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INTERNAL ASSESSMENT TEST – I

Sub:	DIGITAL SIGNAL PROCESSING							Code:	18EC52
Date:	04 / 11 / 2022	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE

Answer any 5 full questions

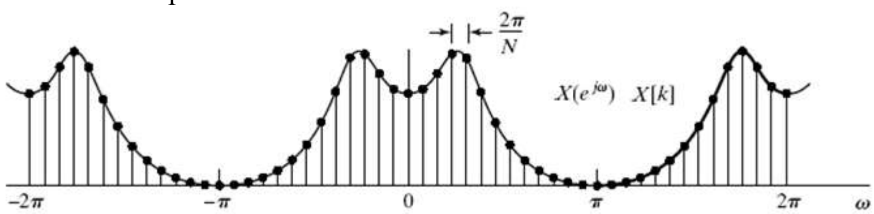
		Marks	CO	RBT
1	Why is it necessary to perform frequency domain sampling? With a neat diagram, show that sampling of DTFT results in N-point DFT.	[10]	CO2	L3
2	Compute the 8-point DFT of the sequence $x[n] = [4,3,2,1]$. Plot the magnitude spectrum and the phase spectrum.	[10]	CO2	L3
3(a)	Compute the 4-point DFT of $x[n] = [8,4,2,6]$ using matrix method. Plot the magnitude spectrum and the phase spectrum.	[06]	CO2	L2
3(b)	Compute the IDFT of $X[k] = [7.8, -1.65 + j0.9526, -1.65 - j0.9526]$ using matrix method.	[04]	CO2	L2
4	The first 4 samples of 6-point DFT of a real 6-point sequence are as follows. $$X[k] = [21, -3 + j5.1962, -3 + j1.7321, -3]$$ Determine the remaining samples of $X[k]$. Evaluate the following without explicitly determining $x[n]$. i) $x[0]$ ii) $x[3]$ iii) $\sum_{n=0}^{5} x[n]$ iv) $\sum_{n=0}^{5} (-1)^n x[n]$	[10]	CO2	L3
5	With proof, explain the nature of DFT for the following cases. i) $x(n)$ is real and circularly even ii) $x(n)$ is real and circularly odd iii) x(n) is imaginary and circularly even iv) x(n) is imaginary and circularly odd	[10]	CO2	L2
6	Compute the N-point DFT of the following sequences i. $x_1(n) = e^{j\frac{2\pi}{N}ln}, 0 \leq n \leq N-1$ ii. $x_2(n) = \cos\left(\frac{2\pi}{N}ln\right), 0 \leq n \leq N-1$ iii. $x_3(n) = \sin\left(\frac{2\pi}{N}ln\right), 0 \leq n \leq N-1$	[10]	CO2	L2
7	Prove that if x(n) is real, then, prove that i)X(0) is real (ii) X(k)=X*(N-k) (iii) X(N/2) is real for n even.	[10]	CO2	L2

Solution.

Q1. Frequency analysis of discrete-time signals is usually and most conveniently performed on a digital signal processor which may be a general-purpose digital computer or specially designed digital hardware. To perform frequency analysis on a discrete-time signal *x*(n) we convert that time-domain sequence into an equivalent frequency-domain representation. We know that such a representation is given by the Fourier transform *X(ω)* of the sequence *x*(n). However *X(ω)* is a continuous function of frequency and therefore it is not a computationally convenient representation of the sequence *x*(n)**.** So we consider the representation oi a sequence *x*(n) by samples of its spectrum *X(ω).*

The FT of DT aperiodic signal is represented by $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$

- Suppose that we sample X(ω) periodically in frequency domain at a spacing of δω between two successive samples .



- Since X(ω) is periodic with period 2π , only samples in period 0 to 2π are necessary. Replacing ω by $\omega_k = \frac{2\pi k}{N}$
- $X(\omega_k) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi k}{N}n}$
- $X\left(\frac{2\pi k}{N}\right) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\frac{2\pi k}{N}n}$
- The summation can be divided in to infinite number of summations with N terms in each summation
- $X\left(\frac{2\pi k}{N}\right) = \cdots\ldots + \sum_{n=-N}^{-1} x(n)\, e^{-j\frac{2\pi k}{N}n} + \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k}{N}n} + \sum_{n=N}^{2N-1} x(n)\, e^{-j\frac{2\pi k}{N}n}$+..........
- $X\left(\frac{2\pi k}{N}\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n)\, e^{-j\frac{2\pi k}{N}n}$ Replace *n* by *n-lN*
- $X\left(\frac{2\pi k}{N}\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi k}{N}(n-lN)}$
- $X\left(\frac{2\pi k}{N}\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN)\, e^{-j\frac{2\pi k}{N}n} \quad \because e^{j\frac{2\pi k}{N}lN} = 1$
- Interchanging the order of summation
- $\because X\left(\frac{2\pi k}{N}\right) = X(k) = \sum_{n=0}^{N-1} \sum_{l=-\infty}^{\infty} x(n-lN)\, e^{-j\frac{2\pi k}{N}n}$
- Let us define $x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$
- $\therefore X(k) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi k}{N}n}$
- Where $x_p(n)$ is periodic extension of *x*(n) for every N samples for finite duration length sequence *x*(n) with length L<N.
- Then $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi k}{N}n}$ computes the DFT of sequence *x*(n). The expression obtained by sampling the X(ω) is called as discrete Fourier transform (DFT)
- $x_p(n)$ can be expanded using FS as $x_p(n) = \sum_{k=0}^{N-1} c_k\, e^{j\frac{2\pi k}{N}n}$
- With FS coefficients $c_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi k}{N}n} = \frac{1}{N} X\left(\frac{2\pi k}{N}\right)$
- Now substituting for c_k in expression of $x_p(n)$. $x_p(n) = \sum_{k=0}^{N-1} \frac{1}{N} X\left(\frac{2\pi k}{N}\right) e^{j\frac{2\pi k}{N}n}$
- $x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi k}{N}\right) e^{j\frac{2\pi k}{N}n}$ Since $x_p(n)$ is periodic extension of *x*(n). *x*(n) can be recovered from $x_p(n)$ obtained by above expression if there is no aliasing (L<N) in the time domain.
- Where $x(n)=x_p(n)$ for $0 \le n \le N-1$
- $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi k}{N}n}$ This equation computes IDFT of *X*(k).

Q2. Given x(n)=[4,3,2,1], Note to compute 8 point DFT four zeros are padded at the end of given the sequence so x(n)=[4,3,2,1,0,0,0,0]

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \text{ where } W_N = e^{-j\frac{2\pi}{N}}$$

For k=0, $X(k) = \sum_{n=0}^{8-1} x(n)\, W_8^0 = \sum_{n=0}^{7} x(n) = x(0)+x(1)+x(2)+x(3)$ =4+3+2+1 =10 as samples values from x(4) to x(7) are 0. $X(0) = 10$.

For k=1, $X(1) = \sum_{n=0}^{7} x(n)\, W_8^n = x(0)\, W_8^0 + x(1)\, W_8^1 + x(2)\, W_8^2 + x(3)\, W_8^3$ samples values from x(4) to x(7) are 0

$$W_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}},\ W_8^2 = -j,\ W_8^3 = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}},\ W_8^4 = -1,\ W_8^5 = W_8^{*3}, W_8^6 = W_8^{*2}, W_8^7 = W_8^{*1}$$

$\therefore X(1) = 4+3\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)+2(-j)+1\left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = 5.5414 - j\,4.828$

For k=2, $X(2) = \sum_{n=0}^{7} x(n)\, W_8^{2n} = x(0)\, W_8^0 + x(1)\, W_8^2 + x(2)\, W_8^4 + x(3)\, W_8^6$ samples values from x(4) to x(7) are 0

$\therefore X(2) = 4+3(-j)+2(-1)+1(j) = 2 - j2$

For k=3, $X(3) = \sum_{n=0}^{7} x(n)\, W_8^{3n} = x(0)\, W_8^0 + x(1)\, W_8^3 + x(2)\, W_8^6 + x(3)\, W_8^9$ samples values from x(4) to x(7) are 0, $W_8^9 = W_8^1$

$\therefore X(3) = 4+3\left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)+2(j)+1\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) = 2.5858 - j0.8284$

For k=4, $X(4) = \sum_{n=0}^{7} x(n)\, W_8^{4n} = x(0)\, W_8^0 + x(1)\, W_8^4 + x(2)\, W_8^8 + x(3)\, W_8^{12}$ samples values from x(4) to x(7) are 0, $W_8^8 = W_8^0$, $W_8^{12} = W_8^4$

$\therefore X(4) = 4+3(-1)+2(1)+1(-1) = 2$

For k=5, $X(5) = \sum_{n=0}^{7} x(n)\, W_8^{5n} = x(0)\, W_8^0 + x(1)\, W_8^5 + x(2)\, W_8^{10} + x(3)\, W_8^{15}$ samples values from x(4) to x(7) are 0, $W_8^{10} = W_8^2$, $W_8^{15} = W_8^7$

$\therefore X(5) = 4+3\left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)+2(j)+1\left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = 2.5858 + j0.8284$

For k=6, $X(2) = \sum_{n=0}^{7} x(n)\, W_8^{6n} = x(0)\, W_8^0 + x(1)\, W_8^6 + x(2)\, W_8^{12} + x(3)\, W_8^{18}$ samples values from x(4) to x(7) are 0, $W_8^{12} = W_8^4$, $W_8^{18} = W_8^2$

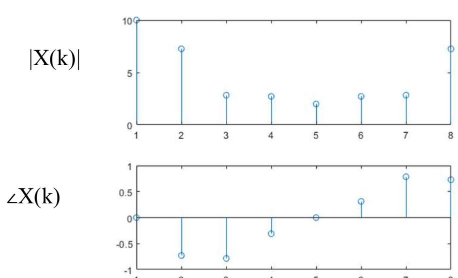
$\therefore X(2) = 4+3(j)+2(-1)+1(-j) = 2 + j2$

For k=7, $X(7) = \sum_{n=0}^{7} x(n)\, W_8^{7n} = x(0)\, W_8^0 + x(1)\, W_8^7 + x(2)\, W_8^{14} + x(3)\, W_8^{21}$ samples values from x(4) to x(7) are 0, $W_8^{14} = W_8^6$, $W_8^{21} = W_8^5$

$\therefore X(7) = 4+3\left(\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right)+2(j)+1\left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) = 5.5414 + j\,4.828$

$X(k)$ = {10∠0 , 7.2545∠-0.7283, 2.8284∠-0.7854 , 2.7153∠0.3100, 2∠0,2.7153∠0.312.8284∠0.7854, 7.2545∠0.7283}

Angles are in radians.



Q3a. To compute the 4-point DFT of $x[n] = [8,4,2,6]$ using matrix method $X = W_{4\times4}x$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 8 \\ 4 \\ 2 \\ 6 \end{bmatrix}$$ Row of W in to column of x.

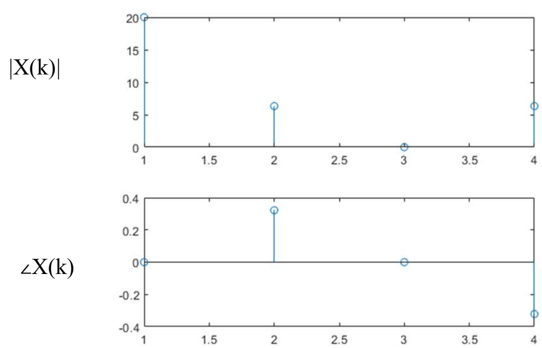
X[0]=8+4+2+6 =20

X[1]=8+4($-j$)+2(-1)+6(j) =6+2j

X[2]=8+4(-1)+2+6 (-1)=0

X[3]=8+4(j)+2(-1)+6($-j$) =6-2j

$X(k) = \{20\angle 0\ , 6.3245\angle 0.321, 0\angle 0\ , 6.3245\angle -0.321\}$ Angles are in radians.



Q3.b. To compute the 4-point IDFT of $X[k] = [7.8, -1.65 + j0.9526, -1.65 - j0.9526]$ using matrix

method. $x = \frac{1}{3}W^*{}_{3\times3}X$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^3 \end{bmatrix} \quad W *_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{*1} & W_3^{*2} \\ 1 & W_3^{*2} & W_3^{*3} \end{bmatrix}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 + j0.866 & -0.5 - j0.866 \\ 1 & -0.5 - j0.866 & -0.5 + j0.866 \end{bmatrix} \begin{bmatrix} 7.8 \\ -1.65 + j0.9526 \\ -1.65 - j0.9526 \end{bmatrix}$$

$\therefore x[0] = \frac{1}{3}\{7.8 - 1.65 + j0.9526 - 1.65 - j0.9526\}$ =1.5

$\therefore x[1] = \frac{1}{3}\{7.8 + (-0.5 + j0.866) \cdot (-1.65 + j0.9526) + (-0.5 - j0.866) \cdot (-1.65 - j0.9526)\}$ =2.6

$\therefore x[2] = \frac{1}{3}\{7.8 + (-0.5 - j0.866) \cdot (-1.65 + j0.9526) + (-0.5 + j0.866) \cdot (-1.65 - j0.9526)\}$ =3.7

Q.4. Given the first 4 samples of 6-point DFT of a real 6-point sequence

$X[k] = [21, -3 + j5.1962, -3 + j1.7321, -3]$ the remaining samples are computed by X[k]=X*[6-k]

X[4]=X*[6-4]=X*[2]= $-3 - j1.7321$ and X[5]=X*[6-5]=X*[1]= $-3 - j5.1962$.

i) To find x(0) let us use IDFT equation $x(n) = \frac{1}{6}\sum_{k=0}^{5} X(k)\, e^{j\frac{2\pi k}{6}n}$; In this equation if we substitute n=0 we get $x(0) = \frac{1}{6}\sum_{k=0}^{5} X(k)\, e^{0}$;$= \frac{1}{6}$ {X[0]+X[1]+X[2]+X[3]+X[4]+X[5]}

$\therefore x(0) = \frac{1}{6}\{21 - 3 + j5.1962\ - 3 + j1.7321\ - 3 - 3 - j1.7321 - 3 - j5.1962\ \}=1$

ii) Now in IDFT equation if we substitute n=3 we get $x(3) = \frac{1}{6}\sum_{k=0}^{5} X(k)\, e^{j\frac{2\pi k}{6}3}$

$\therefore x(3) = \frac{1}{6}\sum_{k=0}^{5} X(k)\,(-1)^{k}$

$\therefore x(3)= \frac{1}{6}$ {X[0]-X[1]+X[2]-X[3]+X[4]-X[5]}

$\therefore x(3) = \frac{1}{6}\{21 + 3 - j5.1962\ - 3 + j1.7321 + 3 - 3 - j1.7321 + 3 + j1.7321\}=\frac{24}{6}\ =4$

iii) To find $\sum_{n=0}^{5} x[n]$ let use DFT equation $X(k) = \sum_{n=0}^{5} x(n)\, e^{-j\frac{2\pi k}{6}n}$ and substitute k=0 in that

$\therefore X[0] = \sum_{n=0}^{5} x[n] = 21$

iv) To find $\sum_{n=0}^{5}(-1)^{n}\, x[n]$ substitute k=3 in DFT equation

$\therefore X[3] = \sum_{n=0}^{5}(-1)^{n} x[n] = -3$

Q5. We now that DFT $x(n) \overset{DFT}{\longleftrightarrow} X[k]$ and $x\langle(-n)\rangle_N \overset{DFT}{\longleftrightarrow} X^{*}[k]$

$X[k] = DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n) cos\left(\frac{2\pi}{N}\right) - j\sum_{n=0}^{N-1} x(n) sin\left(\frac{2\pi nk}{N}\right)$ ……………………(1)

$X^{*}[k] = DFT\{x\langle(-n)\rangle_N\} = \sum_{n=0}^{N-1} x(n) cos\left(\frac{2\pi nk}{N}\right) + j\sum_{n=0}^{N-1} x(n) sin\left(\frac{2\pi nk}{N}\right)$…………………..(2)

i) If x(n) is real and even then $DFT\{x_e(n)\} = \frac{1}{2}\{DFT\{x(n)\} + DFT\{x\langle(-n)\rangle_N\}\}$

From equation (1) and (2) $DFT\{x_e(n)\} = \frac{1}{2} 2\sum_{n=0}^{N-1} x(n) cos\left(\frac{2\pi nk}{N}\right)$=$\sum_{n=0}^{N-1} x(n) cos\left(\frac{2\pi nk}{N}\right)$

ii) If x(n) is real and odd then $DFT\{x_0(n)\} = \frac{1}{2}\{DFT\{x(n)\} - DFT\{x\langle(-n)\rangle_N\}\}$

From equation (1) and (2) $DFT\{x_0(n)\} = \frac{1}{2}(-2j)\sum_{n=0}^{N-1} x(n) sin\left(\frac{2\pi nk}{N}\right)$=$-j\sum_{n=0}^{N-1} x(n) sin\left(\frac{2\pi nk}{N}\right)$

iii) If x(n) is purely imaginary and even then $DFT\{x_e(n)\} = \frac{1}{2}\{DFT\{x(n)\} + DFT\{x\langle(-n)\rangle_N\}\}$

From equation (1) and (2) $DFT\{x_e(n)\} = \frac{1}{2} 2\sum_{n=0}^{N-1} jx(n) cos\left(\frac{2\pi nk}{N}\right)$=$\sum_{n=0}^{N-1} jx(n) cos\left(\frac{2\pi nk}{N}\right)$

iv) If x(n) is purely imaginary and odd then $DFT\{x_0(n)\} = \frac{1}{2}\{DFT\{x(n)\} - DFT\{x\langle(-n)\rangle_N\}\}$

From equation (1) and (2) $DFT\{x_0(n)\} = \frac{1}{2}(-2j)\sum_{n=0}^{N-1} jx(n) sin\left(\frac{2\pi nk}{N}\right)$=$\sum_{n=0}^{N-1} x(n) sin\left(\frac{2\pi nk}{N}\right)$

Q.6 By definition of DFT we have $X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$ where $W_N = e^{-j\frac{2\pi}{N}}$

i) $X_1(k) = \sum_{n=0}^{N-1} e^{j\frac{2\pi ln}{N}}\, e^{-j\frac{2\pi nk}{N}}$=$\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(l-k)}$ now expressing its closed form of equation

$\therefore X_1(k) = \frac{1-e^{j\frac{2\pi N}{N}(l-k)}}{1-e^{j\frac{2\pi}{N}(l-k)}} = 0$ for $l \neq k$ and for $l = k$ $\therefore X_1(k)$=$\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(0)}$=N

$\therefore X_1(k)$=$N\delta(l-k)$

ii) $X_2(k) = \frac{1}{2}\sum_{n=0}^{N-1}\left\{e^{j\frac{2\pi ln}{N}} + e^{-j\frac{2\pi ln}{N}}\right\} e^{-j\frac{2\pi nk}{N}}$=$\frac{1}{2}\left\{\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(i-k)} + \sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(i+k)}\right\}$ now expressing its closed form of equation

$\therefore \sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(i-k)} = \frac{1-e^{j\frac{2\pi N}{N}(l-k)}}{1-e^{j\frac{2\pi}{N}(l-k)}} = 0$ for $l \neq k$ and for $l = k$ $\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(i-k)}$=$\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(0)}$=N i.e. $N\delta(l-k)$

Similarly $\sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(i+k)} == \frac{1-e^{j\frac{2\pi N}{N}(l+k)}}{1-e^{j\frac{2\pi}{N}(l+k)}} = 0$ for $l \neq -k$ and for $l = -k$ $\sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(0)}$=N i.e. $N\delta(l+k)$

$\therefore X_2(k) = \frac{N}{2}\{\delta(l-k) + \delta(l+k)\}$

ii) $X_3(k) = \frac{1}{2j}\sum_{n=0}^{N-1}\left\{e^{j\frac{2\pi ln}{N}} - e^{-j\frac{2\pi ln}{N}}\right\} e^{-j\frac{2\pi nk}{N}} = \frac{1}{2j}\left\{\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(i-k)} - \sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(i+k)}\right\}$ now expressing its closed form of equation

$\therefore \sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(i-k)} = \frac{1-e^{j\frac{2\pi N}{N}(l-k)}}{1-e^{j\frac{2\pi}{N}(l-k)}} = 0$ for $l \neq k$ and for $l = k$ $\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(i-k)} = \sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(0)}$=N i.e. $N\delta(l-k)$

Similarly $\sum_{n=0}^{N-1} e^{-j\frac{2\pi n}{N}(i+k)} == \frac{1-e^{j\frac{2\pi N}{N}(l+k)}}{1-e^{j\frac{2\pi}{N}(l+k)}} = 0$ for $l \neq -k$ and for $l = -k$ $\sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(0)}$=N i.e. $N\delta(l+k)$

$\therefore X_3(k) = \frac{N}{2j}\{\delta(l-k) - \delta(l+k)\}$

Q.7. By definition of DFT we have $X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$ where $W_N = e^{-j\frac{2\pi}{N}}$

i) If we substitute k=0 in this equation we get $X[0] = \sum_{n=0}^{5} x[n]\, W_N^{0} = \sum_{n=0}^{5} x[n]$.

If x(n) is real then X[0] is real and is $\sum_{n=0}^{5} x[n]$

ii) For X[N-k] DFt equation shall be written as $X(N-k) = \sum_{n=0}^{N-1} x(n)\, W_N^{n(N-k)}$

$X(N-k) = \sum_{n=0}^{N-1} x(n) W_N^{-nk}\, W_N^{nN} = \sum_{n=0}^{N-1} x(n) W_N^{-nk}$ Now taking conjugate of this equation

$X^*(N-k) = \left(\sum_{n=0}^{N-1} x(n) W_N^{-nk}\right)^*$ as x*(n)= x(n) $\therefore X^*(N-k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$

$X^*(N-k) = X[k]$

iii) If N is even and if we substitute k=N/2 in DFT equation then we have $X(\frac{N}{2}) = \sum_{n=0}^{N-1} x(n)\, W_N^{n(N/2)}$

$\therefore X(\frac{N}{2}) = \sum_{n=0}^{N-1}(-1)^n x(n)$ $\because W_N^{n(N/2)} = e^{-j\frac{2\pi n N}{2N}} = e^{-j\pi} = (-1)^n$

So if x(n) is real and N is even then evaluation of $\therefore X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1}(-1)^n x(n)$ is real.