USN 1CR18ME061

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Internal Assessment Test 3 – Dec 2022

Sub:	CONTROL ENGINEERING			Sub Code:	18ME71	Branch:	ME	
Date:	26/12/22	Duration:	90 min's	Max Marks: 50	Sem / Sec:	7^{th}/A & B		OBE

Use your own semi log graph sheets for Bode Plot.

		MARKS	CO	RBT
1	Examine the stability of a control system with the following characteristic equation using RH method. $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$	[10]	CO4	L3
2	The open loop transfer function of a certain unity feedback system is given by $G(s) = \frac{K}{s(s+2)(s+20)}$ Construct Bode Plot and determine: i) Limiting value of K for system to be stable. ii) Value of K for gain margin to be 10dB. iii) Value of K for phase margin to be 50^0.	[20]	CO5	L3
3	For a system $G(s)H(s)=\frac{242(s+5)}{s(s+1)(s^2+5s+121)}$ Sketch the Bode Plot and determine gain margin, phase margin, gain crossover frequency, phase crossover frequency. Comment on stability of the system.	[20]	CO5	L3

1)

Sol. :

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	0	0	0	0 ⟸ Special case 2

Row of zeros

$A(s) = 2s^4 + 4s^2 + 2 = 0$ i.e. $s^4 + 2s^2 + 1 = 0$

$\frac{dA(s)}{ds} = 4s^3 + 4s$

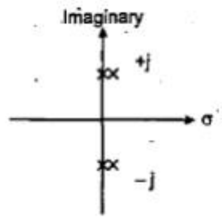
s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	4	4	0	0
s^2	2	2	0	0
s^1	0	0	0	0 ⟸ Special case 2

Row of zeros again

$\therefore \quad A'(s) = 2s^2 + 2 = 0$ i.e. $\frac{dA'(s)}{ds} = 4s = 0$

s^6	1	4	5	2
s^5	3	6	3	0
s^4	2	4	2	0
s^3	4	4	0	0
s^2	2	2	0	0
s^1	4	0	0	0
s^0	2	0	0	0

- No sign change, hence **no root is located in R.H.S. of s-plane.** As row of zeros occur, system may be marginally stable or unstable. To examine that find the roots of first auxiliary equation.



$A(s) = s^4 + 2s^2 + 1 = 0 \qquad s^2 = \frac{-2 \pm \sqrt{4-4}}{2} = -1$

$s^2 = -1, \quad s^2 = -1, \quad s_{1,2} = \pm j, \quad s_{3,4} = \pm j$

- The roots of $A'(s) = 0$ are the roots of $A(s) = 0$. So do not solve second auxiliary equation. Predict the stability from the nature of roots of first auxiliary equation.
- **As there are repeated roots on imaginary axis, system is unstable.**

2)

Sol. :

Step 1 : G(s)H(s) in time constant form.

$$G(s)H(s) = \frac{K/40}{s(1+0.5s)(1+0.05s)}$$

$$= \frac{A}{s(1+0.5s)(1+0.05s)}$$

Step 2 :

i) $A = \frac{K}{40}$, it is unknown.

ii) One pole at origin, $\frac{1}{s}$, straight line of slope −20 dB/dec passing through intersection of ω=1 and 0 dB.

iii) $\frac{1}{1+0.5s}$, simple pole, $T_1 = 0.5$, $\omega_{C1} = \frac{1}{T_1} = 2$, straight line of slope −20 dB/dec for ω≥2.

iv) $\frac{1}{1+0.05s}$, simple pole, $T_2 = 0.05$, $\omega_{C2} = \frac{1}{T_2} = 20$, straight line of slope −20 dB/dec for ω≥20

Resultant slope table is,

Range of ω→	$0<\omega<2$	$2\le\omega<20$	$20\le\omega<\infty$
Slope in dB/dec	−20	−20−20 = −40	−40−20 = −60

Step 3 : Phase angle table

$$G(j\omega)H(j\omega) = \frac{A}{j\omega(1+0.5j\omega)(1+0.05j\omega)}$$

ω	$\frac{1}{j\omega}$	$-\tan^{-1}0.5\omega$	$-\tan^{-1}0.05\omega$	ϕ_R
0.2	−90°	−5.71°	−0.57°	−96.3°
2	−90°	−45°	−5.71°	−140.7°
10	−90°	−78.7°	−26.56°	−195.3°
20	−90°	−84.3°	−45°	−219.3°
∞	−90°	−90°	−90°	−270°

Step 4 : Draw the Bode Plot without considering effect of A, as shown in the Fig. 11.7.4.

i) For system to be stable, $\omega_{pc} = \omega_{gc} = 7$ rad/sec. Hence magnitude plot must be shifted from B to G i.e. shift BG = 28 dB upwards. This must be contribution by 20 Log A.

∴ 20 Log A = 28 i.e. A = 25.11

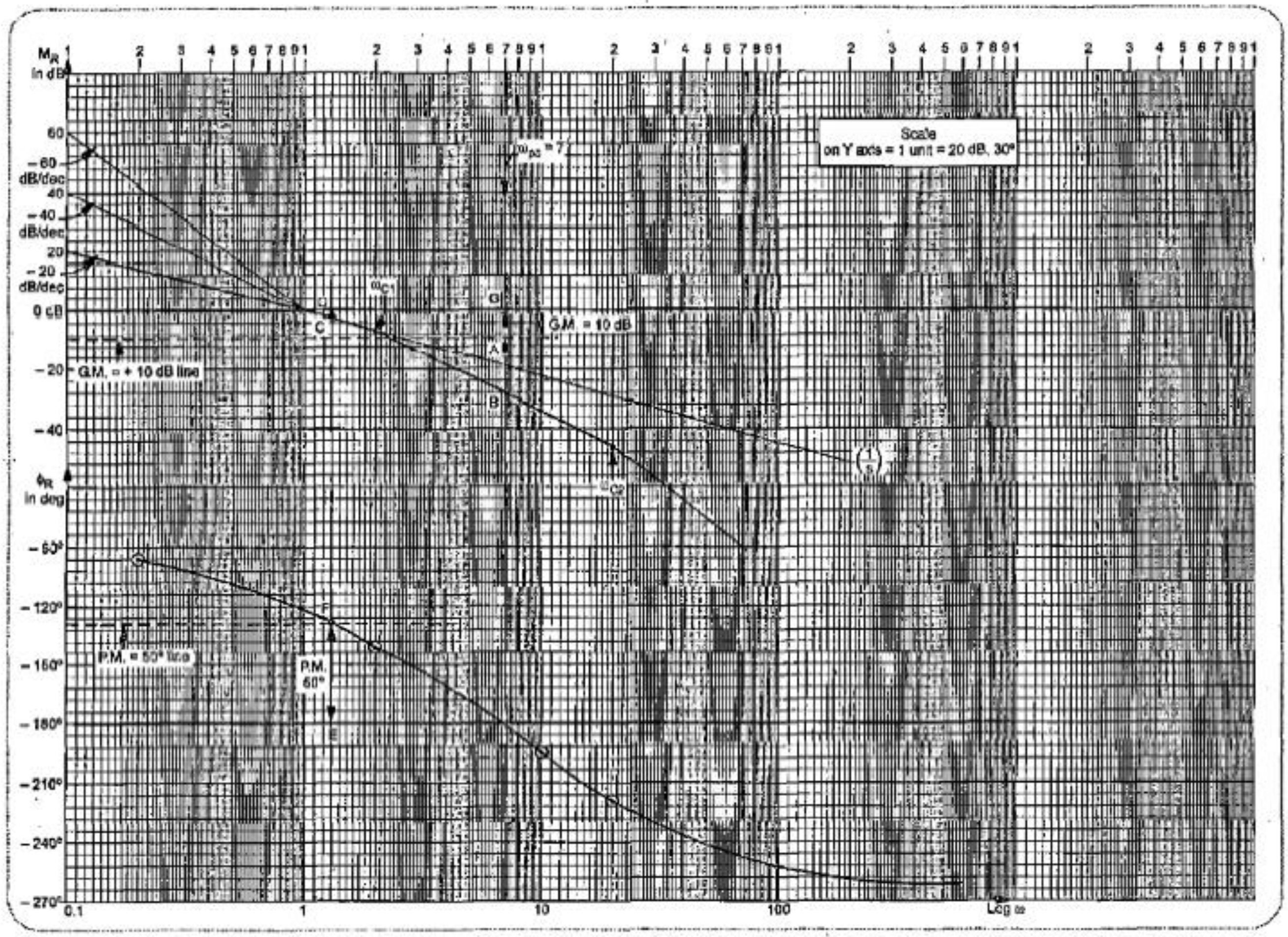
∴ $K_{mar} = 40\times A = 1004.75$

ii) Value of K for G.M. = 10 dB.

- From the plot, for G.M. = +10 dB, the shift AB is to be contributed by 20 Log A.

∴ 20 Log A = shift BA ↑ = 18 dB

∴ A = 7.943 i.e. K = 40 A = 317.7 for G.M. 10 dB.



M_R in dB
ϕ_R in deg
Scale on Y axis = 1 unit = 20 dB, 30°
−60 dB/dec
−40 dB/dec
−20 dB/dec
ω_{pc} = 7
ω_{C1}
ω_{C2}
G.M. = 10 dB
G.M. = + 10 dB line
P.M. = 60° line
P.M. 60°
Log ω

3)

As G(s)H(s) includes a quadratic pole, comparing it with $\frac{1}{s^2+\xi\omega_n s+\omega_n^2}$*, decide* ξ *and* ω_n*. These values give us proper correction to be applied at* ω_n *while sketching, magnitude plot of it.*

∴ Comparing

$$\frac{1}{s^2+5s+121} = \frac{1}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\omega_n^2 = 121 \qquad \therefore \omega_n = 11$$

$$\text{and} \quad 2\xi\omega_n = 5 \qquad \therefore \xi = \frac{5}{2\times 11} = 0.22$$

Referring to correction Table 11.4.2 discussed earlier in quadratic factor analysis, approximate correction required at $\omega_n = 11$ is +7.95 dB upwards, for $\xi = 0.2$.

or correction can be calculated precisely as

$$\text{Correction} = -20\,\text{Log}\sqrt{4\xi^2}$$

$$= -20\,\text{Log}\,2\xi = -20\,\text{Log}\,(2\times 0.22)$$

$$= +7.13\ \text{dB}$$

Step 2 : Factors.

i) Constant K = 10,

ii) 1 pole at the origin, 1/s

iii) Simple pole, $\frac{1}{1+s}$, $T_1 = 1$

$$\therefore \omega_{C1} = \frac{1}{T_1} = 1\ \text{rad/sec.}$$

iv) Simple zero, $1+\frac{s}{5}$, $T_2 = \frac{1}{5}$

$$\therefore \omega_{C2} = \frac{1}{T_2} = 5\ \text{rad/sec.}$$

v) Quadratic pole, $\frac{1}{(1+0.0415+\frac{s^2}{121})}$,

$$\therefore \omega_{C3} = \omega_n = 11\ \text{rad/sec.}$$

Step 3 : Magnitude plot analysis

i) Contribution due to K = 10 is 20 Log K = 20 dB.

ii) 1 pole at origin, magnitude plot is straight line of slope –20 dB/decade, passing through intersection point of $\omega = 1$ and 0 dB line.

iii) Shift this intersection point on 20 Log K line and draw parallel line to –20 dB/decade. This line represents addition of K = 10 and 1/s. So starting slope is – 20 dB/decade. This will continue till $\omega_{C1} = 1$.

iv) At $\omega_{C1} = 1$, simple pole occurs so it contributes –20 dB/decade individually and hence resultant will have slope – 20 – 20 = – 40 dB/decade from '1' onwards and will continue till $\omega_{C2} = 5$.

v) At $\omega_{C2} = 5$, simple zero occurs so it contributes + 20 dB/decade individually and hence resultant will have slope – 40 + 20 = – 20 dB/decade from 5 onwards and will continue till $\omega_{C3} = 11$.

vi) At $\omega_{C3} = 11$, quadratic pole with $\xi = 0.22$ occurs so it contributes – 40 dB/decade individually and hence resultant will have slope $-20-40 = -60$ dB/decade exhibiting + 7.95 dB upward shift at $\omega_{C3} = 11$ and will continue till ∞ as there is no other factor.

Step 4 : Phase angle plot

$$G(j\omega)H(j\omega) = \frac{10\left(1+j\frac{\omega}{5}\right)}{j\omega(1+j\omega)\left(1+0.041\,j\omega+\frac{(j\omega)^2}{121}\right)}$$

$$= \frac{10\left(1+j\frac{\omega}{s}\right)}{j\omega(1+j\omega)\left(1+0.041\,j\omega-\frac{\omega^2}{121}\right)}$$

As $j^2 = -1$

$$\therefore \angle G(j\omega)H(j\omega) = \frac{\angle 10+j0\,\angle(1+j\frac{\omega}{5})}{\angle j\omega\,\angle 1+j\omega\,\angle 1+0.041\,j\omega-\frac{\omega^2}{121}}$$

$$\angle 10+j0 = 0°,\ 1+j\frac{\omega}{5} = +\tan^{-1}\frac{\omega}{5}$$

$$\angle\frac{1}{j\omega} = -90° \text{ as 1 pole at origin.}$$

$$\angle\frac{1}{1+j\omega} = -\tan^{-1}\omega,$$

$$\angle\frac{1}{1+0.041\,j\omega-\frac{\omega^2}{121}} = -\tan^{-1}\left\{\frac{0.041\,\omega}{1-\frac{\omega^2}{121}}\right\}$$

∴ Phase angle table

ω	$\frac{1}{j\omega}$	$+\tan^{-1}\frac{\omega}{5}$	$-\tan^{-1}\omega$	$-\tan^{-1}\left[\frac{0.041\,\omega}{1-\frac{\omega^2}{121}}\right]$	ϕ_R
0.1	– 90°	+ 1.14°	– 5.7°	– 0.23°	– 94.7°
1	– 90°	+ 113°	– 45°	– 23.6°	– 126.0°
5	– 90°	+ 45°	– 78.6°	– 14.4°	– 138°
8	– 90°	+ 58°	– 82.8°	– 34.8°	– 149.8°
10	– 90°	+ 63.4°	– 84.2°	– 67.0°	– 177.8°
20	– 90°	+ 75.9°	– 87.13°	+ 19.5° – 180° = – 160.4°	– 261.63°
∞	– 90°	+ 90°	– 90°	– 180°	– 270°

Step 5 : Sketch the Bode plot and obtain the solution.