CBCS SCHEME

22MCA11

USN | | | | | | | | | | |

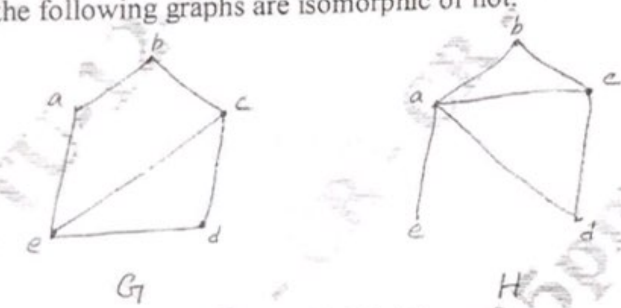
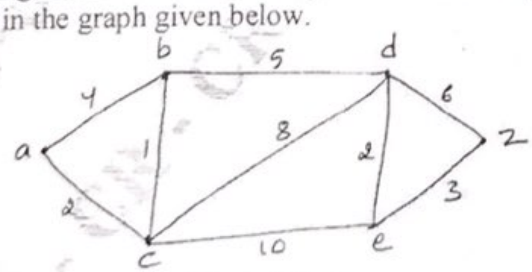
First Semester MCA Degree Examination, Jan./Feb. 2023
Mathematical Foundation for Computer Applications

Time: 3 hrs.   Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

			M	L	C
		Module – 1			
Q.1	a.	Define, cardinality of a set, singleton set and universal set with example.	6	L2	CO1
	b.	Define union and intersection of two sets with example.	4	L2	CO1
	c.	Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$.	10	L2	CO1
		OR			
Q.2	a.	Define matrix. Explain different types of matrices with example.	8	L2	CO1
	b.	Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{6, 7, 8, 9, 10\}$ and $f : A \to B$ be a function defined by $f = \{(1, 7)(2, 7)(3, 8)(4, 6)(5, 9)(6, 9)\}$. Determine $f^{-1}(6)$ and $f^{-1}(9)$. Also if $B_1 = \{7, 8\}$, $B_2 = \{8, 9, 10\}$ then find $f^{-1}(B_1)$ and $f^{-1}(B_2)$.	4	L1	CO1
	c.	In a class of 52 students, 30 are studying C++, 28 are studying pascal and 13 are studying both languages. How many in this class are studying at least one of these languages? How many are studying neither of these languages?	6	L2	CO1
	d.	State and explain Pigeon hole principle.	2	L1	CO1
		Module – 2			
Q.3	a.	State the laws of logic.	8	L2	CO2
	b.	Write the contra positive, converse and the inverse of the conditional statement. "If oxygen is a gas then Gold is compound".	6	L1	CO2
	c.	Define Tautology. Show that the compound proposition, $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a Tautology.	6	L3	CO2
		OR			
Q.4	a.	Prove the following is valid argument : $$\begin{array}{l} p \to r \\ \neg p \to q \\ q \to s \\ \hline \therefore \neg r \to s \end{array}$$	8	L2	CO2

<table>
<tr><td></td><td>b.</td><td>Give the direct proof of the following statement "If n is an odd integer, then n^2 is odd".</td><td>6</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>What is a proposition? Let p and q be the propositions "Swimming in the New Jersy sea shore is allowed and sharks have been near the sea shore." Express each of the following compound propositions as an English sentence.
(i) $p \to \sim q$ (ii) $\sim p \to \sim q$ (iii) $p \leftrightarrow q$</td><td>6</td><td>L1</td><td>CO2</td></tr>
<tr><td colspan="6">Module – 3</td></tr>
<tr><td>Q.5</td><td>a.</td><td>Let A = {1, 2, 3, 4}, R = {(1, 3)(1, 1)(3, 1)(1, 2)(3, 3)(4, 4)} be a relation on A. Determine whether R is reflexive, symmetric, asymmetric and write matrix representation.</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>If A = {1, 2, 3, 4} and R = {(1, 2)(1, 3)(2, 4)(4, 4)},
S = {(1, 1)(1, 2)(1, 3)(1, 4)(2, 3)(2, 4)} be relations on A then find RoS, SoR, R^2 and S^2. Also write their matrices.</td><td>8</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Discuss briefly on partitions and equivalence classes with example.</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.6</td><td>a.</td><td>Show that the set A={1, 2, 3, 4, 6, 8, 12} is a POSET with respect to the relation R defined as {(a, b) : a divides b} and draw its Hasse diagram.</td><td>8</td><td>L3</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Draw the directed graph of relation,
R = {(1, 1)(1, 3)(2, 1)(2,3)(2, 4)(3, 1)(3, 2)(4, 1)} on the set {1, 2, 3, 4}. Also find in-degree and out-degree of each vertex.</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Define lattices. Determine whether the POSET ({1, 2, 3, 4, 5}, |) is lattice or not.</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>Q.7</td><td>a.</td><td>A random variable X has the following probability distribution.
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>P(X)</td><td>K</td><td>3K</td><td>5K</td><td>7K</td><td>9K</td><td>11K</td><td>13K</td></tr></table>(i) Find K.
(ii) Evaluate $P(X<4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$.
(iii) Find the minimum value of K so that $P(X \leq 2) > 0.3$</td><td>10</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that, (i) Exactly two will be defective (ii) at least two will be defective (iii) none will be defective.</td><td>10</td><td>L2</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>Given that 2% of the fuses manufactured by a firm are defective. Find by using Poisson distribution, the probability that a box containing 200 fuses has (i) no defective fuses (ii) 3 or more defective fuses (iii) at least one defective fuse.</td><td>8</td><td>L2</td><td>CO4</td></tr>
</table>

	b.	The length of a telephone conversation has an exponential distribution with a mean of 3 minutes. Find the probability that a call (i) Ends in less than 3 minutes. (ii) Takes between 3 and 5 minutes.	6	L2	CO4
	c.	Find the constant C such that, $f(x) = \int_{-\infty}^{\infty} \begin{cases} Cx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$ is a probability density function. Also compute $P(1 < X < 2)$.	6	L2	CO4
		Module – 5			
Q.9	a.	Define the following with suitable examples: (i) Simple graph (ii) Complete graph (iii) Bipartite graph	6	L2	CO5
	b.	Verify the following graphs are isomorphic or not.  Fig.Q9 (b)	6	L2	CO5
	c.	Explain the Konigsberg bridge problem.	8	L1	CO5
		OR			
Q.10	a.	Determine $\|V\|$ for the graph $G = (V, E)$ if G has 10 edges with two vertices of degree 4 and others of degree 3.	4	L2	CO6
	b.	Define the following with suitable examples : (i) Euler's graph (ii) Hamilton graph	6	L2	CO6
	c.	Use Dijkstra's algorithm to find the length of a shortest path between the vertices a and z in the graph given below.  Fig. Q10 (c)	10	L2	CO6

* * * * *

Q1. a. Singleton set : A set having only one element is called a singleton set.

Eg:- The set of all integers b/w 2 and 8 that are perfect squares.

Universal set : Suppose in a discussion, all sets that we consider are subsets of a certain set U. This set U, is called the universal set or the universe for that discussion.

set of all integers is the universe for a study concerned with integers.

b. Consider 2 sets A and B. Then the set consisting of all elements that belong to A or B is called the union of A and B & is denoted by $A \cup B$. Thus $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Given two sets A and B, the set consisting of all elements that belong to both A and B is called intersection of A and B. & is denoted by $A \cap B$. Thus, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Given $A = \{1, 3, 5\}$, $B\{3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5\}$

1c. $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ characteristic eq^n is : $|A - \lambda I| = 0$

$$\begin{vmatrix} (7-\lambda) & 3 \\ 3 & (-1-\lambda) \end{vmatrix} = 0$$

$\lambda^2 - 6\lambda - 16 = 0$

$(\lambda+2)(\lambda-8) = 0 \quad \Rightarrow \lambda = -2, \lambda = 8$

For ~~λ = -2~~

$(A-\lambda I)X = 0$

$$\begin{pmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$(7-\lambda)x + 3y = 0$

$3x - (1+\lambda)y = 0$

when $\lambda = -2$

$9x + 3y = 0$

$9x = -3y$

$\frac{x}{\cancel{3}} = \frac{y}{\cancel{9}3}$

$(-1,3)$ is the eigen vector corresponding to $\lambda = -2$

For $\lambda = 8$

$-x + 3y = 0 \quad \Rightarrow x = 3y$

$3x - 9y = 0 \quad \frac{x}{3} = \frac{y}{1}$

$(3,1)$ is the eigen vector.

Q2 a. Matrix: A rectangular array of $m \times n$ numbers (real or complex) in the form of m horizontal lines and n vertical lines is called a matrix of order m by n.

Eg: $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 5 & 0 \end{bmatrix}$

Types:

1. Row Matrix: A matrix having only one row is called a row matrix.

Eg: $\begin{bmatrix} 1 & 3 & -4 \end{bmatrix}$

2. Column Matrix: A matrix having only one column

Eg: $\begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$

3. Zero or Null Matrix: If all elements in a matrix are zeros, then it is called a zero matrix. Eg:- $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

4. Singleton Matrix: A matrix having only one element.

Eg:- $[3]$

5. Square Matrix: A matrix in which the number of rows and columns are equal. Eg:- $\begin{bmatrix} 5 & 7 \\ -3 & 6 \end{bmatrix}$

Q2 b. $f^{-1}(6) = 4$

$f^{-1}(9) = 6$

$B_1 = \{7, 8\}$ $B_2 = \{8, 9, 10\}$

$f^{-1}(B_1) = \{1, 2, 3\}$ $f^{-1}(B_2) = \{3, 5, 6\}$

c. Let U be the set of all students in a class. $|U| = 52$.
Let A and B be the set of all students studying C^{++} & Pascal respectively. $|A| = 30$, $|B| = 28$. $|A \cap B| = 13$

wkt $|A \cup B| = |A| + |B| - |A \cap B|$

$= 30 + 28 - 13$

$= 45$

45 students study at least one of these languages.

$|\bar{A} \cap \bar{B}| = |\overline{A \cup B}| = |U| - |A \cap B|$

$= 52 - 45 = 7$

7 students study none of the languages.

d. ~~If the~~ Pigeon-hole Principle:
"If there are m pigeons and n pigeon-holes with $m > n$, then atleast one pigeon-hole will contain $p+1$ or more pigeons in it where $p = \left\lfloor \frac{m-1}{n} \right\rfloor$."

Eg:- If there are 8 people, treat them as 8 pigeons & 7 days of a week as pigeon-holes. ~~At~~ At least 2 of them must have born on same day of the week.

Q3 a) Laws of Logic:

Let p: Oxygen is a gas q: Gold is compound.

Given $p \to q$

Converse: $q \to p$

If Gold is a compound then oxygen is a gas.

Inverse: $\neg p \to \neg q$

If oxygen is not a gas then gold is not a compound.

Contrapositive: $\neg q \to \neg p$

If gold is not a compound then oxygen is not a gas.

c)

p	q	r	$q \to r$	$p \to (q \to r)$ (x)	$(p \to q)$	$(p \to r)$	$(p \to q) \to (p \to r)$ (y)	$x \to y$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1		1	1
0	1	0	0	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	1	1	0	0	1	1
1	0	1	1	1	0	1	1	1
1	1	0	0	0	1	0	0	1
1	1	1	1	1	1	1	1	1

Since all the entries of the last column are 1's, given compound proposition is a tautology.

Q4 a)

$$\begin{array}{l} p \to r \\ \neg p \to q \\ q \to s \\ \hline \therefore \neg r \to s \end{array} \Rightarrow \begin{array}{l} p \to r \\ \neg p \to s \\ \hline \therefore \neg r \to s \end{array}$$ Rule of syllogism.

$p \to r \Leftrightarrow \neg r \to \neg p$

$$\Rightarrow \begin{array}{l} \neg r \to \neg p \\ \neg p \to s \\ \hline \therefore \neg r \to s \end{array}$$

This is a valid argument in view of rule of syllogism.

b) Let p: n is an odd integer. q: n^2 is odd.

Given $p \to q$

Assume p is true.

$\Rightarrow$ n is odd integer.

$\Rightarrow n = 2k+1 \Rightarrow k \in Z$

$\Rightarrow n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1 = 2(2k^2+2k) + 1$

$= 2l+1$; $l = 2k^2+2k \in Z$

which is odd.

$\Rightarrow n^2$ is odd.

$\Rightarrow q$ is true.

$\therefore p \to q$ is true.

c. A statement which is either true or false, but not both in a given context, is called a proposition.

(i) $p \rightarrow \sim q$

If swimming in the New Jersy sea shore is allowed then sharks have not been near the sea shore.

(ii) $\sim p \rightarrow \sim q$

If swimming in the New Jersy sea shore is not allowed then sharks have not been near the sea shore.

(iii) $p \leftrightarrow q$

Swimming in the new Jersy sea shore is allowed if and only if sharks have been near the sea shore.

5a) $A = \{1, 2, 3, 4\}$

$R = \{(1,3), (1,1), (3,1), (1,2), (3,3), (4,4)\}$

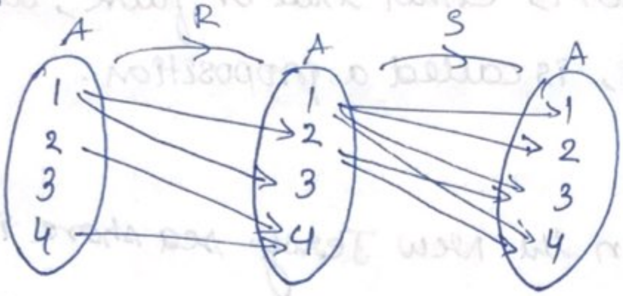
R is not reflexive as $(2,2) \notin R$.

R is not asymmetric as $(1,2) \in R$ & $(2,1) \notin R$

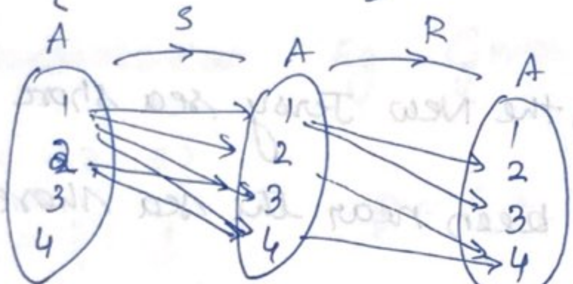
R is asymmetric as it is not symmetric.

$$M(R) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

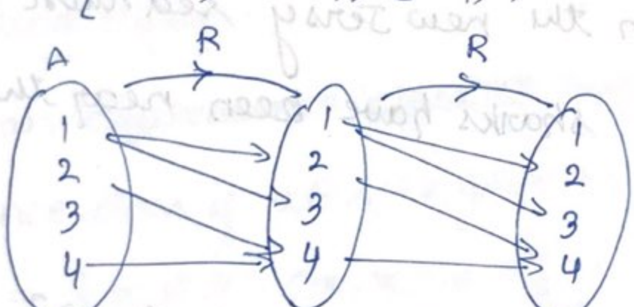
5b) RoS



$R \circ S = \{(1,3), (1,4)\}$



$S \circ R = \{(1,2), (1,3), (1,4), (2,4)\}$



$R^2 = \{(1,4), (2,4), (4,4)\}$

$S^2 = \{(1,1), (1,2), (1,3), (1,4)\}$

C. Partitions:

Given a set A, if we have subsets of A, say $A_1, A_2 \dots A_k$ such that (i) $A_1 \cup A_2 \cup \dots \cup A_k = A$

(ii) $A_i \cap A_j = \phi$ for $1 \le i, j \le k$; $i \ne j$

Then $P = \{A_1, A_2, \dots A_k\}$ is called the partition of A.

~~Equivalence class~~ Eg: Given $A = \{1,2,3,4,5\}$

$A_1 = \{3, 5\}$

$A_2 = \{1\}$

$A_3 = \{2,4\}$

$P = \{A_1, A_2, A_3\}$ is a partition of A.

Equivalence classes:

Given an equivalence ~~class~~ relation R on a set A, equivalence class of $a \in A$ is given by

$[a] = \{x \in A \mid (a,x) \in R\}$

Eg: $A = \{1,2,3\}$

& $R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$ be equi rel^n on A.

Then $[1] = \{1, 2\}$

$[2] = \{1, 2\}$

$[3] = \{3\}$

6. a) $A = \{1, 2, 3, 4, 6, 8, 12\}$

$R = \{(1,1), (1,2), (1,3) \ldots (1,12), (2,2), (2,4), (2,6), (2,8), (2,12), (3,3), (3,6), (3,12), (4,4), (4,8), (4,12), (6,6), (6,12), (8,8), (12,12)\}$

wkt a divides a, $\forall a \in A$.

$\therefore$ R is reflexive.

Suppose $(a, b) \in R$

$\Rightarrow$ a divides b

$\Rightarrow$ b doesn't divide a ; $a \neq b$

$\Rightarrow (b,a) \notin R$ for $a \neq b$.

$\therefore$ R is anti-symmetric.

Suppose $(a,b) \in R$ & $(b,c) \in R$

$\Rightarrow$ a divides b & b divides c

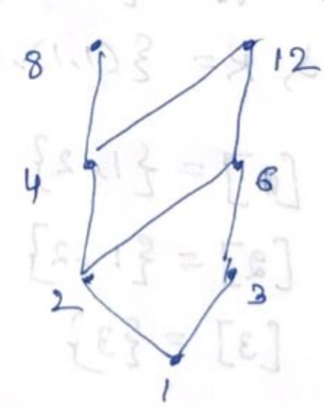
$\Rightarrow$ a divides c.

$\Rightarrow (a,c) \in R$.

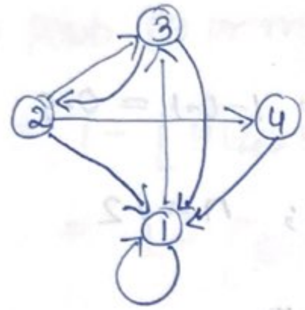
$\therefore$ R is anti-symmetric.

$\therefore$ R is a POSET

Hasse Diagram.



6 b.



vertex	In-deg	Out-deg
1	4	2
2	1	3
3	2	2
4	1	1

6c. Let (A, R) be a poset. This POSET is called a lattice if every two-element subset of A has a least ~~bound~~ upper bound and a greatest lower bound in A.

7a (i) The prob dist is valid if $p(x) \geq 0$ & $\Sigma p(x) = 1$

$\Rightarrow K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$\Rightarrow K = 1/49$

(ii) $P(x \geq 5) = P(5) + P(6) = 24/49$

$P(x < 4) = 1/49 + 3/49 + 5/49 + 7/49 = 16/49$

$P(3 < x \leq 6) = P(4) + P(5) + P(6) = 33/49$

(iii) $P(x \leq 2) = P(0) + P(1) = K + 3K = 4K$

$P(x \leq 2) > 0.3 \Rightarrow 4K > 0.3$

$\Rightarrow K > \frac{3}{40}$

7b Prob of a def pen $= 1/10 = 0.1$

— u — non-def pen $= q = 1-p = 1-0.1 = 0.9$

We have $p(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$; $n = 12$

(i) $P(x=2) = {}^{12}C_{2} (0.1)^2 (0.9)^{10}$

$= 0.2301$

(ii) $P(x \geq 2) = 1 - P(x<2) = 1 - [P(0) + P(1)]$

$= 1 - \left[{}^{12}C_0 (0.1)^0 (0.9)^{12} + {}^{12}C_1 (0.1)^1 (0.9)^{11} \right]$

$= 0.341$

(iii) Prob(no def) $= P(x=0)$

$= {}^{12}C_0 (0.1)^0 (0.9)^{12}$

$= 0.2824$

Q8 a) $p =$ prob(def fuse) $= 2/100 = 0.02$

$m = np = 200 \times 0.02 = 4$

$$P(x) = \frac{m^x e^{-m}}{x!} = \frac{4^x e^{-4}}{x!}$$

(i) Prob $(x=0) = \dfrac{4^0 e^{-4}}{0!} = e^{-4}$

(ii) prob (3 or more def fuse)

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[0.0183 + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!}\right]$

$= 0.7621$

(iii) $\text{Prob}(x \geq 1) = 1 - P(x < 1) = 1 - P(0)$

$= 1 - 0.0183 = 0.9817$

b) We have $f(x) = \alpha e^{-\alpha x}, \; x > 0$

Mean $= 1/\alpha = 3 \Rightarrow \alpha = 1/3$

(i) $P(x < 3) = \int_0^3 f(x)\,dx = \int_0^3 \frac{1}{3} e^{-x/3}\,dx$

$= \frac{1}{3}\left[\frac{e^{-x/3}}{-1/3}\right]_0^3 = -\left[e^{-x/3}\right]_0^3 = 0.6321$

(ii) $P(3 < x < 5) = \int_3^5 \frac{1}{3} e^{-x/3}\,dx = -\left[e^{-x/3}\right]_3^5$

$= -\left[e^{-5/3} - e^{-1}\right] = 0.179$

c $f(x) \geq 0$ if $c \geq 0$

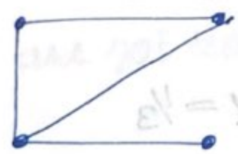
Also $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{\infty} Cx^2\,dx = 1$

$\Rightarrow c\left[\frac{x^3}{3}\right]_0^3 = 1 \Rightarrow c = \frac{1}{9}$

$P(1 < x < 2) = \int_1^2 f(x)\,dx = \int_1^2 \frac{x^2}{9}\,dx = \left[\frac{x^3}{27}\right]_1^2 = \frac{7}{27}$

Q9. a) (i) Simple Graph - Simple graph is a graph that has no multiple edges, parallel edges and loops.

Eg:-



(ii) Complete graph : It is a simple graph with minimum 2 vertices that has an edge between every pair of vertices.

Eg:-

(iii) Bipartite graph: A graph $G(V,E)$ where V has two non-empty, disjoint subsets V_1 & V_2 such that $V_1 \cup V_2 = V$ and every edge in the graph has one end vertex in V_1 & other in V_2.

Eg:-

$V = \{A, B, C, D, E\}$

$V_1 = \{A, B, C\}$

$V_2 = \{D, E\}$

b. Both the graphs G and H have 5 vertices.

______ " ______ have 6 edges.

But, graph H has a vertex 'a' of degree 4 and graph G has no vertex of degree 4.

$\therefore$ G and H are not isomorphic to each other.

10. a Let
$|E| = m = 10$, $|V| = n$.

From hand-shaking property, $\sum deg(v_i) = 2|E|$

$$2(4) + (n-2)3 = 2(10)$$

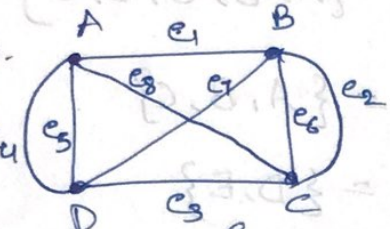
$$8 + 3n - 6 = 20$$

$$3n = 18$$

$$n = 6$$

b. (i) Euler's graph - Euler's graph is a graph that contains Eulers circuit. It is a graph that contains a circuit that has all the edges present in the graph.

Eg:-

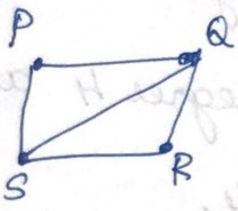


G

$A e_1 B e_6 C e_2 B e_7 D e_3 C e_8 A e_4 D e_5 A$ is an Euler circuit

$\therefore$ G is an Euler circuit.

(ii) Hamilton graph :- is a graph containing a cycle that has all the vertices of the graph.



G

PQRSP is a Hamilton cycle.

$\therefore$ G is a Hamilton graph.

3a) Laws of Logic

Here T_0 denotes tautology & F_0 denotes contradiction

1. Law of double negation

For any proposition p, $\neg(\neg p) \Leftrightarrow p$

2. Idempotent laws

For any proposition p,

(a) $(p \vee p) \Leftrightarrow p$ (b) $(p \wedge p) \Leftrightarrow p$

3. Identity Laws

For any proposition p,

(a) $(p \vee F_0) \Leftrightarrow p$ (b) $(p \wedge T_0) \Leftrightarrow p$

4. Inverse Laws

For any proposition p,

(a) $(p \vee \neg p) \Leftrightarrow T_0$ (b) $(p \wedge \neg p) \Leftrightarrow F_0$

5. Domination Laws

For any propn p,

(a) $(p \vee T_0) \Leftrightarrow T_0$ (b) $(p \wedge F_0) \Leftrightarrow F_0$

6. Commutative Laws

For any 2 propns p & q.

(a) $(p \vee q) \Leftrightarrow (q \vee p)$ (b) $(p \wedge q) \Leftrightarrow (q \wedge p)$

7. Absorption Laws

For any propns p & q,

(a) $[p \vee (p \wedge q)] \Leftrightarrow p$ (b) $[p \wedge (p \vee q)] \Leftrightarrow p$

8. Demorgan's Laws

(a) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ (b) $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

9. Associative Laws

(a) $p \vee (q \vee r) = (p \vee q) \vee r$ (b) $p \wedge (q \wedge r) = (p \wedge q) \wedge r$

10. <u>Distributive Laws</u>

For any proposition p, q, r.

(a) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

(b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Prove with the aid of truth tables.

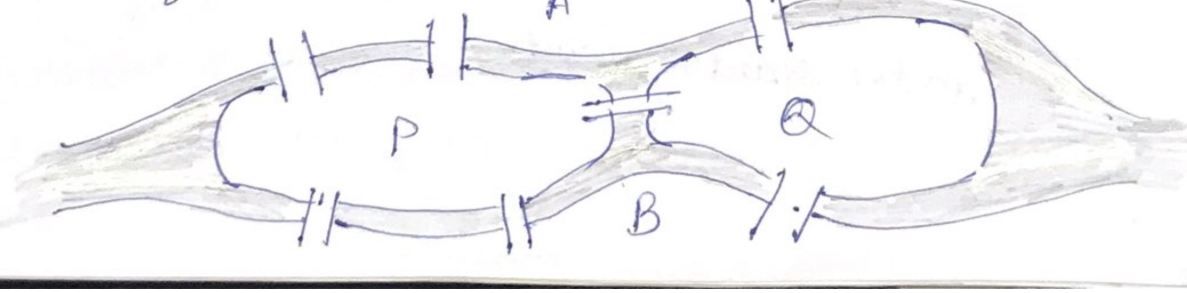
Laws for the <u>Negation of a conditional.</u>

9c

The Köonigsberg Bridge Problem

In 18th century, the city of Konigsberg in Prussia was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. The problem was, by starting at any of the four land areas, can we return to that area after crossing each of the seven bridges exactly once?

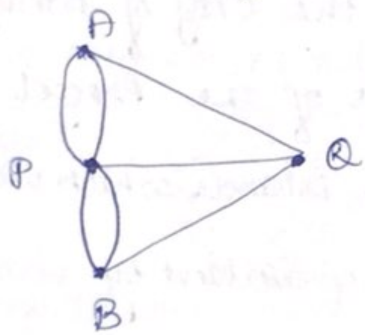
[four land areas – two of these parts are the banks of the river and two are islands]



This is the starting point for the development of graph theory. In 1736, Euler analyzed this problem with the help of a graph and gave the sol?. This problem is known as Konigsberg Bridge Problem.

Let the land areas be denoted by A, B, P, Q. A, B are banks of the river & P, Q are islands.

Treat four land areas as four vertices and 7 bridges as 7 edges. So, we get the graph –



We note that, in this graph $\deg(A) = 3$

$\deg(B) = 3$, $\deg(P) = 5$, $\deg(Q) = 3$

which are not even. ∴ graph doesn't have an Euler circuit. ∴ It is not possible to walk over each of the seven bridges exactly once and return to the starting point.

a

Q10 c) First, let us rename the vertices as indicated below:

a:1, b:2, c:3, d:4, e:5, f:6, g:7

First Iteration:

Now let $P=\{1\}$ and $t_j = q_{1j}$ for $j=2,3,4,5,6,7$

From Fig., $t_2 = 14$, $t_3=\infty$, $t_4=\infty$, $t_5=\infty$, $t_6=10$, $t_7=17$

Step1: t_j is minimum for $j=6$, the minimum being $t_6=10$.

$\therefore$ we label the arc $(1,6)$ as p_1. Also, we adjoin 6 to P so that $P=\{1,6\}$ for the next step.

Step2: We have $P=\{1,6\}$ and $t_6=10$.

new $t_2 = \min\{t_2, t_6+q_{62}\} = \min\{14, 10+3\} = 13$

new $t_3 = \min\{t_3, t_6+q_{63}\} = \min\{\infty, 10+\infty\} = \infty$

new $t_4 = \min\{t_4, t_6+q_{64}\} = \min\{\infty, 10+\infty\} = \infty$

new $t_5 = \min\{t_5, t_6+q_{65}\} = \min\{\infty, 10+4\} = 14$

new $t_7 = \min\{t_7, t_6 + q_{67}\} = \min\{17, 10+6\} = 16$

Second Iteration:

For this iteration, $P = \{1, 6\}$ and $t_6 = 10$. Also $t_2 = 13$, $t_3 = \infty$, $t_4 = \infty$, $t_5 = 14$, $t_7 = 16$

Step1: Among new t_j's, t_j is minimum for $j = 2$, the minimum being $t_2 = 13$. Among the arcs from the vertices in P to the vertex 2, the arc (6, 2) has the least weight. Let us label this arc (6, 2) as p_2. Also, we adjoin 2 to P so that $P = \{1, 6, 2\}$ for the next step.

Step 2: We have $P = \{1, 6, 2\}$, $t_6 = 10$, $t_2 = 13$.

New $t_3 = \min\{t_3, t_2 + q_{23}\} = \min\{\infty, 13+9\} = 22$

New $t_4 = \min\{t_4, t_2 + q_{24}\} = \min\{\infty, 13+\infty\} = \infty$

new $t_5 = \min\{t_5, t_2 + q_{25}\} = \min\{14, 13+\infty\} = 14$

new $t_7 = \min\{t_7, t_2 + q_{27}\} = \min\{16, 13+\infty\} = 16$

Third Iteration:

For this iteration, $P = \{1, 6, 2\}$, $t_6 = 10$, $t_2 = 13$.

Also, $t_3 = 22$, $t_4 = \infty$, $t_5 = 14$, $t_7 = 16$

Step1: Among new t_j's, t_5 is minimum for $j = 5$. Among the arcs from the vertices in P to the vertex 5, the arc (6, 5) has the least weight. Label this arc (6, 5) as p_3. Also, we adjoin 5 to P so that $P = \{1, 6, 2, 5\}$.

Step2: We have $P = \{1, 6, 2, 5\}$, $t_6 = 10$, $t_2 = 13$, $t_5 = 14$.

We choose new $t_3 = \min\{t_3, t_5 + q_{53}\} = \min\{22, 14+\infty\} = 22$

new $t_4 = 21$, new $t_7 = 15$

Fourth Iteration

Now $P = \{1, 6, 2, 5\}$, $t_6 = 10$, $t_2 = 13$, $t_5 = 14$.

Also, $t_3 = 22$, $t_4 = 21$, $t_7 = 15$.

Step1: Among new t_j's, $t_7 = 15$ is minimum for $j = 7$.

Among the arcs from the vertices in P to the vertex 7, the arc (5,7) has the least weight. Let us label this arc (5,7) as P_4. Also, we adjoin 7 to P so that $P = \{1, 6, 2, 5, 7\}$ for the next step.

Step2: We have $P = \{1, 6, 2, 5, 7\}$, $t_6 = 10$, $t_2 = 13$, $t_5 = 14$,

$t_7 = 15$.

new $t_3 = \min\{t_3, t_7 + q_{73}\} = \min\{22, 15+\infty\} = 22$

new $t_4 = \min\{t_4, t_7 + q_{74}\} = \min\{21, 15+\infty\} = 21$

Fifth Iteration

Now $P = \{1, 6, 2, 5, 7\}$, $t_6 = 10$, $t_2 = 13$, $t_5 = 14$, $t_7 = 15$. Also,

$t_3 = 22$, $t_4 = 21$.

Among new t_j's, $t_4 = 21$ is minimum for $j = 4$.

Among the arcs from the vertices in P to the vertex 4, the arc $(5,4)$ has the least weight. Let us label this arc $(5,4)$ as P_5. Also add 4 to P for the next step.

Final Step: At this stage, the vertex left over in P is 3.

Among the arcs from the vertices in P to the vertex 3, the arc $(4,3)$ has the least weight. Let us label this arc $(4,3)$ as P_6.

Further we adjoin 3 to P, so that $P = \{1, 6, 2, 5, 7, 4, 3\}$. Now P covers all vertices. ∴ stop the process.

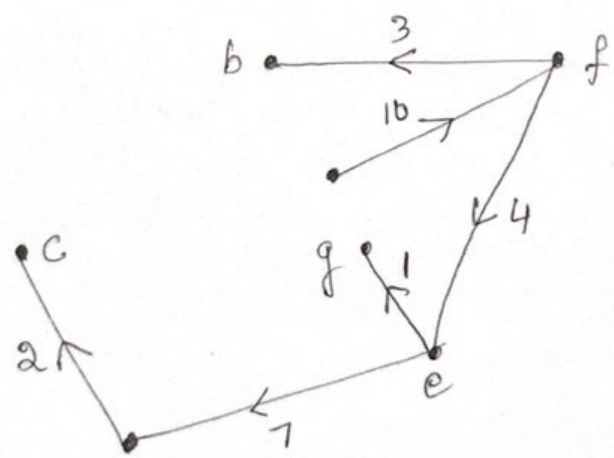
The arcs that we have labeled as $P_1, P_2, \cdots P_6$ are

$(1,6), (6,2), (6,5), (5,7), (5,4), (4,3)$

From the fig, these arcs are

$(a,f), (f,b), (f,e), (e,g), (e,d), (d,c)$

The shortest path arborescence formed by these arcs is :



From the above fig,

following are the shortest paths from a to other vertices:

(1) Path from a to f: $a \rightarrow f$; distance 10.

(2) Path from a to b: $a \rightarrow f \rightarrow b$; distance 13.

(3) — " — a to e: $a \rightarrow f \rightarrow e$; — " — 14

(4) — " — a to g: $a \rightarrow f \rightarrow e \rightarrow g$; — " — 15

(5) — " — a to d: $a \rightarrow f \rightarrow e \rightarrow d$; — " — 21

(6) — " — a to c: $a \rightarrow f \rightarrow e \rightarrow d \rightarrow c$; — " — 23