ITC - IAT-2 Solutions

1.

Symbol	probability	n_i	F_i	Binary expansion	C_i up to n_i
aa	0.25	2	0	0.00	00
ab	0.15	3	0.25	0.010	010
ba	0.15	3	0.40	0.011	011
ac	0.10	4	0.55	0.1000	1000
ca	0.1	4	0.65	0.1010	1010
bb	0.09	4	0.75	0.1100	1100
bc	0.06	5	0.84	0.11010	11010
cb	0.06	5	0.90	0.11100	11100
cc	0.04	5	0.96	0.11110	11110

Efficiency $\eta = \frac{H(S)}{H\hat{N}} \times 100$

$H(S^2)$: 2.9709 bits/sym

$H\hat{N} = \sum_{i=1}^{9} n_i p_i$ = 8.36 bits/sym

η : 88.41 %.

Redundancy: $1-\eta$: 11.59 %.

2.

s_1	7/8	0
s_2	1/8	1

$H\hat{N}$: 1 bit/sym

$H(S)$: 0.54356

efficiency : 54.356 %.

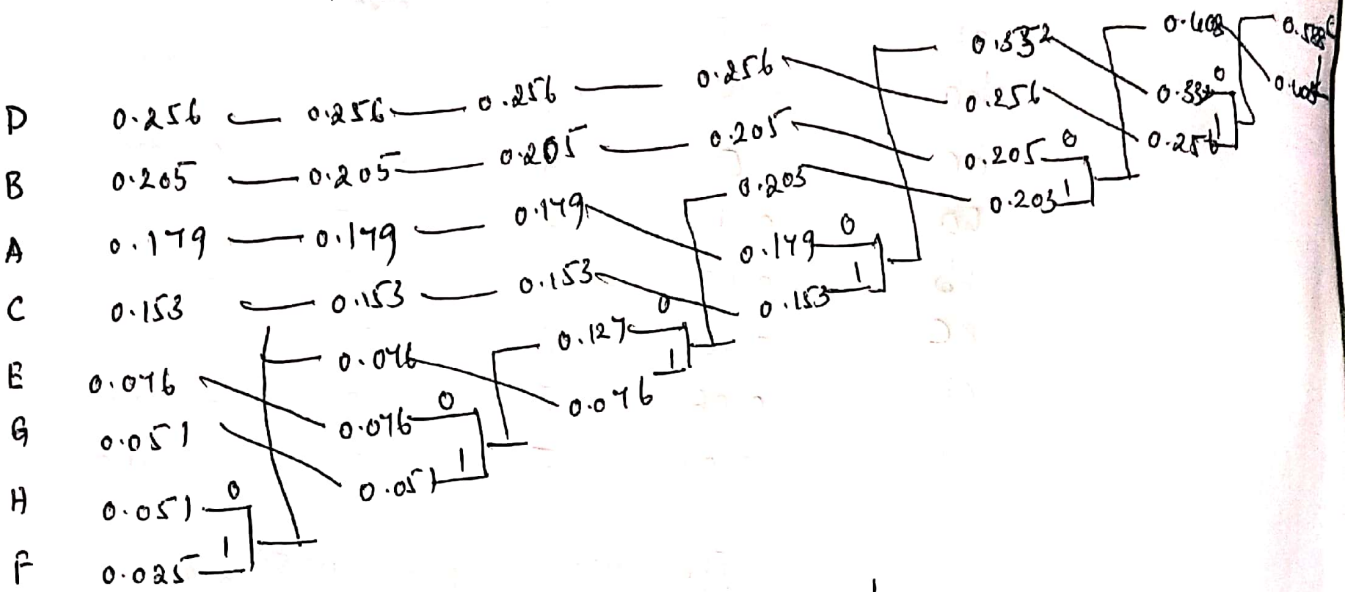
2nd extension

						n_i
$s_1 s_1$	49/64	0				1
$s_1 s_2$	7/64	1	0			2
$s_2 s_1$	7/64	1	1	0		3
$s_2 s_2$	1/64	1	1	1		3

H_N^{λ} = 1.859375 bits/sym

$H(s^2)$ = 1.08712 bits/sym

$\eta = \frac{1.08}{1.35} = 80\%$.



D — 01

B — 10

A — 000

C — 001

E — 1100

G — 1101

H — 1110

F — 1111

$H(s) = 2.468$ bits/sym

$H_N^{\lambda} = 2.698$

$e = 91.47\%$.

$R = 8.55\%$.

$$P\left(\frac{y}{\lambda}\right) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.2 & 0.5 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \end{array}$$

$H(x) = 1.5$ bits/sym

$$P(xy) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.2 & 0.15 & 0.15 \\ 0.075 & 0.05 & 0.125 \\ 0.025 & 0.1 & 0.125 \end{bmatrix} \end{array}$$

$P(y_1) = 0.3$ $P(y_2) = 0.3$ $P(y_3) = 0.4$

$H(y) = 1.57$ bits/sym

$H(xy) = 2.997$ b/sym

$$P\left(\frac{x}{y}\right) = \begin{bmatrix} \frac{2}{3} & \frac{1}{2} & 3/8 \\ \frac{1}{4} & \frac{1}{6} & \frac{5}{11} \\ \frac{1}{12} & \frac{1}{3} & \frac{5}{14} \end{bmatrix}$$

$H\left(\frac{x}{y}\right) = 1.427$

$H\left(\frac{y}{\lambda}\right) = 1.496$

$I(xy) = 0.074$

5.

0.3 — 0
0.3 — 1
0.12 — 2 0
0.12 — 2 1
0.06 — 2 2 0
0.06 — 2 2 1
0.04 — 2 2 2

$\hat{H_N}$ = 1.56

H(s) = 2.4491

$H_r(s) = \frac{H(s)}{\log_2^3} = 1.5452$

$\eta = \frac{1.5452}{1.56} = 0.9905 = 99.05\%$

6.

$$P(xy) = \begin{bmatrix} 0.05 & 0 & 0.2 & 0.05 \\ 0 & 0.1 & 0.1 & 0 \\ 0 & 0 & 0.2 & 0.1 \\ 0.05 & 0.05 & 0 & 0.1 \end{bmatrix}$$

$P(y_1) = 0.1$ $P(y_2) = 0.15$ $P(y_3) = 0.5$ $P(y_4) = 0.25$

$P(x_1) = 0.3$ $P(x_2) = 0.2$ $P(x_3) = 0.3$ $P(x_4) = 0.2$

H(x) = 1.971 bits/sym H(y) = 1.743 bits/sym

H(xy) = 3.122 bits/sym $H(\frac{x}{y}) = 1.379$ bits/sym

$H(\frac{y}{x}) = 1.151$

$$P(\frac{y}{x}) = \begin{bmatrix} \frac{1}{6} & 0 & \frac{2}{3} & \frac{1}{6} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{2} \end{bmatrix}$$

I(xy) = 0.592