CMR
INSTITUTE OF
TECHNOLOGY

USN

Internal Assessment Test - III

Sub:	SIGNALS AND SYSTEMS							Code:	18EE54
Date:	23/01/23	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	Determine x[n] using partial fraction method for the given ROC $X(z)=(-1+5z^{-1})/(1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2})$ with ROC a. $\|z\|>1$ b. $\|z\|<\frac{1}{2}$	10	CO5	L3
2	Solve the diffrence equation.The initial conditions are y(-1)=1, y(-2)=-1 with the input $x(n)=3^n u(n)$. $y(n)-\frac{1}{4}y(n-1)-\frac{1}{8}y(n-2)=x(n)+x(n-1)$	10	CO5	L3
3	Describe the properties of Region of convergence	10	CO5	L1
4	State and prove the following continous time Fourier transform properties: a.Parsevals theorem b.Frequency shift property	10	CO4	L2
5	Find the Fourier transform of the following using appropriate properties: $x(t)=e^{-3t}\sin 2t$.	10	CO4	L3
6	Using partial expansion,determine the Inverse Fourier transform of $X(j\omega)=\frac{5j\omega+12}{(j\omega)^2+5j\omega+6}$	10	CO4	L3

INTERNAL ASSESSMENT - 3.

1. $x(z) = \dfrac{-1 + 5z^{-1}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$

On applying partial fractions,

$(1-z^{-1})(1-\frac{1}{2}z^{-1})$

$1 - \frac{1}{2}z^{-1} - z^{-1} + \frac{1}{2}z^{-2}$

$x(z) = \dfrac{-1 + 5z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}$

$x(z) = \dfrac{A}{1-z^{-1}} + \dfrac{B}{1-\frac{1}{2}z^{-1}}$

$-1 + 5z^{-1} = A(1 - \frac{1}{2}z^{-1}) + B(1 - z^{-1})$

Let $z^{-1} = 2$

$-1 + 5(2) = B(1-2)$

$-1 + 10 = -2B$

$9 = -B$

$\boxed{B = -9}$

Let $z^{-1} = 1$

$-1 + 5(1) = A(1 - \frac{1}{2})$

$4 = \frac{1}{2}A$

$\boxed{A = 8}$

$\therefore x(z) = 8\left(\dfrac{1}{1-z^{-1}}\right) - 9\left(\dfrac{1}{1-\frac{1}{2}z^{-1}}\right)$

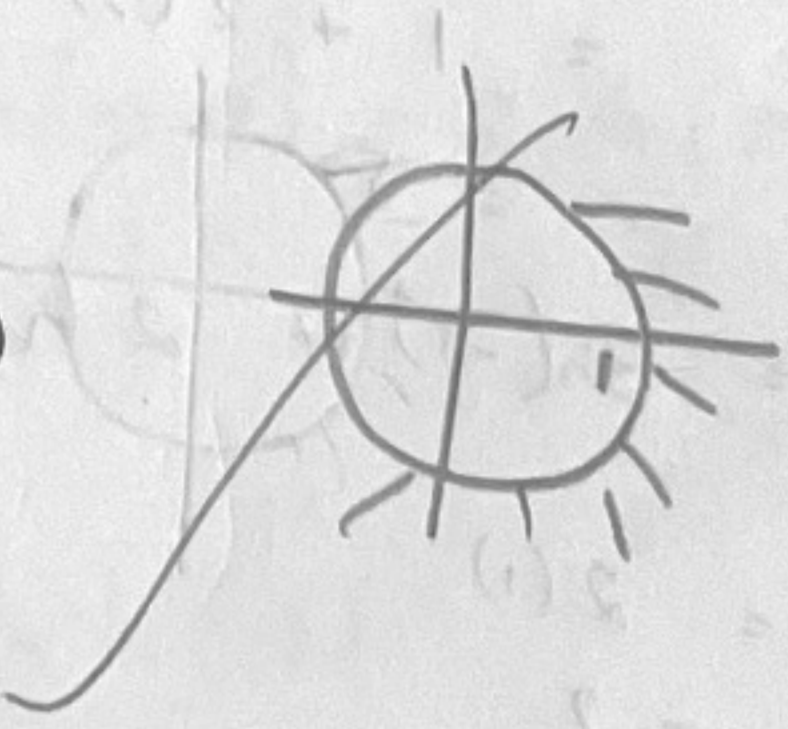
Poles $\Rightarrow 1$ and $\frac{1}{2}$

ROC > Pole
$\Downarrow$
Right sided
$\Downarrow$
$a^n u(n)$

(i) ROC $\Rightarrow |z| > 1$

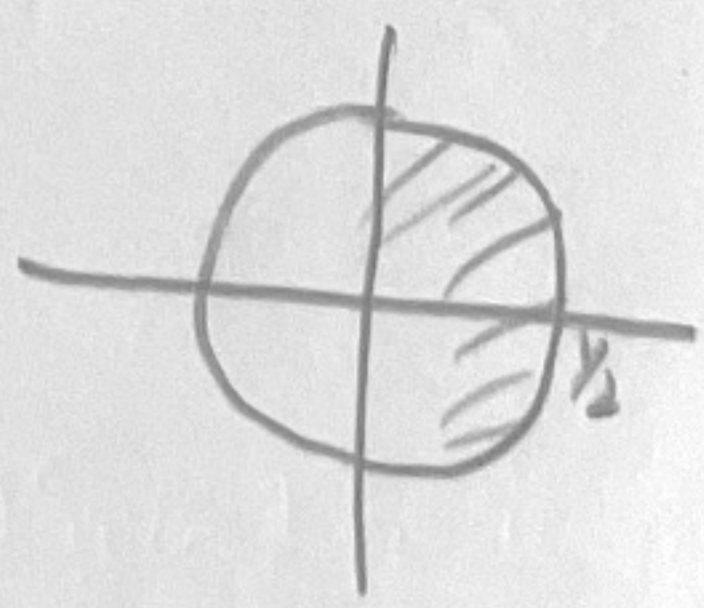
$\cancel{x(n) = -8(1)^n u(-n-1) + 9(\frac{1}{2})^n u(-n-1)}$

$x(n) = 8(1)^n u(n) - 9(\frac{1}{2})^n u(n)$



(ii) ROC $\Rightarrow |z| < \frac{1}{2}$

$x(n) = -8(1)^n u(-n-1) + 9(\frac{1}{2})^n u(-n-1)$



$y(-1) = 1$, $y(-2) = -1$

$x(n) = 3^n u(n)$

$y(n) - \frac{1}{4} y(n-1) - \frac{1}{8} y(n-2) = x(n) + x(n-1)$

On applying z transform,

$Y[z] - \frac{1}{4}\left[y(-1) + z^{-1}Y(z)\right] - \frac{1}{8}\left[y(-2) + z^{-1}y(-1) + z^{-2}Y(z)\right] =$

$X(z) + \left[x(-1) + z^{-1}X(z)\right]$

On substituting initial conditions,

$Y[z] - \frac{1}{4}\left[1 + z^{-1}Y[z]\right] - \frac{1}{8}\left[+1 + z^{-1}(-1) + z^{-2}Y[z]\right] =$

$X[z] + \left[x(-1) + z^{-1}X(z)\right]$

$x(n) = 3^n u(n)$

$u(n) \leftrightarrow \frac{1}{1-z^{-1}}$

Also $\alpha^n x(n) \leftrightarrow X(z/\alpha)$

$\therefore X(z) = \frac{1}{1-(z/3)^{-1}} = \frac{1}{1-\frac{3}{z}} = \boxed{\frac{1}{1-3z^{-1}}} = X(z)$

$x(-1) = 3^{-1}u(-1)$

$X(z) = \sum_{n=-\infty}^{\infty} 1/3\, u(-1)$

$u(-1) = 0$

$\therefore \boxed{x(-1) = 0}$

$Y[z]\left[1 - \frac{z^{-1}}{4} + \frac{z^{-2}}{8}\right] - \frac{1}{4} + \frac{1}{8} - \frac{1}{8}z^{-1} = \frac{1}{1-3z^{-1}} + \left[z^{-1}\left(\frac{1}{1-3z^{-1}}\right)\right]$

$Y[z]\left[1 - \frac{1}{4}z^{-1} + \frac{1}{8}z^{-2}\right] - \frac{1}{8} - \frac{1}{8}z^{-1} = \frac{1}{1-3z^{-1}} + \frac{z^{-1}}{1-3z^{-1}}$

$Y[z]\left[1 - \frac{1}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = \frac{1+z^{-1}}{1-3z^{-1}} + \frac{1}{8}\left[1+z^{-1}\right]$

$$Y[z]\left[1-\frac{1}{4}z^{-1}+\frac{1}{8}z^{-2}\right] = \frac{8(1+z^{-1})+(1-3z^{-1})(1+z^{-1})}{8(1-3z^{-1})}$$

$$Y[z] = \frac{8+8z^{-1}+1+z^{-1}-3z^{-1}+3z^{-2}}{(8-24z^{-1})(1-\frac{1}{4}z^{-1}+\frac{1}{8}z^{-2})}$$

$$Y[z] = \frac{9+6z^{-1}+3z^{-2}}{(8-24z^{-1})(1-2z^{-1})(1-4z^{-1})}$$

$$Y[z] = \frac{A}{(8-24z^{-1})} + \frac{B}{(1-2z^{-1})} + \frac{C}{(1-4z^{-1})}$$

$$9+6z^{-1}+3z^{-2} = A(1-2z^{-1})(1-4z^{-1}) \cancel{+8B(1-3z^{-1})+C(}$$
$$+\,8B(1-3z^{-1})(1-4z^{-1}) + 8C(1-3z^{-1})(1-2z^{-1})$$

Let $z^{-1} = \frac{1}{2}$

$$9+6\left(\frac{1}{2}\right)+3\left(\frac{1}{4}\right) = 8B\left(1-\frac{3}{2}\right)(1-2)$$

$$9+3+\frac{3}{4} = 8B\left(-\frac{1}{2}\right)(-1)$$

$$\frac{51}{4} = 4B$$

$$\boxed{B = \frac{51}{16}}$$

$z^{-1} = \frac{1}{4}$

$$9+6\left(\frac{1}{4}\right)+3\left(\frac{1}{16}\right) = 8C\left(1-\frac{3}{4}\right)\left(1-\frac{2}{4}\right)$$

$$9+\frac{3}{2}+\frac{3}{16} = 8C\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$$

$$11 = 3C$$

$$\boxed{C = \frac{11}{3}}$$

Let $z^{-1} = \frac{1}{3}$

$$9+6\left(\frac{1}{3}\right)+3\left(\frac{1}{3}\right) = A\left(1-\frac{2}{3}\right)\left(1-\frac{4}{3}\right)$$

$$9+2+1 = A\left(\frac{1}{3}\right)\left(-\frac{1}{3}\right)$$

$$12 = -\frac{A}{9}$$

$$\boxed{A = -108}$$

$$y(n) = -\frac{108}{8}(3)^n u(n) + \frac{51}{16}(2)^n u(n) + \frac{11}{3}(4)^n u(n) \quad //$$

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3. Properties of ROC.

- Region on convergence is forms a ring in the z plane with centre at origin
- Region of convergence does not include any poles
- If $x(n)$ is a finite value, then the region of convergence can extend from 0 to ∞ but except 0 and ∞.
- If $x(n)$ is a right sided sequence, then consider a ROC with $|z| = r_0$, then all the points greater than r_0 $|z| > r_0$ lie in the ROC i.e., they lie outside the circle.

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- If $x(n)$ is a left sided sequence, then let $|z| = r_0$, then all the point lesser than r_0, i.e., $|z| < r_0$ lie in the ROC i.e., inside the circle.

- If $x(n)$ is a two sided sequence, then the ROC is formed between the concentric circles of the circles formed by the poles at radius

4.(a) Parseval's theorem.

Parseval's theorem states that

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt \xleftrightarrow{FT} \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega$$

Proof.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \, e^{-j\omega t} \, dt$$

Also, $|x(t)|^2 = x(t) \cdot x^*(t)$

∴ Here,

$$= \int_{-\infty}^{\infty} x(t) \cdot x^*(t) \, e^{-j\omega t} \, dt.$$

Also, $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$

$$\Rightarrow x^*(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(j\omega)\, e^{-j\omega t}\, d\omega$$

$$\therefore\ x(j\omega) = \int_{-\infty}^{\infty} x(t)\, \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(j\omega)\, e^{-j\omega t}\, d\omega\, dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \int_{-\infty}^{\infty} X^*(j\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} (X(j\omega)) . (X^* j\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega$$

$= RHS$ //.

Frequency shift property.

$$e^{j\beta t} x(t) \longleftrightarrow X(j(\omega - \beta))$$

Proof

w.k.t.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Here $\Rightarrow x(t) \longrightarrow e^{j\beta t} x(t)$

$$\therefore LHS = \int_{-\infty}^{\infty} x(t) e^{j\beta t} . e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j(\omega - \beta)t}\, dt$$

On comparing with original eq^n

$$= X(j(\omega - \beta))$$

$= RHS$ //

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6. $$x(j\omega) = \frac{5j\omega + 12}{(j\omega)^2 + 5j\omega + 6}$$

$$= \frac{5j\omega + 12}{(j\omega+2)(j\omega+3)}$$

$$x(j\omega) = \frac{A}{j\omega+2} + \frac{B}{j\omega+3}$$

$$5j\omega + 12 = A(j\omega+3) + B(j\omega+2)$$

Let $j\omega = -3$

$$5(-3) + 12 = B(-3+2)$$

$$-15 + 12 = -B$$

$$-3 = -B$$

$$\boxed{B = 3}$$

Let $j\omega = -2$

$$5(-2) + 12 = A(-2+3)$$

$$-10 + 12 = A$$

$$\boxed{A = 2}$$

$$\therefore x(j\omega) = 2\left(\frac{1}{j\omega+2}\right) + 3\left(\frac{1}{j\omega+3}\right).$$

w.k.t $e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$

$$\Rightarrow \boxed{x(n) = 2e^{-2t}u(t) + 3e^{-3t}u(t)}$$