Internal Assessment Test III – Jan 2023

Sub: Dynamics of Machinery **Code:** 18ME53

Date: 20/ 01 / 2023 Duration: 90 mins Max Marks: 50 **Sem:** V **Branch:** MECH

Note: Answer **all** questions.

		Marks	OBE CO	OBE RBT
1	State the conditions for the equilibrium of following systems: i. Two force member ii) Three force member	**4**	CO1	L1
2	A four link mechanism is acted upon by forces as shown in the fig (a). Determine the torque T_2 to be applied on link 2 to keep the mechanism in equilibrium. AD=50mm, AB=40mm, BC=100mm, DC=75mm, DE= 35mm.	**16**	CO1	L3

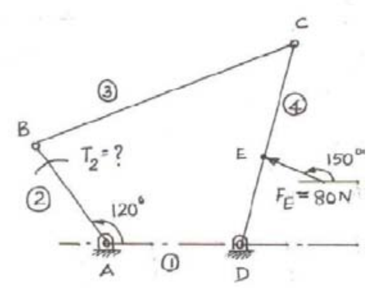

fig (a)

Internal Assessment Test III – Jan 2023

Sub: Dynamics of Machinery **Code:** 18ME53

Date: 20/ 01 / 2023 Duration: 90 mins Max Marks: 50 **Sem:** V **Branch:** MECH

Note: Answer any **five** questions.

		Marks	OBE CO	OBE RBT
1	State the conditions for the equilibrium of following systems: i. Two force member ii) Three force member	**4**	CO1	L1
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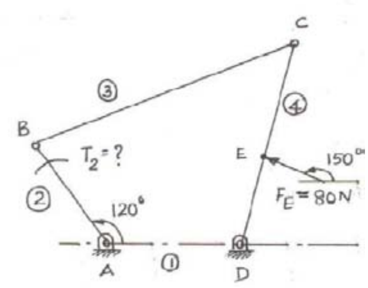

fig (a)

3. Determine i) Critical damping coefficient ii) damping factor (iii) natural frequency of damped vibration (iv) logarithmic decrement (v) ratio of two consecutive amplitudes which consists of mass of 25 kg, a spring of stiffness 15kN/m and a damper. The damping provided is only 15% of critical value. **10** CO5 L3

4 Determine the natural frequency of the system shown in Fig. 1 **10** CO4 L3

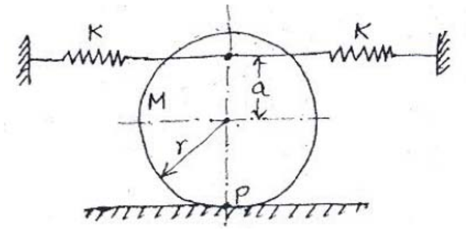

Fig. 1

5 Define logarithmic decrement and show that it can be expressed as $\delta = \frac{1}{n}\log\left(\frac{x_0}{x_n}\right), where$ n = no of cycles, x_0 is initial amplitude and x_n is the amplitude after 'n' cycles. **10** CO5 L3

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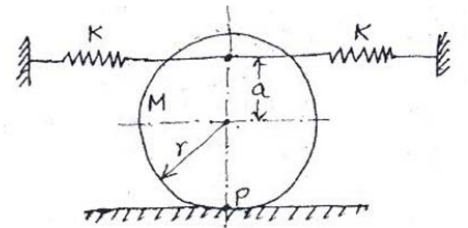

Fig. 1

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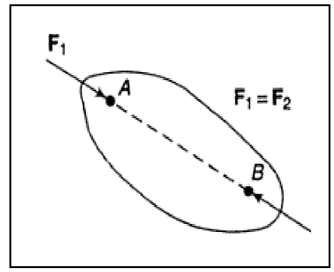
Equilibrium of Two Force Members

A member under the action of two forces will be in equilibrium if

- The forces are of the same magnitude,
- The forces act along the same line, and the forces are in opposite directions



Equilibrium of Three Force Members

A member under the action of three forces will be in equilibrium if

- The resultant of the forces is zero, and
- The lines of action of the forces intersect at a point (known as *point of concurrency).*

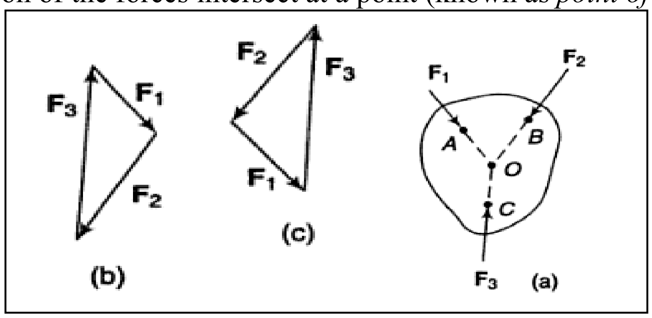


Figure (a) indicates an example for the three force member and (b) and (c) indicates the force polygon to check for the static equilibrium.

Member with two forces and a torque

A member under the action of two forces and an applied torque will be in equilibrium if

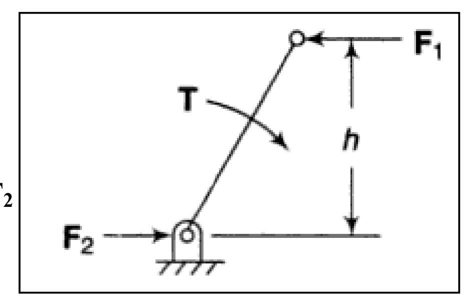
- The forces are equal in magnitude, parallel in direction and opposite in sense and
- The forces form a couple which is equal and opposite to the applied torque.

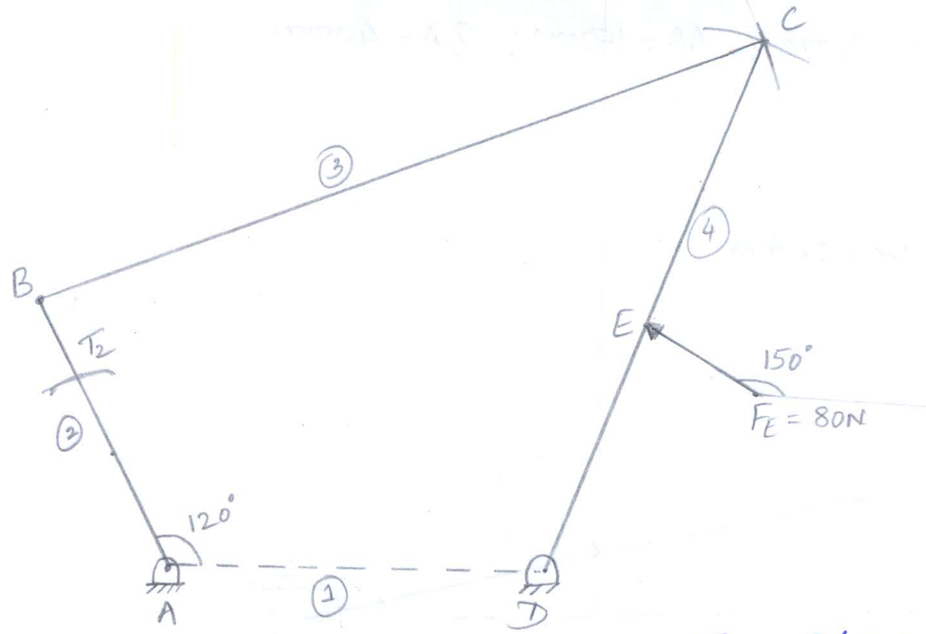
Figure shows a member acted upon by two equal forces $\mathbf{F_1}$, and $\mathbf{F_2}$ and an applied torque **T** for equilibrium,

$$T = F_1 h = F_2 h$$

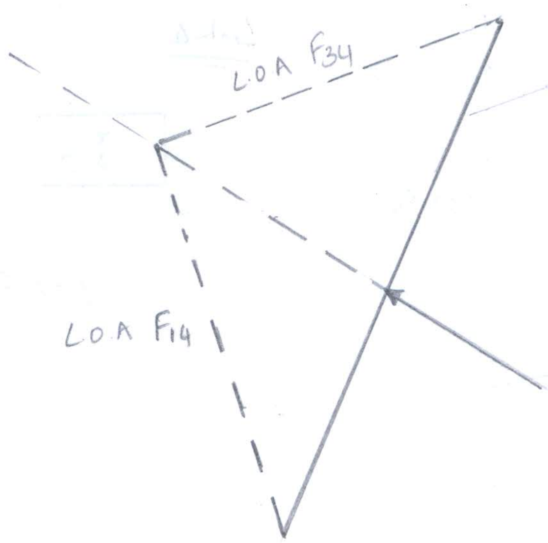
Where *T, F_1 and F_2 are the magnitudes of* ***T***, $\mathbf{F_1}$ *and* $\mathbf{F_2}$ respectively.

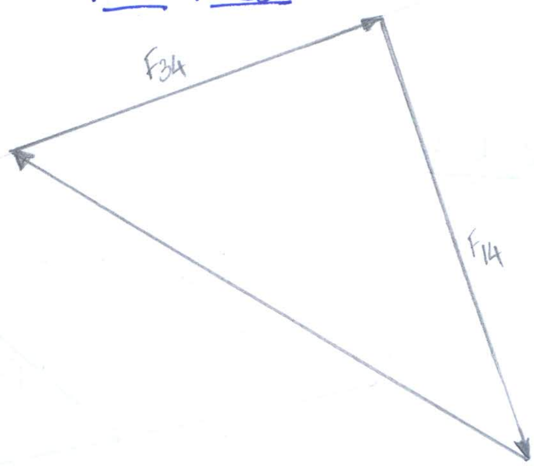
T is clockwise whereas the couple formed by $\mathbf{F_1}$, and $\mathbf{F_2}$ is counter-clockwise.





Force polygon





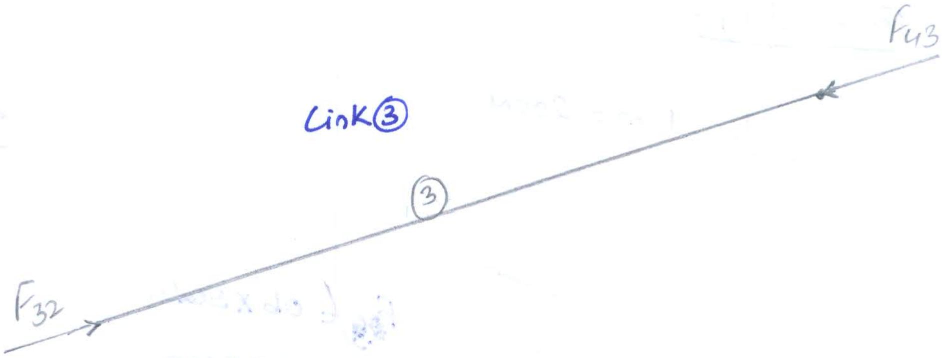
$F_{34} = 50N$

$F_{14} = 61N$

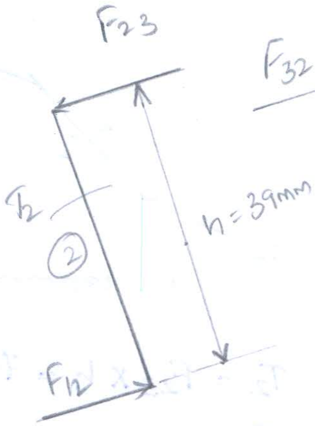
$F_{34} = -F_{43}$

$-F_{32} = +F_{23}$

Link ③



Link ②



$T_2 = F_{23} \times h$

$= 50 \times 39$

$T_2 = 1.9$ N-m

4) $M = 25\ kg$; $K = 15\ kN/m = 15{,}000\ N/m$; $C = 0.15\ C_c$

Damping factor $\xi = \frac{C}{C_c} = \frac{0.15\ C_c}{C_c} = 0.15$

Critical damping Coeff. $C_c = 2 m \omega_n = 2 \times 25 \times \sqrt{\frac{15{,}000}{25}}$

$C_c = \qquad N\text{-}s/m$

Logarithmic decrement $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi(0.15)}{\sqrt{1-0.15^2}}$

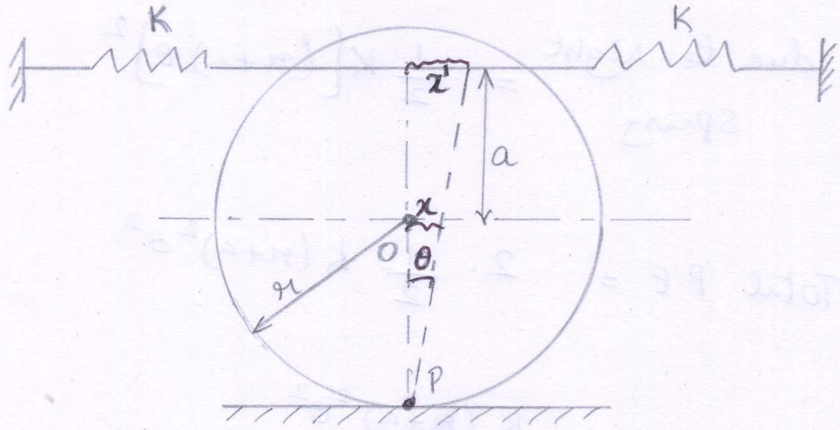
$\delta = 0.9439$

Ratio of two Cons. amp'

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right)$$

$$\frac{x_n}{x_{n+1}} = e^{\delta} = e^{0.9439} = 2.57$$

3.



From fig $x = r\theta$; $x' = (r+a)\theta$

<u>Energy method</u>

$$K.E = \text{Rotational } K.E \; (KE_{Rot}) + (K.E)_{Translation}.$$

$$(K.E)_{Rot} = \frac{1}{2} I \dot{\theta}^2 = \frac{1}{2}\left[\frac{1}{2}Mr^2\right]\dot{\theta}^2$$

$$(K.E)_{Trans.} = \frac{1}{2}M\dot{x}^2 = \frac{1}{2}M(r\dot{\theta})^2 = \frac{1}{2}Mr^2\dot{\theta}^2$$

$$\therefore \text{ Total } K.E = \frac{1}{2}\left[\frac{1}{2}Mr^2\right]\dot{\theta}^2 + \frac{1}{2}Mr^2\dot{\theta}^2$$

$$= \frac{1}{4}Mr^2\dot{\theta}^2 + \frac{1}{2}Mr^2\dot{\theta}^2$$

Potential Energy = P.E due to left spring + P.E due to right Spring

P.E due to left Spring $= \frac{1}{2}K\left[(r+a)\theta\right]^2$

P.E due to right spring $= \frac{1}{2} K\left[(r+a)\,\theta\right]^2$

$\therefore$ Total P.E $= 2 \cdot \frac{1}{2} K (r+a)^2 \theta^2$

$= K (r+a)^2 \theta^2$

According to energy method,

$$K.E + P.E = \text{Constant}$$

$$\frac{d}{dt}(K.E + P.E) = 0$$

$$\frac{d}{dt}\left\{\left[\frac{1}{4} Mr^2 \dot{\theta}^2 + \frac{1}{2} Mr^2 \dot{\theta}^2\right] + K(r+a)^2 \theta^2\right\} = 0$$

$$\frac{1}{4} Mr^2\, 2\dot{\theta}\ddot{\theta} + \frac{1}{2} Mr^2\, 2\dot{\theta}\ddot{\theta} + K(r+a)^2\, 2\theta\dot{\theta} = 0$$

$$\left[\frac{1}{2} Mr^2 + Mr^2\right]\ddot{\theta} + 2K(r+a)^2\theta = 0$$

$$\left[\frac{3Mr^2}{2}\right]\ddot{\theta} + 2K(r+a)^2\theta = 0$$

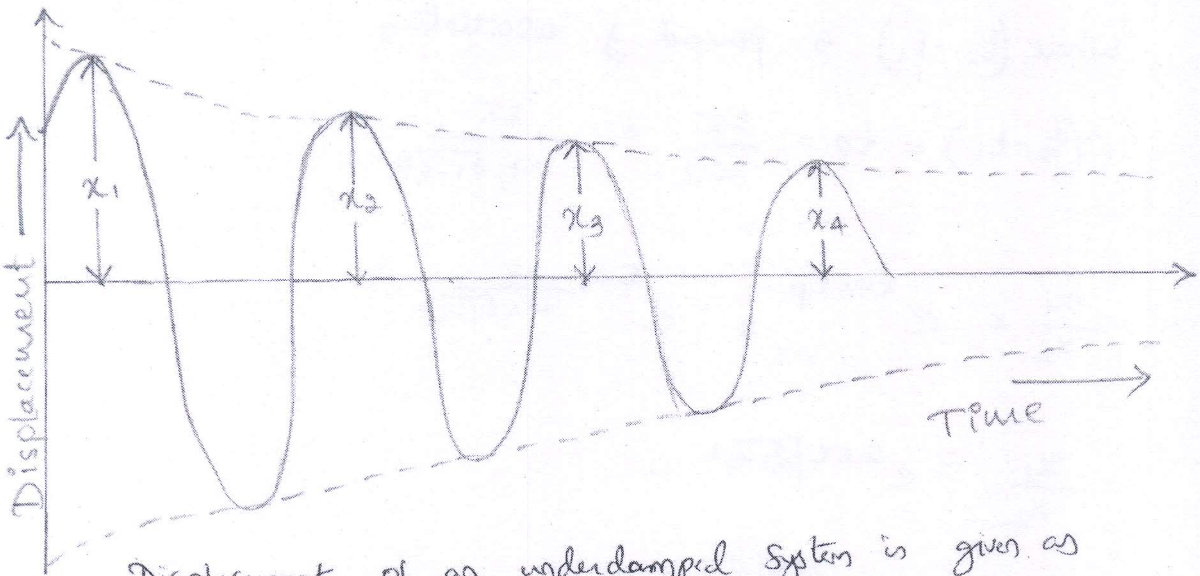
$$\ddot{\theta} + \frac{4K(r+a)^2}{3Mr^2}\theta = 0$$

$$\therefore \omega_n = \sqrt{\frac{4K(r+a)^2}{3Mr^2}} \text{ r/s.}$$

Natural frequency $f_n = \frac{1}{2\pi} \cdot \omega_n = \frac{1}{2\pi}\sqrt{\frac{4K(r+a)^2}{3Mr^2}}$ Hz.

4. Logarithmic Decrement

It is defined as the natural log. of the ratio of any two successive amplitudes on the same side of the mean position in an underdamped system.



Displacement of an underdamped system is given as

$$x = X e^{-\xi \omega_n t} \sin(\omega_d t + \theta)$$

where $X e^{-\xi\omega_n t}$ – Amplitude

ω_d – Angular frequency

When $\sin(\omega_d t + \phi) = 1$, the amplitude is maximum

Now, Max' amplitude $x = X e^{-\xi \omega_n t}$

Let x_1 be max' amplitude when the time is t_1

x_2 be max' amplitude when the time is t_2

$$\therefore \quad x_1 = X e^{-\xi\omega_n t_1}$$

$$x_2 = X e^{-\xi\omega_n t_2}$$

$$\frac{x_1}{x_2} = \frac{Xe^{-\xi\omega_n t_1}}{Xe^{-\xi\omega_n t_2}} = e^{-\xi\omega_n t_1 - (\xi\omega_n t_2)} = e^{\xi\omega_n(t_2-t_1)}$$

Where (t_2-t_1) is period of oscillation

$$(t_2-t_1) = t_p = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\frac{x_1}{x_2} = e^{\xi\omega_n t_p} = e^{\xi \cancel{\omega_n} \frac{2\pi}{\cancel{\omega_n}\sqrt{1-\xi^2}}}$$

$$\frac{x_1}{x_2} = e^{2\pi\xi/\sqrt{1-\xi^2}}$$

$|||^{ly}$ $\frac{x_2}{x_3} = e^{2\pi\xi/\sqrt{1-\xi^2}}$

$$\therefore \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \cdots\cdots \frac{x_n}{x_{n+1}} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

Taking natural log on both sides

$$\ln\left(\frac{x_1}{x_2}\right) = 2\pi\xi/\sqrt{1-\xi^2}$$

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

When ξ is very small $\delta \simeq 2\pi\xi$

$$\frac{x_0}{x_n} = \left(\frac{x_0}{x_1}\right)^n \; ; \quad \left(\frac{x_0}{x_1}\right) = \left(\frac{x_0}{x_n}\right)^{1/n} \Rightarrow \delta = \ln\left(\frac{x_0}{x_1}\right)$$

$$\boxed{\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right)}$$