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INTERNAL ASSESSMENT TEST – III

Sub:	BASIC SIGNAL PROCESSING							**Code:**	21EC33
Date:	09/02/ 2022	**Duration:**	90 mins	**Max Marks:**	50	**Sem:**	III	**Branch:**	ECE

Answer any 5 full questions

	Questions	Marks	CO	RBT
1	a) State and explain the different properties of Region of Convergence (RoC) of Z-transform. b) State the condition for LTI system to be (a) Causal (b) Stable and (c) Causal as well as Stable in terms of ROC of H(Z)	[5+5]	CO4	L1
2	Find the Z-transform and ROC of the following: a) $x[n] = \left(-\frac{1}{2}\right)^n u[-n] + 2\left(\frac{1}{4}\right)^n u[n]$ b) $x[n] = -u[-n-1] + \left(\frac{1}{2}\right)^n u[n]$	[5+5]	CO4	L3
3	Find the Z-transform of following signal using properties of the Z-transform: a) $x[n] = n\left(-\frac{1}{2}\right)^n u[n] + \delta[n-k],\ for\ k > 0$ b) $x[n] = u[n-2] * \left(\frac{2}{3}\right)^n u[n]$	[5+5]	CO4	L3
4	State and prove the following properties of the Z-transform: a) Multiplication by exponential sequence in time domain b) Convolution in time domain	[5+5]	CO4	L2
5	Use the method of partial fractions to obtain the time domain signals corresponding to the following Z-transforms: a) $X[z] = \frac{\left(\frac{1}{4}\right)z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)},\quad ROC: \frac{1}{4} < \lvert z\rvert < \frac{1}{2}$ b) $X[z] = \frac{z+1}{3z^2-4z+1},\quad ROC: \lvert z\rvert > 1$	[6+4]	CO4	L3

6 Find the transfer function and impulse response of a causal LTI system if input to the system is $x[n] = (-\frac{1}{3})^n u[n]$ and output is $y[n] = 3(-1)^n u[n] + (\frac{1}{3})^n u[n]$. [10] CO4 L3

7 Determine the impulse response corresponding to the following transfer functions if (i) System is stable and (ii) System is causal [10] CO4 L3

$$H[z] = \frac{5z^2}{z^2 - z - 6} \quad , |z| > 0$$

CCI HoD

1a

(10)

Proporties of RoC

1) RoC cannot contain any poles
2) For a finite causal seq $x[n]$ RoC is the entire z-plane except at $z=0$
3) If $x(n)$ is a left sided finite seq then RoC is entire z-plane except $z=\infty$.
4) If $x(n)$ is a finite double sided seq, then RoC is entire z plane except at $z=0$ and $z=\infty$
5) If $x(n)$ is a infinte duration right sided seq then RoC is of the form $|z|>r_+$
6) RoC of left sided infinite duration signal is $|z|<r_-$
7) RoC of two sided signal is of the form $r_+<|z|<r_-$

To Determine stability of the system using ROC

If ROC contains unit circle $|z|=1$ then the system is considered as stable system,

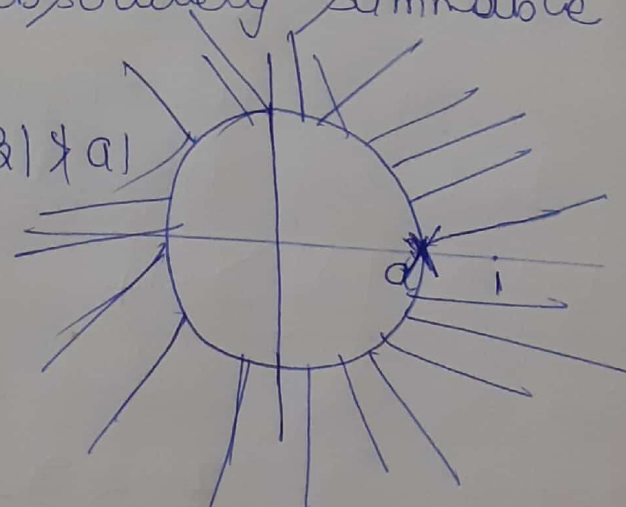
Discrete LTI system $= h(n)$

$\Rightarrow$ stable if $h(n)$ is absolutely summable.

Eg:- $h(n) = a^n u(n)$

$h(n)$ is absolutely summable if $|a|<1$

$$H(z) = \frac{z}{z-a}; \quad |z| > |a|$$



$|a| < 1$ i.e ROC contains $|z| = 1$

$\therefore$ System is stable

3) Multiplication by an exponential

If $Z\{x(n)\} = X(z)$ with ROC: $R_1 < |z| < R_2$

then $Z\{a^n x(n)\} = X(z)\Big|_{z=z/a} = X\left(\frac{z}{a}\right)$

ROC: $|a| R_1 < |z| < |a| R_2$

Proof:- $Z\{a^n x(n)\} = \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n}$

$= \sum_{n=-\infty}^{\infty} x(n) \left(\frac{z}{a}\right)^{-n}$ ROC: $R_1 < \left|\frac{z}{a}\right| < R_2$

ROC $= R_1|a| < |z| < |a| R_2$

$Z\{a^n x(n)\} = X\left(\frac{z}{a}\right)$; ROC $= R_1|a| < |z| < R_2|a|$

Hence the proof

5) Convolution in time domain

If $Z\{x_1(n)\} = X_1(z)$ ROC: $r_1^- < |z| < r_1^+$

and $Z\{x_2(n)\} = X_2(z)$ ROC: $r_2^- < |z| < r_2^+$

then $Z\{x_1(n) * x_2(n)\} = X_1(z) X_2(z)$ with

ROC: $\left(r_1^- < |z| < r_1^+\right) \cap \left(r_2^- < |z| < r_2^+\right)$

Proof: WKT $x_1(n) * x_2(n) = \sum_{k=-\infty}^{\infty} x_1(k) x_2(n-k)$

$$Z\{x_1(n) * x_2(n)\} = \sum_{n=-\infty}^{\infty} \left[x_1(n) * x_2(n)\right] z^{-n}$$

$$Z\{x_1(n) * x_2(n)\} = \sum_{n=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} x_1(k) x_2(n-k)\right] z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) \sum_{n=-\infty}^{\infty} x_2(n-k) z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) \sum_{n=-\infty}^{\infty} x(n-k) z^{-n}$$

$$= \sum_{k=-\infty}^{n} x_1(k) \left[Z(x(n-k))\right] = \sum_{k=-\infty}^{\infty} x_1(k) X_2(z) z^{-k}$$

$$= X_2(z) \sum_{k=-\infty}^{\infty} x_1(k) z^{-k} = X_2(z) X_1(z)$$

2a) $X(z) = Z\{(-1/2)^n u(-n)\} + Z\{2(1/4)^n u(n)\}$

$$= \frac{1}{1+2z} + \frac{2z}{z-1/4}$$

$$= \frac{3z - 1/4}{(1+2z)(z-1/4)} \quad ROC: (|z| < 1/2) \cap (|z| > 1/4)$$

2b) $x(n) = u(-n-1) + (1/2)^n u(n)$

$X(z) = Z\{u(-n-1)\} + Z\{(1/2)^n u(n)\}$

$$= \frac{z}{z-1} + \frac{z}{z-1/2} \quad ROC: (z<1) \cap (|z| > 1/2)$$

$$= \frac{2z^2 - 3/2 z}{(z-1)(z-1/2)} \quad ROC: (|z| > 1/2) \cap (|z| < 1)$$

3a) $= Z\{n(-1/2)^n u(n)\} + Z\{\delta(n-k)\}$

$$= -z \frac{d}{dz}\left[\frac{z}{z+1/2}\right] + z^{-k}$$

$$= \frac{-1/2 z}{(z+1/2)^2} + z^{-k} = \frac{-z + 2z^{-k}(z+1/2)^2}{2(z+1/2)^2} \quad k>0$$

3b

$$Z\{u(n-2) * (2/3)^n u(n)\} =$$

$$Z\{u(n-2)\}\, Z\{(2/3)^n u(n)\}$$

$$= \left[z^{-2}\frac{z}{z-1}\right]\left[\frac{z}{z-2/3}\right] \quad \text{ROC:} (|z|>1) \cap (|z|<2/3)$$

$$= \frac{1}{(z-1)(z-2/3)} \quad (|z|>1) \cap (z<2/3)$$

5a

$$X(z) = \frac{1/4\, z}{(z-1/2)(z-1/4)} \qquad \frac{X(z)}{z} = \frac{1/4}{(z-1/2)(z-1/4)}$$

$$\frac{X(z)}{z} = \frac{c_1}{z-1/2} + \frac{C_2}{z-1/4} = \frac{1}{z-1/2} - \frac{1}{z-1/4}$$

$$X(z) = \frac{z}{z-1/2} - \frac{z}{z-1/4}$$

$$\boxed{x(n) = -\left(\frac{1}{2}\right)^n u(-n-1) - (1/4)^n u(n)}$$

5b

$$X(z) = \frac{z+1}{3(z^2 - \frac{4}{3}z + 1/3)} \qquad \frac{X(z)}{z} = \frac{z+1}{3z(z-1)(z-1/3)}$$

$$\frac{X(z)}{z} = \frac{c_1}{z} + \frac{c_2}{z-1} + \frac{c_3}{z-1/3} \qquad c_1 = 1 \quad c_2 = 1 \quad c_3 = -2$$

$$X(z) = 1 + \frac{z}{z-1} - \frac{2z}{z-1/3}$$

$$= \delta(n) + u(n) - 2(1/3)^n u(n)$$