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Internal Assesment Test-III									
Sub:	Electromagnetic Waves							Code:	18EC55
Date:	23/01/2023	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	ECE(A,B,C,D)
Answer any **FIVE FULL** Questions									

		Marks	OBE CO	OBE RBT
1.(a)	Define a) Magnetization, b) Permeability	[04]	CO4	L2
1.(b)	Derive the expression for Inductance of a Solenoid.	[06]	CO4	L2
2.(a)	Using Faraday's law, derive an expression for e.m.f. induced in a stationary conductor placed in a time varying magnetic field. Also obtain the expression for motional e.m.f.	[06]	CO5	L3
2.(b)	List the Maxwell equations in point and integral forms for time varying fields.	[04]	CO5	L3
3.	Derive the expression for Wave equation for a Uniform Plane Wave for free space from Maxwell's equation.	[10]	CO5	L3
4.	State Poynting's theorem. Prove that **P = E × H.** Give the expression for power in an electromagnetic wave.	[10]	CO5	L3
5.	Explain displacement current. What is the inconsistency of Ampere's law with the equation of continuity? Derive modified form of Ampere's law for time varying fields.	[10]	CO5	L3
6.(a)	Let $\mu = 10^5$ H/m, $\varepsilon = 4 \times 10^{-9}$ F/m, $\sigma = 0$ and $\rho_v = 0$. Find K so that the following pair of fields satisfy Maxwell's equations: $\boldsymbol{E} = (20y - Kt)\boldsymbol{a_x}\, V/m$ & $\boldsymbol{H} = (y + 2 \times 10^6 t)\boldsymbol{a_z}$ A/m.	[07]	CO5	L3
6.(b)	Calculate skin depth, when f = 10MHz, σ = 10s/m, μ_r = 4.	[03]	CO5	L3
7.	Find the capacitance between the two concentric spheres of radii r = b, and r = a, such that b>a, if the potential V = V_0 at r =a and V = 0 at r = b, using the Laplace's equation.	[10]	CO3	L3
8.(a)	State and prove the Uniqueness theorem.	[08]	CO3	L2
8.(b)	Determine whether or not the given potential field satisfy the Laplace equation: $V = 2x^2 - 3y^2 + z^2$.	[02]	CO3	L3

1)(a) a) Magnetization

Magnetization is defined as the total magnetic dipole moment per unit volume.

$$\boxed{\vec{M} = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{n} m_i}{\Delta V}} \quad (A/m)$$

b) Permeability or Relative Permeability

We know that $\vec{M} = \chi_m \vec{H}$

where χ_m = magnetic susceptibility of the material

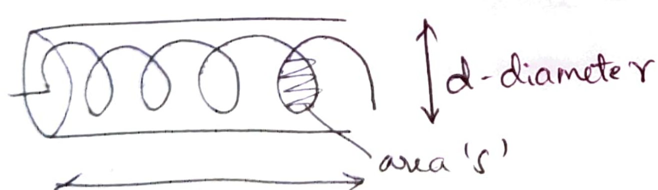
$$\boxed{1 + \chi_m = \mu_r}$$

where μ_r is the relative permeability

Also $\boxed{\mu = \mu_0 \mu_r}$ (H/m)

μ is defined as the ratio of magnetic field intensity per unit l...

1)(b)



$S = \frac{\pi d^2}{4}$

Inductance of a Solenoid:

$L = \frac{N\phi}{I}$ where $\phi = \iint_S \vec{B} \cdot \vec{ds}$

and $\vec{B} = \mu_0 \mu_r \vec{H}$

Magnetic field Intensity due to solenoid,

$\vec{H} = \frac{NI}{l} \cdot \vec{a_l}$

$$\vec{H} = \frac{NI}{l}\vec{a_l}$$

$$\therefore \vec{B} = \mu_0 \mu_r \frac{NI}{l}\vec{a_l}$$

$$\therefore \phi = \iint_s \vec{B}\cdot\vec{ds} = B\iint_s ds = \frac{\mu_0 \mu_r N I S}{l}$$

where $S = \frac{\pi d^2}{4}$

$$\therefore \boxed{\phi = \frac{\mu_0 \mu_r N I S}{l}}$$

$$\therefore L = \frac{N\phi}{I} = \frac{N\phi}{I} = \frac{\mu_0 \mu_r N^2 S}{l}$$

$$\therefore \boxed{L = \frac{\mu_0 \mu_r N^2 S}{l}} \quad (H)$$

Faraday's law → statement in words. ??

$$\boxed{V_{induced} = -N\frac{d\phi}{dt}}$$

i) Transformer emf :

Transformer emf is induced when the coil is stationary and field is time varying.

$$V_{induced} = -N\frac{d\phi}{dt}$$

$$V_{induced} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\iint_s \vec{B}\cdot\vec{ds}\right)$$

$$V_{induced} = -\iint_s \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}$$

iii) Field is time varying and coil is moving.

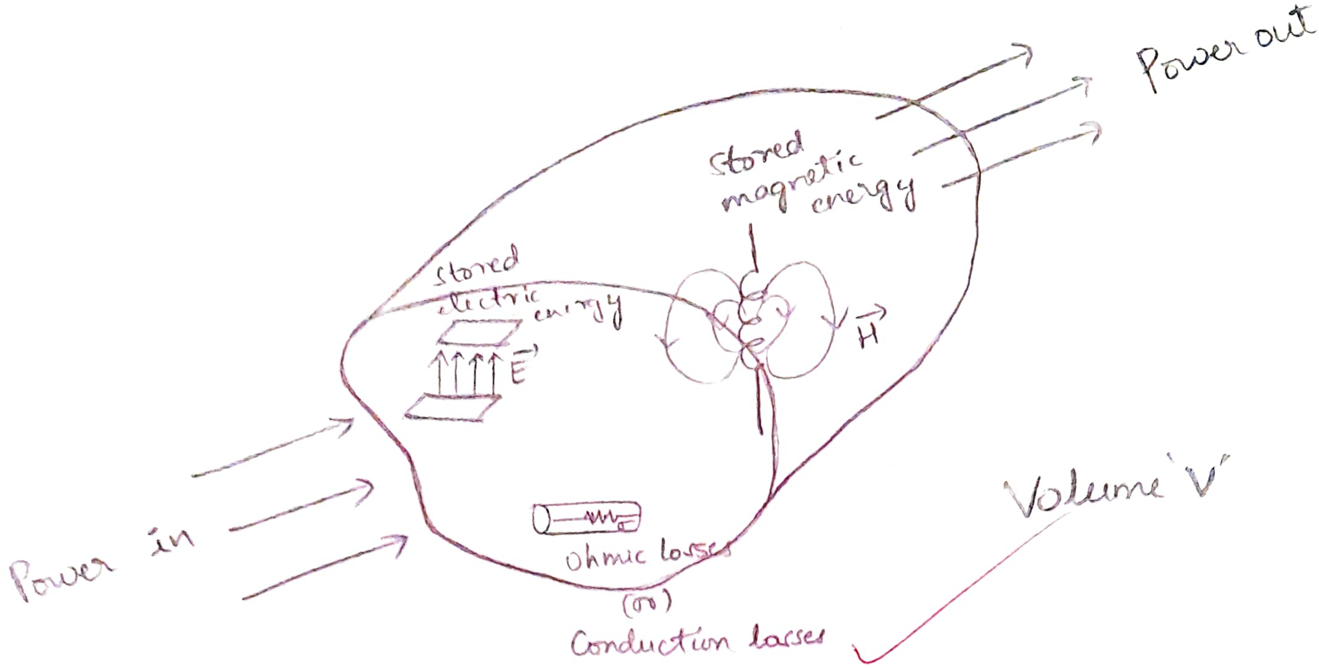
emf = transformer emf + motional emf

$$\therefore \boxed{V_{induced}\left(\text{induced emf}\right) = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} + \oint_L \vec{V} \times \vec{B} \cdot \vec{dl}}$$

2)(b)

	Integral form	Point form
① Faraday's law	$\oint_L \vec{E} \cdot \vec{dl} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$	$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$
② Ampere's law	$\oint_L \vec{H} \cdot \vec{dl} = \iint_S \sigma \vec{E} \cdot \vec{ds} + \iint_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$	$\vec{\nabla} \times \vec{H} = \vec{J}_c + \vec{J}_D$ $\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$
③ Gauss's law ($\vec{E}$-field)	$\oint\!\!\oint_S \vec{D} \cdot \vec{ds} = Q_{enc} = \iiint_V \rho_v \, dV$	$\vec{\nabla} \cdot \vec{D} = \rho_v$
④ Gauss's law ($\vec{H}$-field)	$\oint\!\!\oint_S \vec{B} \cdot \vec{ds} = 0$	$\vec{\nabla} \cdot \vec{B} = 0$

4) Poynting's theorem states that the net power flowing out of the given volume is equal to the time rate of decrease in stored energy within the volume minus conduction ~~loos~~ losses (ohmic losses).



Maxwell's equation :

① $\vec{\nabla} \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t}$

② $\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \dfrac{\partial \vec{E}}{\partial t}$

③ $\vec{\nabla} \cdot \vec{E} = \dfrac{\rho_v}{\varepsilon}$

④ $\vec{\nabla} \cdot \vec{H} = 0$

Vector Identity :

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} (\vec{\nabla} \times \vec{H})$$

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) - \vec{E}\left(\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = -\mu \left(\vec{H} \cdot \frac{\partial \vec{H}}{\partial t}\right) - \sigma \vec{E} \cdot \vec{E} - \varepsilon \left(\vec{E} \cdot \frac{\partial \vec{E}}{\partial t}\right)$$

$$\frac{\partial |\vec{H}|^2}{\partial t} = 2\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \quad \& \quad \frac{\partial |\vec{E}|^2}{\partial t} = 2\vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \sigma E^2 - \frac{\varepsilon}{2}\frac{\partial E^2}{\partial t}$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right] - \sigma E^2$$

Integrating the above equation with given volume

$$\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dV = -\frac{\partial}{\partial t}\left[\iiint_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right]dV\right] - \iiint_V \sigma E^2\,dV$$

Using Divergence theorem in LHS

$$\iiint_V \vec{\nabla}\cdot\vec{D}\,dV = \oiint_S \vec{D}\cdot\vec{ds}$$

$$\boxed{\therefore \underbrace{\oiint_S (\vec{E}\times\vec{H})\cdot\vec{ds}}_{\text{Net Power flowing out of volume}} = \underbrace{-\frac{\partial}{\partial t}\left[\frac{1}{2}\iiint_V \mu H^2\,dV + \frac{1}{2}\iiint_V \varepsilon E^2\,dV\right]}_{\text{Rate of decrease in stored energy}} - \iiint_V \underbrace{\sigma E^2\,dV}_{\text{ohmic losses}}}$$

↳ This is Integral form of Poynting's theorem

Energy ~~stored~~ stored in the E-field, $W_E = \frac{1}{2}\iiint_V \varepsilon E^2\,dV$

Energy stored in the H-field, $W_H = \frac{1}{2}\iiint_V \mu H^2\,dV$

Power in the electromagnetic wave is given by

$$\boxed{P = \oiint_S (\vec{E}\times\vec{H})\cdot\vec{ds}} \quad (W)$$

Power density vector

$$\boxed{\vec{S} = \vec{P} = \vec{E} \times \vec{H}}$$

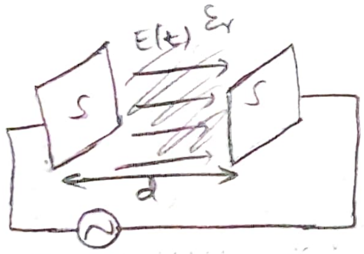
(or) $$\boxed{\vec{P} = \vec{E} \times \vec{H}} \quad W/m^2$$

where $\vec{P} \rightarrow$ Poynting's vector

5) Displacement current density,

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} \quad (A/m^2)$$

Displacement current, $$\boxed{I_D = \iint_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}}$$



$V(t) = V_0 \cos wt$

$$I_D = \iint_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds} = ?$$

$$D(t) = \varepsilon\, E(t)$$

$$E(t) = \frac{V(t)}{d} = \frac{V_0 \cos wt}{d}$$

$$D(t) = \frac{\varepsilon_0 \varepsilon_r V_0 \cos wt}{d}$$

$$\frac{\partial D(t)}{\partial t} = \frac{\varepsilon_0 \varepsilon_r V_0}{d}(-w \sin wt)$$

$$\therefore I_D = \iint_S \frac{\varepsilon_0 \varepsilon_r V_0 (-w \sin wt)}{d} \vec{ds}$$

$$I_D = \frac{-\varepsilon_0 \varepsilon_r V_0 w \sin wt \cdot S}{d} \qquad \because C = \frac{\varepsilon_0 \varepsilon_r S}{d}$$

$$\therefore \boxed{I_D = -wCV_0 \sin wt} \ (A)$$

$\therefore$ Displacement current in terms of capacitor is given by

$$\boxed{I_D = -\omega C V_0 \sin \omega t} \quad (A)$$

Continuity equation of current:

$$\boxed{\vec{\nabla}\cdot\vec{J} = \frac{-\partial \rho_v}{\partial t}}$$

Ampere's Circuital law:

Integral form

$$\oint_L \vec{H}\cdot\vec{dl} = I_{enc}$$

Differential form

$$\vec{\nabla}\times\vec{H} = \vec{J}$$

Take divergence on both sides

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J}$$

(Divergence of curl of a vector is always zero)

$$0 \neq \vec{\nabla}\cdot\vec{J}$$

$$\boxed{0 \neq \frac{-\partial \rho_v}{\partial t}}$$

$\therefore$ Ampere's circuital law is inconsistent for time varying fields.

Modified Ampere's law for time varying fields,

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \vec{G}}$$ where $\vec{G} \rightarrow$ unknown vector

Take divergence on both sides

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{G}$$

$$0 = \frac{-\partial \rho_v}{\partial t} + \vec{\nabla}\cdot\vec{G}$$

$$\frac{\partial \rho_v}{\partial t} = \vec{\nabla}\cdot\vec{G}$$

We know $\rho_v = \vec{\nabla}\cdot\vec{D}$

$$\frac{\partial \vec{\nabla}\cdot\vec{D}}{\partial t} = \vec{\nabla}\cdot\vec{G}$$

$$\vec{\nabla}\cdot\frac{\partial \vec{D}}{\partial t} = \vec{\nabla}\cdot\vec{G}$$

$$\therefore \boxed{\vec{G} = \frac{\partial \vec{D}}{\partial t}}$$

$\therefore$ Modified Ampere's law will be

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

6) (a) Given : $\mu = 10^5$ H/m

$\varepsilon = 4\times10^{-9}$ F/m

$\sigma = 0$, $\rho_v = 0$

$$\vec{E} = (20y - kt)\,\vec{a_x} \quad \text{V/m}$$

$$\vec{H} = (y + 2\times10^6 t)\,\vec{a_z} \quad \text{A/m}$$

$$\vec{\nabla}\times\vec{H} = \cancel{\sigma\vec{E}}_{0} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & (y + 2\times10^6 t) \end{vmatrix}$$

$$\vec{\nabla} \times \vec{H} = \vec{a_x}\left[\frac{\partial}{\partial y}(y + 2\times10^6 t) - 0\right] - \vec{a_y}(0) + \vec{a_z}(0)$$

$$\vec{\nabla} \times \vec{H} = \vec{a_x}\left[(1+0)\right] = \vec{a_x}$$

$$\therefore \boxed{\vec{\nabla} \times \vec{H} = \vec{a_x}}$$

$$\frac{\partial \vec{D}}{\partial t} = \frac{\partial(\varepsilon \vec{E})}{\partial t} = \frac{\partial}{\partial t}\left[4\times10^{-9} \times (20y - kt)\vec{a_x}\right]$$

$$= 4\times10^{-9}(0 - k)\vec{a_x}$$

$$\frac{\partial \vec{D}}{\partial t} = 4\times10^{-9}(-k)\vec{a_x}$$

$$\therefore \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{a_x} = 4\times10^{-9}(-k)\vec{a_x}$$

$$\therefore k = -\frac{1}{4\times10^{-9}} = 250\times10^6 \text{ V/ms}$$

$$\boxed{k = 2.5\times10^8 \text{ V/ms}}$$

6)(b) $\delta = ?$

$f = 10$ MHz

$\sigma = 10$ S/m

$\mu_r = 4$

Skin depth, $\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$

$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 4$$

$$\therefore \delta = \frac{1}{\sqrt{\pi \times 10 \times 10^{6} \times 4\pi \times 10^{-7} \times 4 \times 10}}$$

$$\delta = 0.025 \text{ m}$$

$\boxed{\delta = 25 \text{ mm}}$ ← Skin depth

$\vec{E}_{induced} \leftarrow$ induced electric field intensity

$$V_{induced} = \oint_L \vec{E}_{induced} \cdot \vec{dl}$$

$$\therefore \boxed{\oint_L \vec{E} \cdot \vec{dl} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}}$$

↳ This is Integral form of Faraday's law.

WKT. Stoke's theorem

$$\oint_L \vec{A} \cdot \vec{dl} = \iint_S \vec{\nabla} \times \vec{A} \cdot \vec{ds}$$

$$\therefore \iint_S \vec{\nabla} \times \vec{E} \cdot \vec{ds} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$$

$$\therefore \boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

→ This is Differential form

Point form of Faraday's law

ii) Generator emf or Motional emf

Field is ~~state~~ static (not changing w.r.t to time) and coil is moving. ($\vec{v}$ - velocity of moving coil)

Force on moving coil, $\vec{F}_m = Q(\vec{v} \times \vec{B})$

$$\vec{E}_{induced} = \frac{\vec{F}_m}{Q} = \vec{v} \times \vec{B}$$

$$V_{induced}\text{ (induced emf)} = \oint_L \vec{E}_{induced} \cdot \vec{dl}$$

$$\therefore \boxed{V_{induced}\text{ (induced emf)} = \oint_L \vec{v} \times \vec{B} \cdot \vec{dl}}$$

using $\oint_L$ $\iint_S$

13. Wave equation for a uniform plane wave

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad — (1)$$

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \qquad \vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad — (2)$$

$$\vec{\nabla} \cdot \vec{E} = \rho_v \quad — (3)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad — (4)$$

$$\vec{E}(z,t) = E(z) e^{j\omega t} \vec{a_x} \qquad \vec{H}(z,t) = H(z) e^{j\omega t} \vec{a_y}$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E(z) & 0 & 0 \end{vmatrix} = \vec{a_x}(0-0) - \vec{a_y}\left(0 - \partial/\partial z \, E\right) + \vec{a_z}(0)$$

$$= \frac{\partial}{\partial z} E \, \vec{a_y} \quad — (3)$$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & H(z) & 0 \end{vmatrix} = \vec{a_x}(0 - \partial/\partial z H)$$

$$\vec{\nabla} \times \vec{H} = -\frac{\partial}{\partial z} H \vec{a_x} \quad \text{—④}$$

By comparing ① & ③

$$-\mu \frac{\partial \vec{H}}{\partial t} = \frac{\partial \vec{E}}{\partial z}$$

Differentiate w.r.t z.

$$-\mu \frac{\partial^2 \vec{H}}{\partial t \partial z} = \frac{\partial^2 \vec{E}}{\partial z^2} \quad \text{—⑤}$$

Differentiate w.r.t t

$$-\mu \frac{\partial^2 \vec{H}}{\partial t^2} = \frac{\partial^2 \vec{E}}{\partial z \partial t} \quad \text{—⑦}$$

By comparing ② and ④

$$\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} = -\frac{\partial \vec{H}}{\partial z}$$

D. w.r.t z

$$\sigma \frac{\partial \vec{E}}{\partial z} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t \partial z} = -\frac{\partial^2 \vec{H}}{\partial z^2} \quad \text{—⑥}$$

D w.r.t t

$$\sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = -\frac{\partial^2 \vec{H}}{\partial z \partial t} \quad \text{—⑧}$$

By comparing ⑧ and ⑧

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \mu \left[\sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \right] = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

— general wave equation in E field

By comparing ⑥ & ⑦

$$-\frac{\partial^2 \vec{H}}{\partial z^2} = \sigma \left[-\mu \frac{\partial H}{\partial t} \right] + \varepsilon \left[-\mu \frac{\partial^2 H}{\partial t^2} \right]$$

$$\frac{\partial^2 H}{\partial z^2} = \sigma\mu \frac{\partial H}{\partial t} + \varepsilon\mu \frac{\partial^2 H}{\partial t^2}$$

— general wave equation in H field

$$\frac{\partial^2 E}{\partial z^2} = \mu \varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 H}{\partial z^2} = \mu \varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\sqrt{\mu_0 \varepsilon_0} = \frac{1}{c}$$

for free space

$$\sqrt{\mu_0 \varepsilon_0} = \frac{1}{3 \times 10^8}$$

$$\mu_0 \varepsilon_0 = \frac{1}{4\pi \times 10^{-7}} = \frac{1}{c^2}$$

$$\mu_0 \varepsilon_0 = \frac{1}{9 \times 10^{16}}$$

$$\varepsilon_0 = \frac{1}{4\pi \times 10^{-7} \times 9 \times 10^{16}}$$

$$= \frac{1}{36 \times \pi \times 10^{-9}}$$

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial z^2}$$

$$\frac{\partial^2 H}{\partial t^2} = c^2 \frac{\partial^2 H}{\partial^2 z^2}$$

$$\vec{E}(z,t) = E(z)\, e^{j\omega t}\, \vec{a_x}$$

$$\vec{H}(z,t) = H(z)\, e^{j\omega t}\, \vec{a_x}$$

for second order equation solution is given by

$$\vec{E}(z,t) = \left[E_0 \cos\left(t - z/V_p\right) + E_0' \cos\left(t + z/V_p\right)\right] \vec{a_z}$$

$$\vec{H}(z,t) = \left[H_0 \cos\left(t - z/V_p\right) + H_0' \cos\left(t + z/V_p\right)\right] \vec{a_y}$$