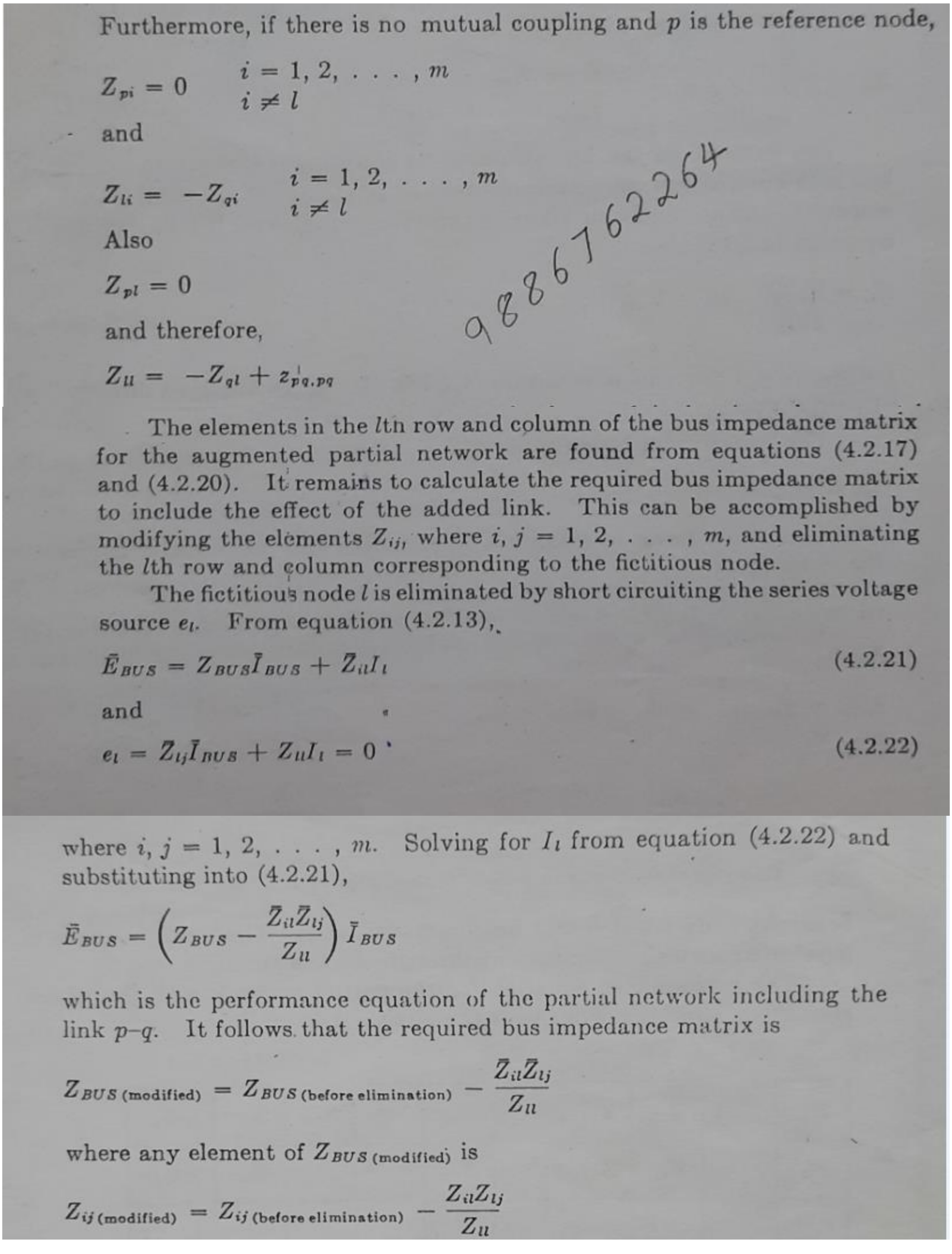
$$z_{pq,pq} = \frac{1}{y_{pq,pq}}$$

It follows from equation (4.2.17) that

$$Z_{li} = Z_{pi} - Z_{qi} \qquad \begin{matrix} i = 1, 2, \ . \ . \ . \ , m \\ i \neq l \end{matrix}$$

and from equation (4.2.20),

$$Z_{ll} = Z_{pl} - Z_{ql} + z_{pq,pq}$$



Furthermore, if there is no mutual coupling and p is the reference node,

$$Z_{pi} = 0 \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ i \neq l \end{array}$$

and

$$Z_{li} = -Z_{qi} \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ i \neq l \end{array}$$

Also

$$Z_{pl} = 0$$

and therefore,

$$Z_{ll} = -Z_{ql} + z_{pq,pq}$$

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The elements in the lth row and column of the bus impedance matrix for the augmented partial network are found from equations (4.2.17) and (4.2.20). It remains to calculate the required bus impedance matrix to include the effect of the added link. This can be accomplished by modifying the elements Z_{ij}, where $i, j = 1, 2, \ldots, m$, and eliminating the lth row and column corresponding to the fictitious node.

The fictitious node l is eliminated by short circuiting the series voltage source e_l. From equation (4.2.13),

$$\bar{E}_{BUS} = Z_{BUS}\bar{I}_{BUS} + \bar{Z}_{il}I_l \qquad (4.2.21)$$

and

$$e_l = \bar{Z}_{lj}\bar{I}_{BUS} + Z_{ll}I_l = 0 \qquad (4.2.22)$$

where $i, j = 1, 2, \ldots, m$. Solving for I_l from equation (4.2.22) and substituting into (4.2.21),

$$\bar{E}_{BUS} = \left(Z_{BUS} - \frac{\bar{Z}_{il}\bar{Z}_{lj}}{Z_{ll}}\right)\bar{I}_{BUS}$$

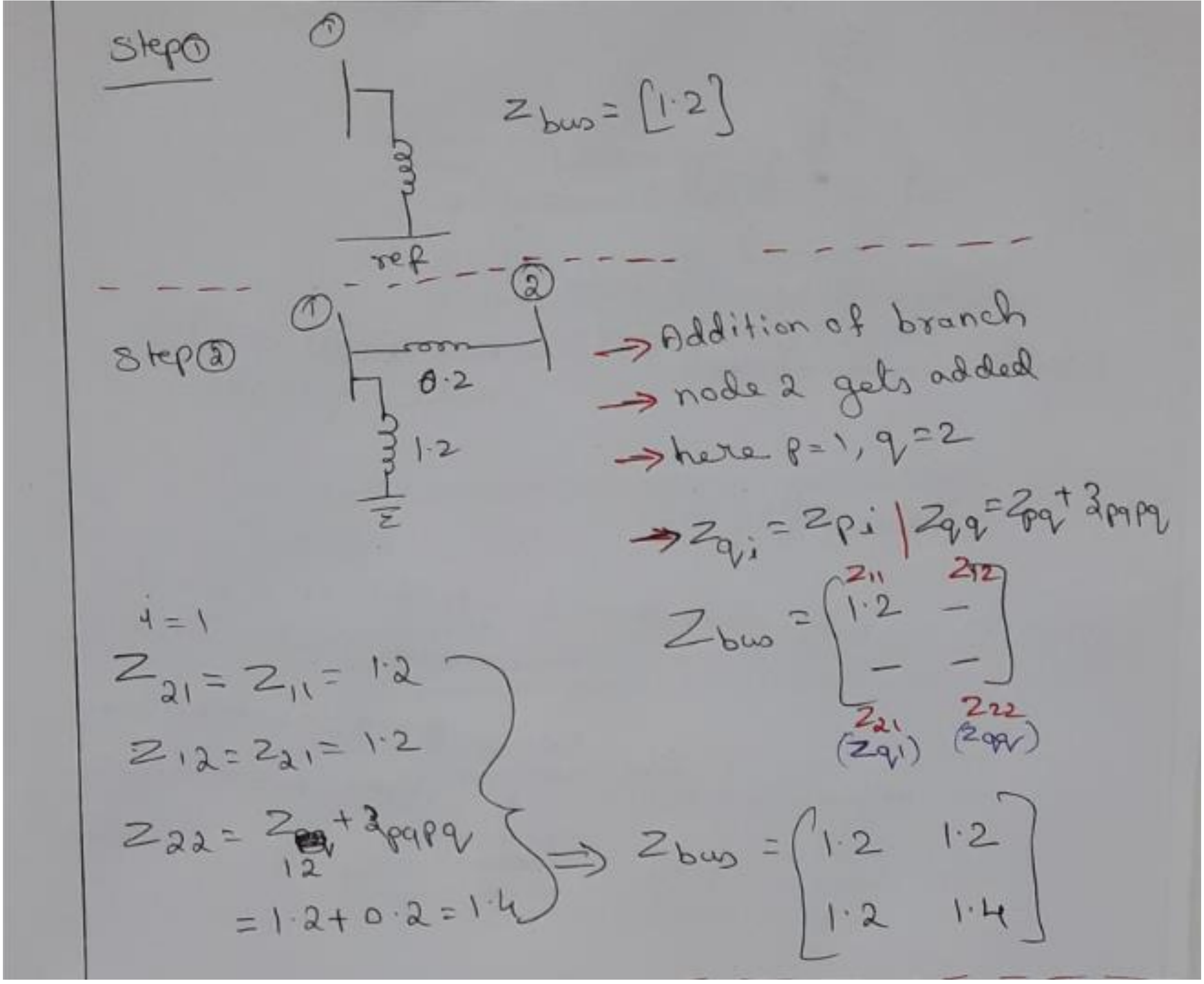
which is the performance equation of the partial network including the link p–q. It follows that the required bus impedance matrix is

$$Z_{BUS\,(\text{modified})} = Z_{BUS\,(\text{before elimination})} - \frac{\bar{Z}_{il}\bar{Z}_{lj}}{Z_{ll}}$$

where any element of $Z_{BUS\,(\text{modified})}$ is

$$Z_{ij\,(\text{modified})} = Z_{ij\,(\text{before elimination})} - \frac{Z_{il}Z_{lj}}{Z_{ll}}$$

5)



Step①

$$Z_{bus} = [1\cdot2]$$

Step②

→ Addition of branch

→ node 2 gets added

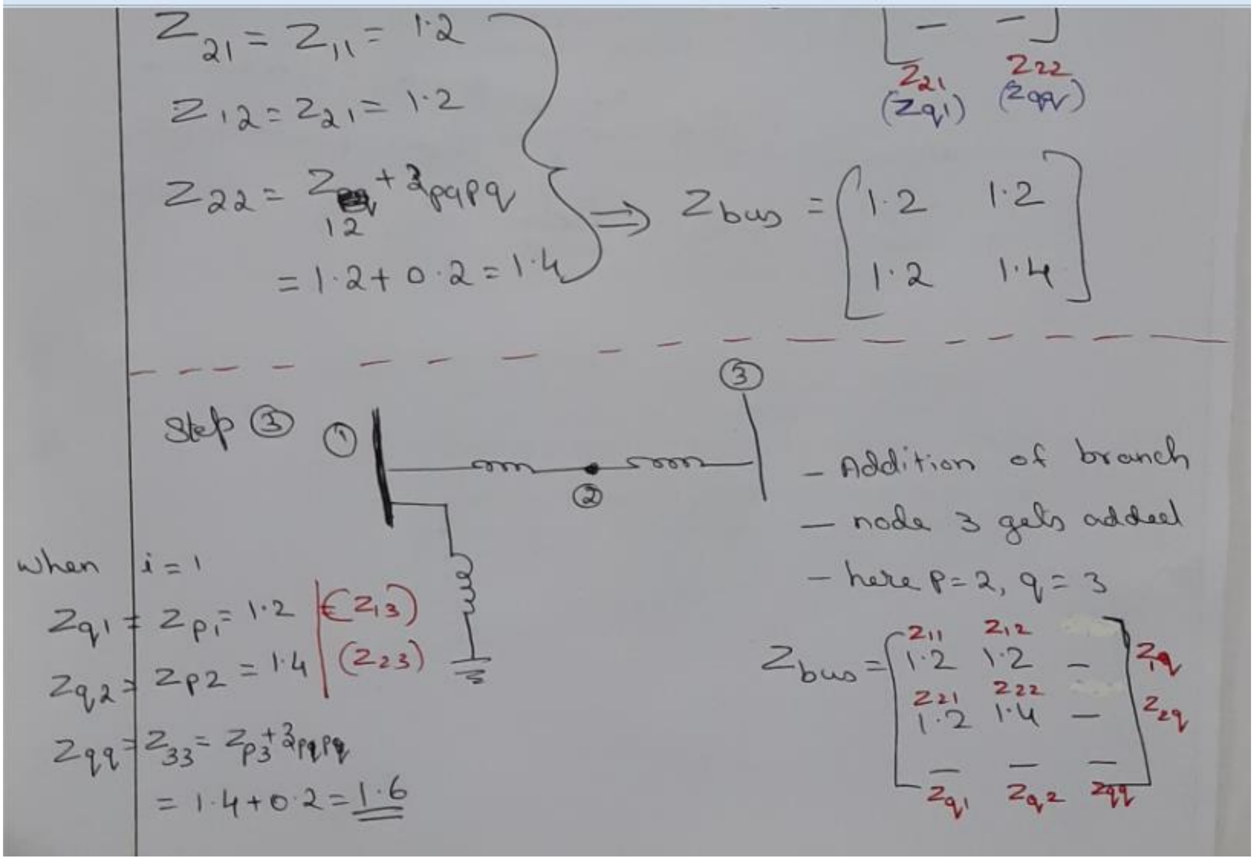
→ here $p = 1, q = 2$

→ $Z_{qi} = Z_{pi} \mid Z_{qq} = Z_{pq} + z_{pqpq}$

$$Z_{bus} = \begin{bmatrix} \overset{Z_{11}}{1\cdot2} & \overset{Z_{12}}{-} \\ \underset{\substack{Z_{21}\\(Z_{q1})}}{-} & \underset{\substack{Z_{22}\\(Z_{qq})}}{-} \end{bmatrix}$$

$i = 1$

$$\left.\begin{aligned} Z_{21} &= Z_{11} = 1\cdot2 \\ Z_{12} &= Z_{21} = 1\cdot2 \\ Z_{22} &= Z_{\underset{12}{\cancel{pq}}} + z_{pqpq} \\ &= 1\cdot2 + 0\cdot2 = 1\cdot4 \end{aligned}\right\} \Rightarrow Z_{bus} = \begin{bmatrix} 1\cdot2 & 1\cdot2 \\ 1\cdot2 & 1\cdot4 \end{bmatrix}$$



$$\begin{array}{cc} - & - \\ Z_{21} & Z_{22} \\ (Z_{q1}) & (Z_{qq}) \end{array}$$

$Z_{21} = Z_{11} = 1.2$

$Z_{12} = Z_{21} = 1.2$

$Z_{22} = \underset{12}{Z_{qq}} + z_{pqpq}$

$= 1.2 + 0.2 = 1.4$

$$\Rightarrow Z_{bus} = \begin{bmatrix} 1.2 & 1.2 \\ 1.2 & 1.4 \end{bmatrix}$$

Step ③

- Addition of branch
- node 3 gets added
- here $p = 2, q = 3$

when $i = 1$

$Z_{q1} = Z_{p1} = 1.2 \quad \leftarrow (Z_{13})$

$Z_{q2} = Z_{p2} = 1.4 \quad (Z_{23})$

$Z_{qq} = Z_{33} = Z_{p3} + z_{pqpq}$

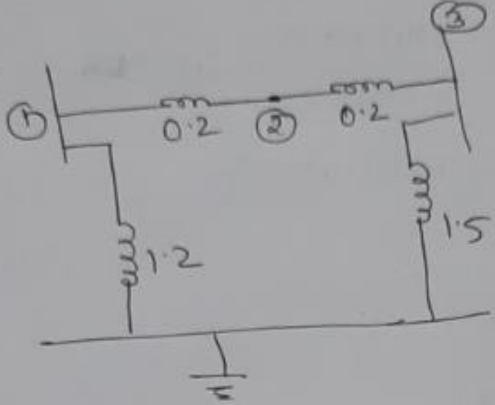
$= 1.4 + 0.2 = \underline{\underline{1.6}}$

$$Z_{bus} = \left[\begin{array}{ccc|c} \overset{Z_{11}}{1.2} & \overset{Z_{12}}{1.2} & - & Z_{1q} \\ \overset{Z_{21}}{1.2} & \overset{Z_{22}}{1.4} & - & Z_{2q} \\ - & - & - & \\ Z_{q1} & Z_{q2} & Z_{qq} & \end{array}\right]$$

CMR

$$Z_{bus} = \begin{bmatrix} 1.2 & 1.2 & 1.2 \\ 1.2 & 1.4 & 1.4 \\ 1.2 & 1.4 & 1.6 \end{bmatrix}$$

Step ④



- Addition of link
- here $p = 0$, $q = 3$

~~$Z_{q\ell} =$~~

- $Z_{\ell i} = -Z_{qi}$

$Z_{\ell\ell} = -Z_{q\ell} + 3_{pqpq}$

$Z_{\ell 1} = -Z_{31} = -1.2$

$Z_{\ell 2} = -Z_{32} = -1.4$

$Z_{\ell 3} = -Z_{33} = -1.6$

$Z_{\ell\ell} = -Z_{3\ell} + 3_{pqpq}$

$= +1.6 + 1.5 = 3.1$

$$Z_{bus} = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{bmatrix} 1.2 & 1.2 & 1.2 & - \\ 1.2 & 1.4 & 1.4 & - \\ 1.2 & 1.4 & 1.6 & - \\ - & - & - & - \end{bmatrix}$$

ℓth node

$Z_{\ell 1}$ $Z_{\ell 2}$ $Z_{\ell 3}$ $Z_{\ell\ell}$

$$Z_{bus} = \begin{array}{c|ccc|c} & & & & ⓛ \\ & 1.2 & 1.2 & 1.2 & -1.2 \\ & 1.2 & 1.4 & 1.4 & -1.4 \\ & 1.2 & 1.4 & 1.6 & -1.6 \\ \hline ⓛ & -1.2 & -1.4 & -1.6 & 3.1 \end{array}$$

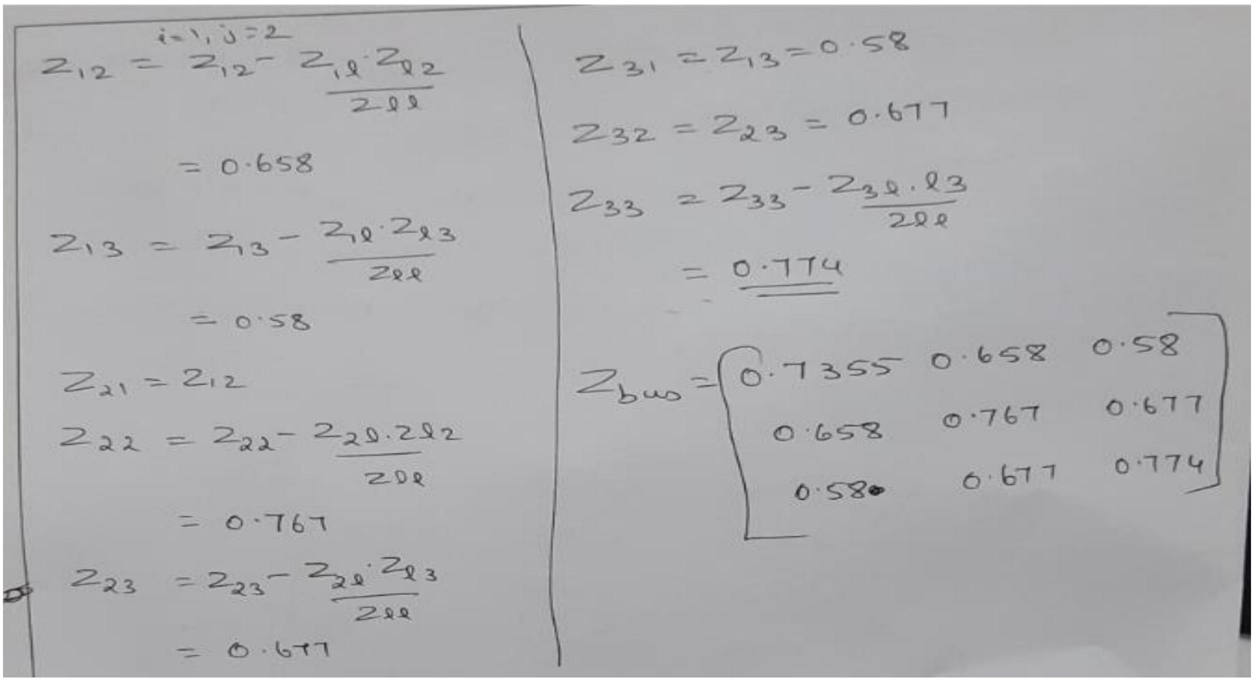
~~$Z_{11} = Z_{11} - \frac{Z_{i\ell} Z_{\ell j}}{Z_{\ell\ell}}$~~

$i = 1, j = 1$

$$Z_{11} = Z_{11} - \frac{Z_{i\ell} Z_{\ell j}}{Z_{\ell\ell}}$$

$$Z_{11} = Z_{11} - \frac{Z_{1\ell} \cdot Z_{\ell 1}}{Z_{\ell\ell}}$$

$= 0.7355$



$i=1, j=2$

$Z_{12} = Z_{12} - \frac{Z_{1l} \cdot Z_{l2}}{Z_{ll}}$

$= 0.658$

$Z_{13} = Z_{13} - \frac{Z_{1l} \cdot Z_{l3}}{Z_{ll}}$

$= 0.58$

$Z_{21} = Z_{12}$

$Z_{22} = Z_{22} - \frac{Z_{2l} \cdot Z_{l2}}{Z_{ll}}$

$= 0.767$

$Z_{23} = Z_{23} - \frac{Z_{2l} \cdot Z_{l3}}{Z_{ll}}$

$= 0.677$

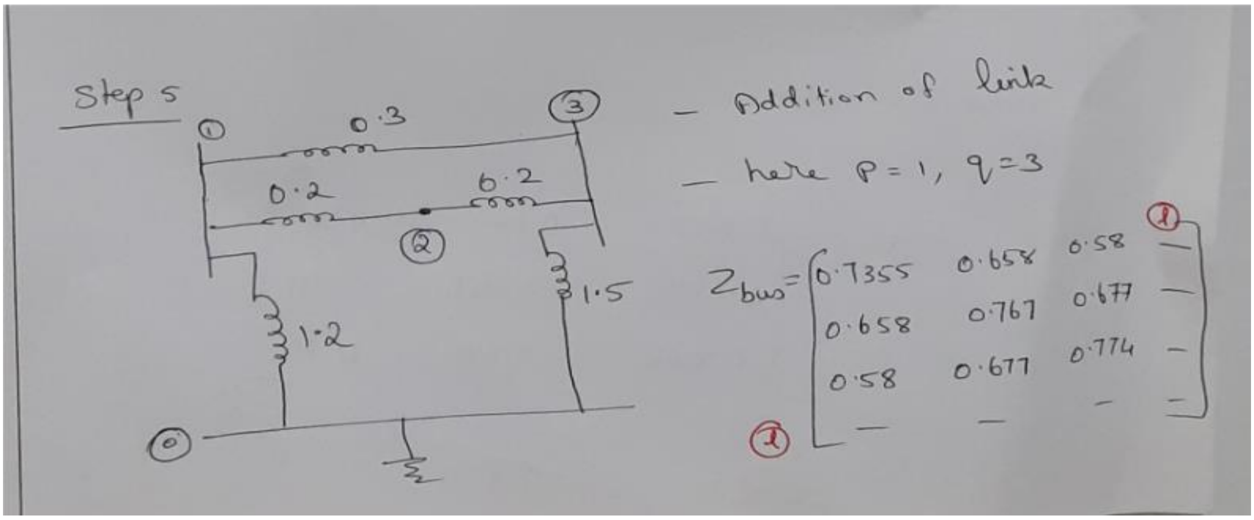
$Z_{31} = Z_{13} = 0.58$

$Z_{32} = Z_{23} = 0.677$

$Z_{33} = Z_{33} - \frac{Z_{3l} \cdot l_3}{Z_{ll}}$

$= \underline{\underline{0.774}}$

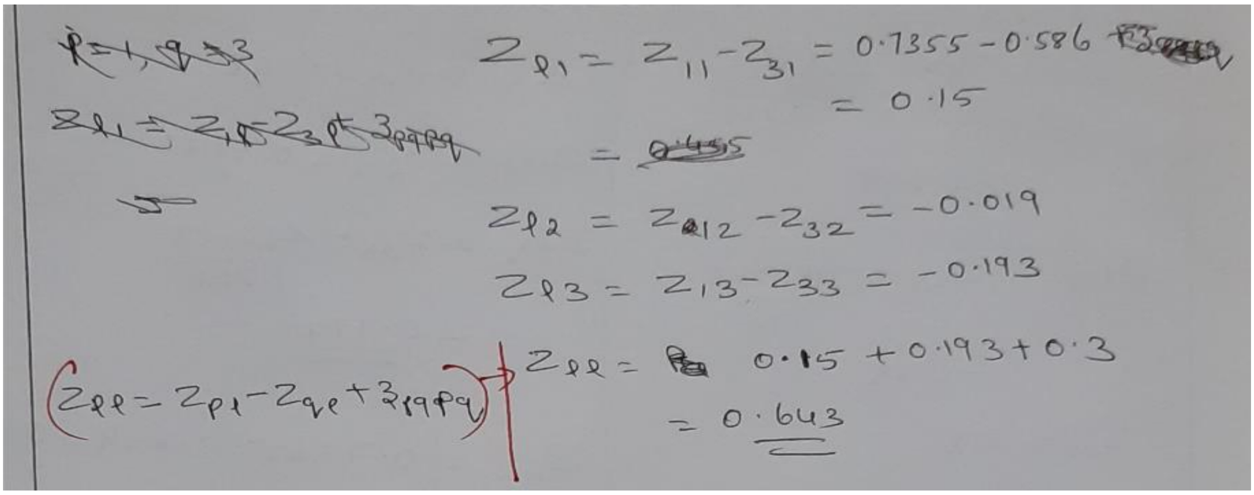
$$Z_{bus} = \begin{bmatrix} 0.7355 & 0.658 & 0.58 \\ 0.658 & 0.767 & 0.677 \\ 0.58 & 0.677 & 0.774 \end{bmatrix}$$



Step 5

- Addition of link
- here $p = 1,\ q = 3$

$$Z_{bus} = \begin{array}{c|cccc} & & & & ① \\ \hline & 0.7355 & 0.658 & 0.58 & - \\ & 0.658 & 0.767 & 0.677 & - \\ & 0.58 & 0.677 & 0.774 & - \\ ① & - & - & - & - \end{array}$$



~~$p=1, q=3$~~

~~$Z_{l1} = Z_{1p} - Z_{3p} + Z_{pqpq}$~~

$Z_{l1} = Z_{11} - Z_{31} = 0.7355 - 0.586$

$= 0.15$

~~$= 0.455$~~

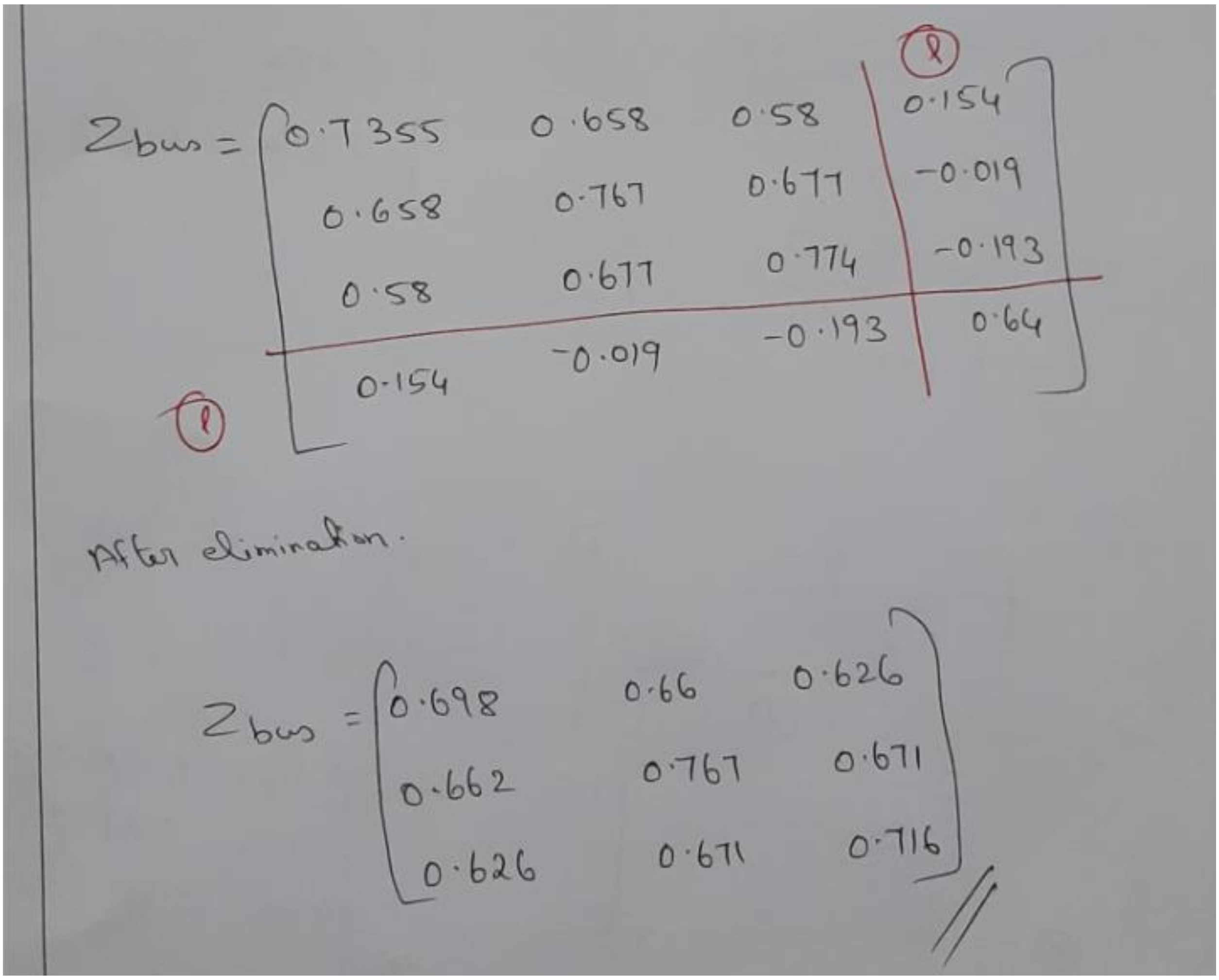
$Z_{l2} = Z_{12} - Z_{32} = -0.019$

$Z_{l3} = Z_{13} - Z_{33} = -0.193$

$(Z_{ll} = Z_{pl} - Z_{ql} + Z_{pqpq}) \rightarrow$

$Z_{ll} = 0.15 + 0.193 + 0.3$

$= \underline{\underline{0.643}}$



$$Z_{bus} = \begin{array}{cc} & \begin{array}{cccc} & & & \textcircled{l} \end{array} \\ \textcircled{l} & \left[\begin{array}{ccc|c} 0.7355 & 0.658 & 0.58 & 0.154 \\ 0.658 & 0.767 & 0.677 & -0.019 \\ 0.58 & 0.677 & 0.774 & -0.193 \\ \hline 0.154 & -0.019 & -0.193 & 0.64 \end{array}\right] \end{array}$$

After elimination.

$$Z_{bus} = \begin{bmatrix} 0.698 & 0.66 & 0.626 \\ 0.662 & 0.767 & 0.671 \\ 0.626 & 0.671 & 0.716 \end{bmatrix}$$

6)

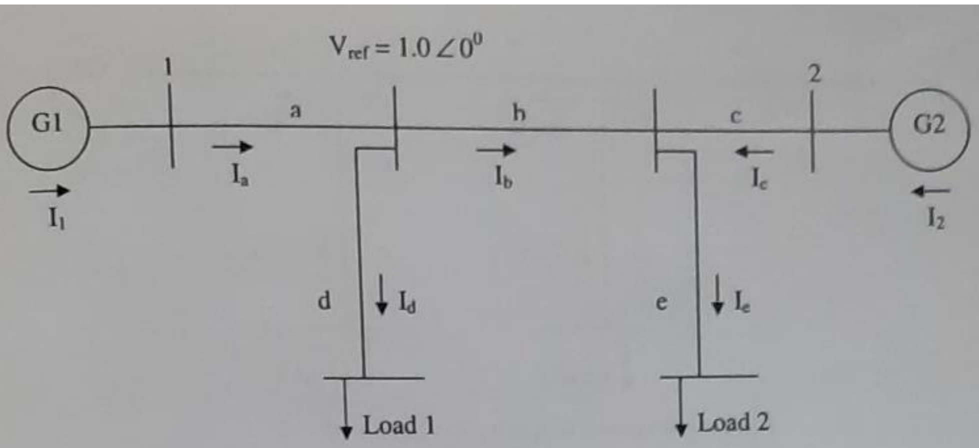


Fig : Example 8

Solution:
Total load current

$I_L = I_d + I_e = 2.0 - j\,0.5 = 2.061\ \angle\text{-}14.03^0 A$

$I_{L1} = I_d = 0.8 - j\,0.2 = 0.8246\ \angle\text{-}14.03^0\ A$

$\frac{I_{L1}}{I_L} = 0.4; \qquad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$

If generator 1, supplies the load then $I_1 = I_L$. The current distribution is shown in Fig a.

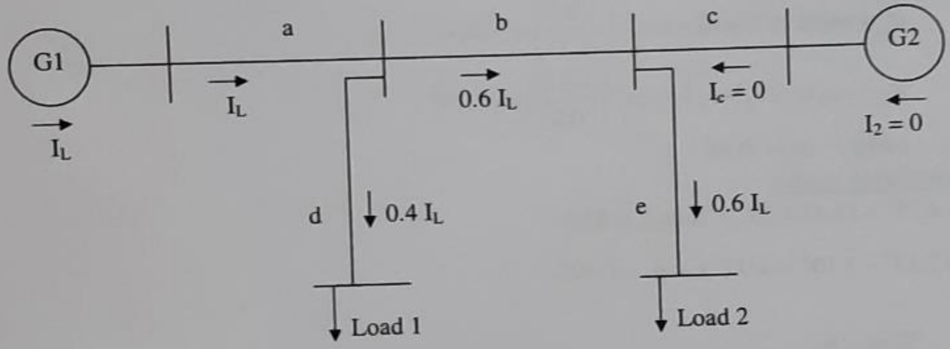


Fig a : Generator 1 supplying the total load

$N_{a1} = \frac{I_a}{I_L} = 1.0;\ N_{b1} = \frac{I_b}{I_L} = 0.6;\ N_{C1} = 0;\ N_{d1} = 0.4;\ N_{e1} = 0.6.$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

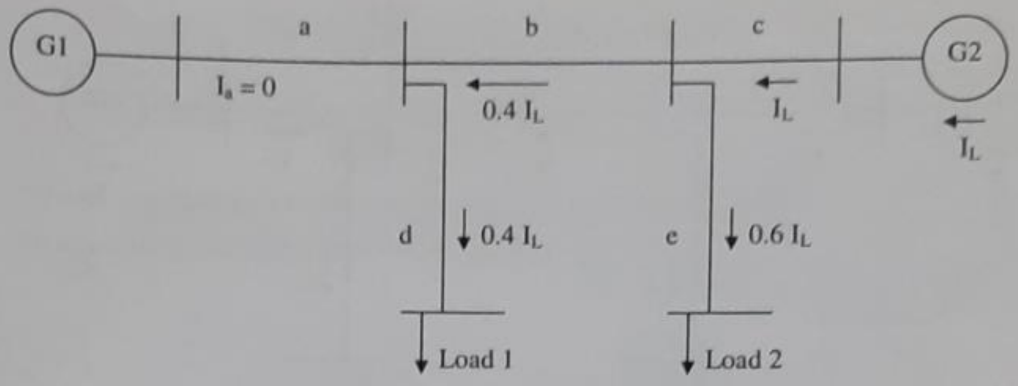


Fig b: Generator 2 supplying the total load

$N_{a2} = 0$; $N_{b2} = -0.4$; $N_{c2} = 1.0$; $N_{d2} = 0.4$; $N_{e2} = 0.6$

From Fig 8.10, $V_1 = V_{ref} + Z_a I_a$

$$= 1\angle 0^0 + (1.2 - j\,0.4)\,(0.02 + j0.08)$$

$$= 1.06\angle 4.78^0 = 1.056 + j\,0.088 \text{ pu.}$$

$V_2 = V_{ref} - I_b\,Z_b + I_c\,Z_c$

$$= 1.0\ \angle 0^0 - (0.4 - j\,0.2)\,(0.08 + j\,0.32) + (0.8 - j\,0.1)\,(0.02 + j\,0.08)$$

$$= 0.928 - j\,0.05 = 0.93\ \angle\text{-}3.10^0 \text{ pu.}$$

Current Phase angles

$$\sigma_1 = \text{angle of } I_1(=I_a) = \tan^{-1}\left(\frac{-0.4}{1.2}\right) = -18.43^0$$

$$\sigma_2 = \text{angle of } I_2(=I_c) = \tan^{-1}\left(\frac{-0.1}{0.8}\right) = -7.13^0$$

$$\cos(\sigma_1 - \sigma_2) = 0.98$$

Power factor angles

$$\phi_1 = 4.78^0 + 18.43 = 23.21^0; \cos\phi_1 = 0.92$$

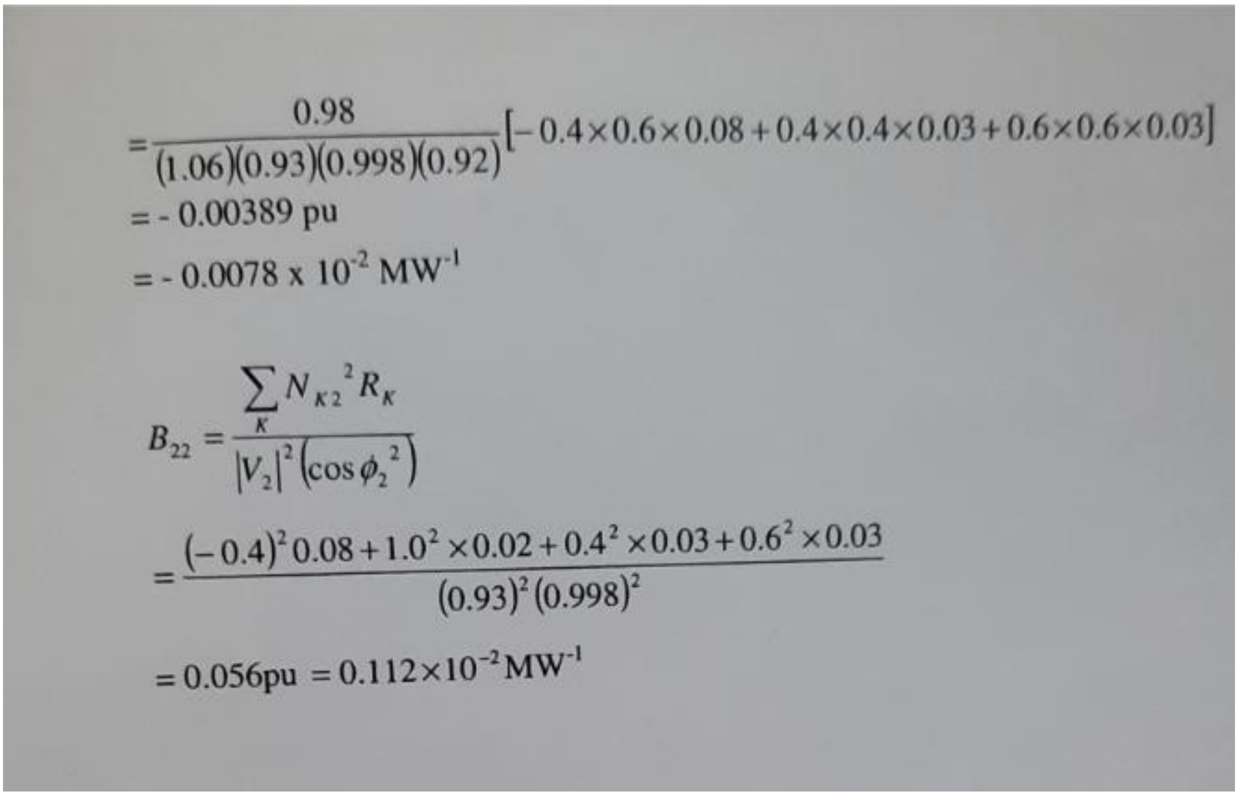
$$\phi_2 = 7.13^0 - 3.10^0 = 4.03^0; \cos\phi_2 = 0.998$$

$$B_{11} = \frac{\sum_K N_{K1}{}^2 R_K}{|V_1|^2 (\cos\phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2 (0.920)^2}$$

$$= 0.0677 \text{ pu}$$

$$= 0.0677 \text{ x } \frac{1}{50} = 0.1354 \times 10^{-2}\,\text{MW}^{-1}$$

$$B_{12} = \frac{Cos(\sigma_1 - \sigma_2)}{|V_1||V_2|(\cos\phi_1)(\cos\phi_2)} \sum_K N_{K1} N_{K2} R_K$$



$$= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)}[-0.4\times0.6\times0.08+0.4\times0.4\times0.03+0.6\times0.6\times0.03]$$

$= - 0.00389$ pu

$= - 0.0078 \times 10^{-2}$ MW^{-1}

$$B_{22} = \frac{\sum_K N_{K2}{}^2 R_K}{|V_2|^2 \left(\cos\phi_2{}^2\right)}$$

$$= \frac{(-0.4)^2 0.08+1.0^2\times0.02+0.4^2\times0.03+0.6^2\times0.03}{(0.93)^2(0.998)^2}$$

$$= 0.056\text{pu} = 0.112\times10^{-2}\text{MW}^{-1}$$