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Internal Assesment Test-I

Sub:	Electromagnetic Waves							Code:	18EC55
Date:	07/11 /2022	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	ECE(A,B,C,D)

Answer any **FIVE FULL** Questions

		Marks	OBE CO	OBE RBT
1.(a)	Transform the following vectors to spherical coordinate system at the following points given: i) 10 $\mathbf{a_x}$ at P(3,2,4) ii) 10 $\mathbf{a_y}$ at Q(5,30°,4).	[05]	CO1	L3

Soln

(a) $r = \sqrt{x^2+y^2+z^2} = \sqrt{9+4+16} = \sqrt{29}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{-3}\right) = -33.69°$

$\therefore \phi = 180° - 33°69° =$

$\cancel{\phi = \tan}$

$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) = \cos^{-1}\left(\frac{4}{\sqrt{29}}\right) = 4$

Let, $\vec{G} = 10\,\hat{a}_x$.

$\therefore G_r = \vec{G}\cdot\hat{a}_r = 10\,\hat{a}_x\cdot\hat{a}_r = 10.\sin\theta\cos\phi$

$= 10\sin(41.6°)\cos(1$

$= 10 \times 0.663 \times (-.83$

$= -5.5$.

$G_\theta = \vec{G}\cdot\hat{a}_\theta = 10\,\hat{a}_x\cdot\hat{a}_\theta$

$= 10.\cos\theta\cos\phi = 10\cos(41.6°)\,co$

$= 10 \times 0.747 \times (-.832) = -6$

Now, $\rho = 5,\ \phi = 30^\circ,\ z = 4$

$\therefore\ x = \rho\cos\phi = 5\cos 30^\circ = 4\cdot33$

$y = \rho\sin\phi = 5\sin 30^\circ = 2\cdot5$

$z = 4$

$\therefore\ r = \sqrt{x^2+y^2+z^2} = 6\cdot40$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2\cdot5}{4\cdot33}\right)$

$= 30^\circ.$

$\theta = \cos^{-1}\left(\frac{z}{\sqrt{(x^2+y^2+z^2)}}\right)$

$= \cos^{-1}\left(\frac{4}{6\cdot4}\right) = 51\cdot31^\circ$

$\therefore\ G_r = 10\sin(51\cdot31^\circ)\sin(30^\circ) = 3\cdot90$

$G_\theta = 10\cos(51\cdot31^\circ)\sin(30^\circ) = 3\cdot12$

$G_\phi = 10\cos(30^\circ) = 8\cdot66$

$\therefore\ \vec{G} = 3\cdot90\,\hat{a}_r + 3$ ... $+$

(b) Transform the vector B = y $\mathbf{a_x}$ – x $\mathbf{a_y}$ + z $\mathbf{a_z}$ into cylindrical coordinates. [05] CO1 L3

$B_\rho = \vec{B}\cdot\hat{a}_\rho = y(\hat{a}_x\cdot\hat{a}_\rho) - x(\hat{a}_y\cdot\hat{a}_\rho)$

$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi$

$= 0$

$B_\phi = \vec{B}\cdot\hat{a}_\phi = y(\hat{a}_x\cdot\hat{a}_\phi) - x(\hat{a}_y\cdot\hat{a}_\phi)$

$= -y\sin\phi - x\cos\phi$

$= -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$

Thus, $\vec{B} = -\rho\,\hat{a}_\phi + z\,\hat{a}_z.$

2.(a) Transform the vector field G=$(\frac{xz}{y})$$\mathbf{a_x}$ into spherical components and variables. [06] CO1 L3

$$
\begin{aligned}
G_r &= \mathbf{G}\cdot\mathbf{a}_r = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_r = \frac{xz}{y}\sin\theta\cos\phi \\
&= r\sin\theta\cos\theta\frac{\cos^2\phi}{\sin\phi} \\
G_\theta &= \mathbf{G}\cdot\mathbf{a}_\theta = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_\theta = \frac{xz}{y}\cos\theta\cos\phi \\
&= r\cos^2\theta\frac{\cos^2\phi}{\sin\phi} \\
G\phi &= \mathbf{G}\cdot\mathbf{a}_\phi = \frac{xz}{y}\mathbf{a}_x\cdot\mathbf{a}_\phi = \frac{xz}{y}(-\sin\phi) \\
&= -r\cos\theta\cos\phi
\end{aligned}
$$

Collecting these results, we have

$$\mathbf{G} = r\cos\theta\cos\phi\,(\sin\theta\cot\phi\,\mathbf{a}_r + \cos\theta\cot\phi\,\mathbf{a}_\theta - \mathbf{a}_\phi)$$

(b) Define electric flux density. Derive the relation between electric flux density and electric field intensity. [04] CO2 L2

$$\mathbf{D}\Big|_{r=a} = \frac{Q}{4\pi a^2}\mathbf{a}_r \qquad \text{(inner sphere)}$$

$$\mathbf{D}\Big|_{r=b} = \frac{Q}{4\pi b^2}\mathbf{a}_r \qquad \text{(outer sphere)}$$

and at a radial distance r, where $a \le r \le b$,

$$\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r$$

If we now let the inner sphere become smaller and smaller, while still retaining a charge of Q, it becomes a point charge in the limit, but the electric flux density at a point r meters from the point charge is still given by

$$\boxed{\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r} \tag{1}$$

for Q lines of flux are symmetrically directed outward from the point and pass through an imaginary spherical surface of area $4\pi r^2$.

This result should be compared with Section 2.2, Eq. (9), the radial electric field intensity of a point charge in free space,

$$\boxed{\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\mathbf{a}_r}$$

In free space, therefore,

$$\boxed{\mathbf{D} = \epsilon_0\mathbf{E}} \qquad \text{(free space only)}$$

3.(a) State and explain Coulomb's law in vector form. Mention the units of each term involved. CO1 L1

[04]

Coulomb stated that the force between two very small objects separated in a vacuum or free space by a distance, which is large compared to their size, is proportional to the charge on each and inversely proportional to the square of the distance between them, or

$$F = k\frac{Q_1 Q_2}{R^2}$$

where Q_1 and Q_2 are the positive or negative quantities of charge, R is the separation, and k is a proportionality constant. If the International System of Units[1] (SI) is used, Q is measured in coulombs (C), R is in meters (m), and the force should be newtons (N). This will be achieved if the constant of proportionality k is written as

$$k = \frac{1}{4\pi\epsilon_0}$$

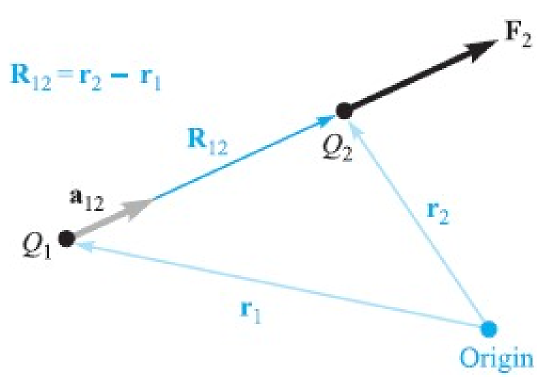
The new constant ϵ_0 is called the *permittivity of free space* and has magnitude, measured in farads per meter (F/m),

$$\boxed{\epsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi}10^{-9} \text{ F/m}} \tag{1}$$

The quantity ϵ_0 is not dimensionless, for Coulomb's law shows that it has the label $C^2/N \cdot m^2$. We will later define the farad and show that it has the dimensions $C^2/N \cdot m$; we have anticipated this definition by using the unit F/m in equation (1).

Coulomb's law is now

$$\boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}} \tag{2}$$



$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}$$

where $\mathbf{a}_{12}$ = a unit vector in the direction of R_{12}, or

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

(b) Two point charges of magnitudes 2mc and -7mc are located at places $P_1(4,7,-5)$ and $P_2(-3,2,-9)$ respectively in free space, evaluate the vector force on charge at P_2. [06] CO1 L3

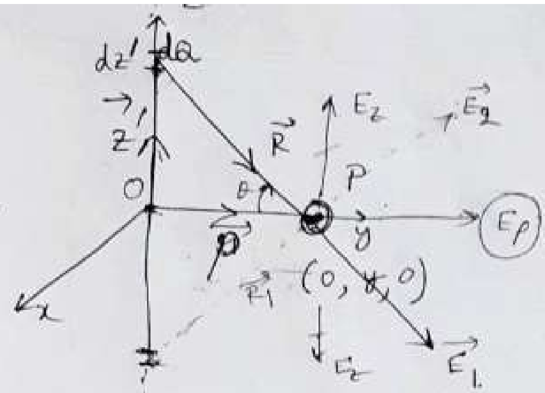
$$\vec{F}_2 = \frac{2\times10^{-3} \times (-7) \times (10^{-3})\,(-7\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)}{4\pi \times 8.854\times10^{-12} \times 90 \times \sqrt{90}}$$

$$= \frac{-14\times10^{6}\,(-7\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)}{4\pi \times 8.854 \times 90 \times 9.48}$$

$$= -147.55\,(-7\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)$$

$$= +1032.8\,\hat{a}_x + 737.75\,\hat{a}_y + 590.2\,\hat{a}_z \text{ Newton}$$

4. State and explain electric field intensity and obtain an expression for electric field intensity due to an infinitely long line charge. [10] CO1 L2



$$\frac{z'}{\rho} = \tan\theta$$
$$z' = \rho \tan\theta$$

ρ_L C/m

Find $\vec{E}$ at P (0, y, 0) because of the line charge along z-axis.

We consider incremental charge

$dQ = \rho_L \, dz'$ . . . (1) [charge on length dz']

$\vec{R} = \vec{\rho} - \vec{z}'$ $\quad (\vec{z}' + \vec{R} = \vec{\rho})$

$\vec{R} = (\rho \hat{a}_\rho - z' \hat{a}_z)$ . . (2) $|\vec{R}| = \sqrt{\rho^2 + z'^2}$

∴ The field at P, because of dQ,

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 |\vec{R}|^2} \hat{a}_R = \frac{\rho_L dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)} \cdot \frac{(\rho \hat{a}_\rho - z' \hat{a}_z)}{(\rho^2 + z'^2)^{1/2}}$$

$[\because |\vec{R}| = \sqrt{\rho^2 + z'^2}]$

$$= \frac{\rho_L dz' . (\rho \hat{a}_\rho - z' \hat{a}_z)}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

We know from the symmetry of the problem - (3)

$$\vec{dE} = \frac{\rho_L dz' \rho \hat{a}_\rho}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

∴ The field at P,

$$\vec{E} = \int d\vec{E} = \int_{z'\to-\infty}^{\infty} \frac{\rho_L\, dz'\, \rho\, \hat{a}_\rho}{4\pi\epsilon_0(\rho^2+z'^2)^{3/2}}\, \hat{a}_\rho$$

$$= \frac{\rho_L\, \rho}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz'}{(\rho^2+z'^2)^{3/2}}\, \hat{a}_\rho$$

note: $\tan\theta = \frac{z'}{\rho}$

$z' = \rho\tan\theta$ | $z' \to \infty,\ \theta = \pi/2$

$dz' = \rho\sec^2\theta\, d\theta$ | $z' \to -\infty,\ \theta = -\pi/2$

$$\therefore \vec{E} = \frac{\rho_L\, \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho\sec^2\theta\, d\theta}{(\rho^2+\rho^2\tan^2\theta)^{3/2}}\, \hat{a}_\rho$$

$$= \frac{\rho_L\, \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho\sec^2\theta\, d\theta}{(\rho^2)^{3/2}(1+\tan^2\theta)^{3/2}}\, \hat{a}_\rho$$

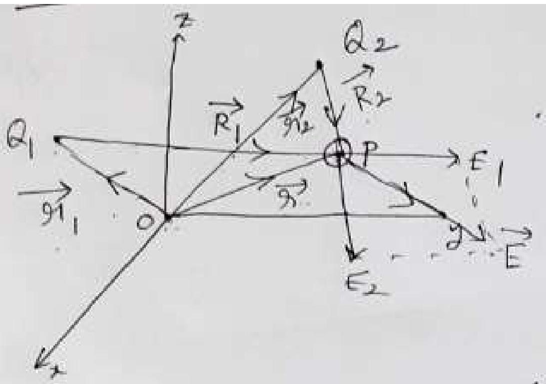
$$= \frac{\rho_L\, \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho\sec^2\theta\, d\theta}{\rho^3(\sec^2\theta)^{3/2}}\, \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta\, d\theta}{\sec^3\theta}\, \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta}\, d\theta\, \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta\, \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} [1+1]\, \hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0\rho}\, \hat{a}_\rho$$

5.(a) Derive the expression for the electric field intensity at a point due to n number of point charges. [04] CO1 L2



$$\therefore \boxed{\vec{E} = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\vec{r}-\vec{r}_m|^2}\,\hat{a}_m \ \text{V/m}}$$

Intensity $\vec{E_1}$ at P due to charge Q_1,

$\vec{E_1} = \dfrac{Q_1}{4\pi\epsilon_0|\vec{R_1}|^2}\,\hat{a}_1$ where, $\hat{a}_1 \rightarrow$ unit vector along $\vec{R_1}$.

$\vec{r_1} + \vec{R_1} = \vec{r}$ or $\vec{R_1} = (\vec{r} - \vec{r_1})$

$$\therefore \vec{E_1} = \frac{Q_1}{4\pi\epsilon_0|\vec{r}-\vec{r_1}|^2}\cdot\frac{\vec{r}-\vec{r_1}}{|\vec{r}-\vec{r_1}|} = \frac{Q_1}{4\pi\epsilon_0|\vec{r}-\vec{r_1}|^2}\cdot\hat{a}_1 \text{ V/m}$$

Now, from ~~ad~~ diagram, $\vec{r_2} + \vec{R_2} = \vec{r}$

or $\vec{R_2} = \vec{r} - \vec{r_2}$

$\therefore$ Field at P due to charge Q_2

$$\vec{E_2} = \frac{Q_2}{4\pi\epsilon_0|\vec{r}-\vec{r_2}|^2}\cdot\hat{a}_2 \text{ V/m}$$

Similarly for the n-th charge, $\vec{E_n} = \dfrac{Q_n}{4\pi\epsilon_0|\vec{r}-\vec{r_n}|^2}\,\hat{a}_n$

$\therefore$ The total field $\vec{E}$,

$$\vec{E} = \vec{E_1} + \vec{E_2} + \dots \vec{E_n} = \frac{Q_1}{4\pi\epsilon_0|\vec{r}-\vec{r_1}|^2}\,\hat{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\vec{r}-\vec{r_2}|^2}\,\hat{a}_2 + \dots \frac{Q_n}{4\pi\epsilon_0|\vec{r}-\vec{r_n}|^2}\,\hat{a}_n$$

(b) Find **D** and **E** at origin due to a point charge 12nC at (2, 0, 6) and a uniform line charge 3nC/m placed along x-axis. [06] CO2 L3

5.(b) 12 nC at $(2,0,6)$, $\vec{R}$ to $(0,0,0)$

$\vec{R} = -2\hat{a}_x - 6\hat{a}_z$

$|\vec{R}| = \sqrt{4+36} = \sqrt{40}$

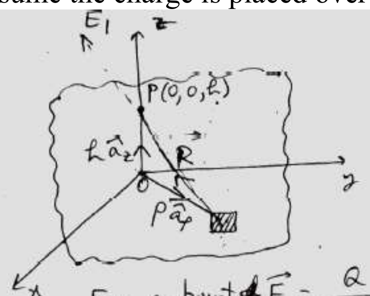
$$\therefore \vec{E} = \frac{12 \times 10^{-9}}{4\pi\epsilon_0\, 40} \cdot \frac{(-2\hat{a}_x - 6\hat{a}_z)}{\sqrt{40}}$$

$$= \frac{12 \times 10^{3}\,(-2\hat{a}_x - 6\hat{a}_z)}{4\pi \times 8.854 \times 40 \times 6.32} = 4.26 \times 10^{-4} \times 10^{3} = 0.426$$

$$\text{or } \vec{E} = 0.426\left(-2\hat{a}_x - 6\hat{a}_z\right) = -0.852\,\hat{a}_x - 2.556\,\hat{a}_z \text{ V/m}$$

$$\vec{D} = \epsilon_0\vec{E} = 3.77\,(-2\hat{a}_x - 6\hat{a}_z) \times 10^{-12} = (-7.54\,\hat{a}_x - 22.62\,\hat{a}_z) \text{ pC/m}^2$$

6. Define surface charge density. Obtain an expression of electric field intensity due to an infinite sheet of charge with uniform surface charge distribution ρ_s C/m^2. Assume the charge is placed over x-y plane. [10] CO1 L2

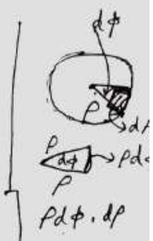


Surface charge density,

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s} \quad \text{C/m}^2$$

For a point charge, $\vec{E} = \dfrac{Q}{4\pi\epsilon_0 |\vec{R}|^2}\,\hat{a}_R$

1) $dQ = ??$
2) $\vec{R} = ??$ $\longrightarrow$ $\vec{\rho} + \vec{R} = h\hat{a}_z$, $\underline{\vec{R} = h\hat{a}_z - \rho\hat{a}_\rho}$
3) $\hat{a}_R = ??$
4) ~~Varied~~ Symmetry of problem
5) Integration.



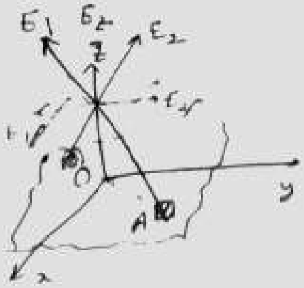
uniform charge density ρ_s C/m^2

$$dQ = \rho_s \cdot |d\vec{A}| = \rho_s \cdot (\rho\, d\rho\, d\phi)$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{(h\hat{a}_z - \rho\hat{a}_\rho)}{\sqrt{h^2+\rho^2}}$$

$\therefore$ The intensity at P due to the chosen infinitesimal surface area,

$$\vec{dE} = \frac{\rho_s \cdot (\rho\, d\rho\, d\phi)}{4\pi\epsilon_0\,(h^2+\rho^2)} \, \frac{(h\hat{a}_z - \rho\hat{a}_\rho)}{\sqrt{h^2+\rho^2}} \cdots$$



From symmetry of the problem,

$$\vec{dE} = \frac{dQ}{4\pi\epsilon_0(\rho^2+h^2)} \cdot \frac{h\,\hat{a}_z}{\sqrt{\rho^2+h^2}}$$

$$\therefore \vec{dE} = \frac{\rho_s\,(\rho\, d\rho\, d\phi)\,(h\,\hat{a}_z)}{4\pi\epsilon_0(\rho^2+h^2)^{3/2}} \quad \cdots (4)$$

$\therefore$ The total electric field intensity at P,

$$\vec{E} = \iint \vec{dE} = \frac{\rho_s\, h}{4\pi\epsilon_0} \int_{\rho=0}^{\infty}\int_{\phi=0}^{2\pi} \frac{\rho\, d\rho\, d\phi}{(\rho^2+h^2)^{3/2}}\,\hat{a}_z$$

$$= \frac{\rho_s\, h}{4\pi\epsilon_0} \int_{\rho=0}^{\infty} \frac{\rho\, d\rho}{(\rho^2+h^2)^{3/2}} \int_{\phi=0}^{2\pi} d\phi \;\; \hat{a}_z$$

$$= \frac{\rho_s\, h}{2\epsilon_0} \int_{\rho=0}^{\infty} \frac{\rho\, d\rho}{(\rho^2+h^2)^{3/2}} \;\; \hat{a}_z$$

Let, $\rho = h\tan\theta$ | when $\rho = 0$, $\theta = 0$

$d\rho = h\sec^2\theta\, d\theta$ | when $\rho \to \infty$, $\theta = \pi/2$

$$\vec{E} = \frac{\rho_s\, h}{2\epsilon_0} \int_0^{\pi/2} \frac{(h\tan\theta)\cdot h\sec^2\theta\, d\theta}{(h^2 + h^2\tan^2\theta)^{3/2}} \;\; \hat{a}_z$$

$$= \frac{\rho_s\, h}{2\epsilon_0} \int_0^{\pi/2} \frac{h^2\tan\theta\sec^2\theta\, d\theta}{h^3 \cdot \sec^2\theta\, \sec\theta} \;\; \hat{a}_z$$

$$= \frac{\rho_l}{2\epsilon_0} \int_0^{\pi/2} \tan\theta \cdot \frac{1}{\sec\theta}\, d\theta\, \hat{a}_z$$

$$= \frac{\rho_l}{2\epsilon_0} \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta} \cdot \cos\theta\, d\theta\, \hat{a}_z$$

$$= \frac{\rho_l}{2\epsilon_0} \int_0^{\pi/2} \sin\theta\, d\theta\, \hat{a}_z$$

$$= \frac{\rho_l}{2\epsilon_0} \left[-\cos\theta\right]_0^{\pi/2} \hat{a}_z = \frac{\rho_l}{2\epsilon_0}\, \hat{a}_z$$

i.e. $\boxed{\vec{E} = \frac{\rho_l}{2\epsilon_0}\, \hat{a}_z}$ V/m.

7.(a) Define line charge density and volume charge density. [03] CO1 L2

Volume charge density: $\boxed{\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}}$

Line charge density: Charge per unit length as the length tends to zero. Unit is C/m.

7.(b) Find **E** at P(1, 1, 1) caused by four identical 3 nC (nanocoulomb) point charges located at $P_1(1, 1, 0)$, $P_2(-1, 1, 0)$, $P_3(-1, -1, 0)$ and $P_4(1, -1, 0)$. [07] CO1 L3

$$\mathbf{E} = 26.96 \left[\frac{\mathbf{a}_z}{1}\frac{1}{1^2} + \frac{2\mathbf{a}_x + \mathbf{a}_z}{\sqrt{5}}\frac{1}{(\sqrt{5})^2} + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{3}\frac{1}{3^2} + \frac{2\mathbf{a}_y + \mathbf{a}_z}{\sqrt{5}}\frac{1}{(\sqrt{5})^2} \right]$$

or

$$\mathbf{E} = 6.82\mathbf{a}_x + 6.82\mathbf{a}_y + 32.8\mathbf{a}_z \text{ V/m}$$