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Internal Assessment Test I – January 2023

Sub:	Engineering Mathematics-I					Sub Code:	22MATS11 / 22MATE11	
Date:	19/01/2023	Duration:	90 mins	Max Marks:	50	Sem / Sec:	I / I to P (CHE CYCLE)	OBE

	Question 1 is compulsory and answer any SIX questions from the rest.	MARKS	CO	RBT
1.	With usual notations prove that for the curve $r = f(\theta)$, $$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$	[08]	CO1	L3
2.	Find the angle between the curves, $r^2 sin2\theta = 4 \ \ and \ r^2 = 16 sin2\theta$.	[07]	CO1	L3
3.	Find the pedal equation of the curve $r^2 = a^2 sec\, 2\theta$	[07]	CO1	L3
4.	Find the rank of the matrix: $\boldsymbol{A} = \begin{bmatrix} \mathbf{2} & -\mathbf{1} & -\mathbf{3} & -\mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{3} & -\mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & -\mathbf{1} \end{bmatrix}$	[07]	CO4	L3

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5.	Investigate the values of λ and μ such that the following system of equations may have a) Unique solutions b) Infinite solutions c) no solutions $\boldsymbol{x+y+z=6}$ $\boldsymbol{x+2y+5z=10}$ $\boldsymbol{2x+3y+\lambda z=\mu}$	[07]	**CO4**	**L3**
6.	Find the values of x, y and z by applying Gauss–Jordan Method: $\boldsymbol{x+2y+z=3}$ $\boldsymbol{2x+3y+3z=10}$ $\boldsymbol{3x-y+2z=13}$	[07]	**CO4**	**L3**
7.	Employ Gauss–Seidel method to solve. $\boldsymbol{5x+2y+z=12}$ $\boldsymbol{x+4y+2z=15}$ $\boldsymbol{x+2y+5z=20}$ Perform 3 iterations by taking initial approximation to the solution as (1, 0, 3)	[07]	**CO4**	**L3**
8.	Find the numerically largest eigenvalue and the corresponding eigenvector of the matrix $\boldsymbol{A=\begin{pmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{pmatrix}}$ by Rayleigh power method taking the initial vector as $[1, 0.8, -0.8]^T$ **(perform 4 iterations)**.	[07]	**CO4**	**L3**

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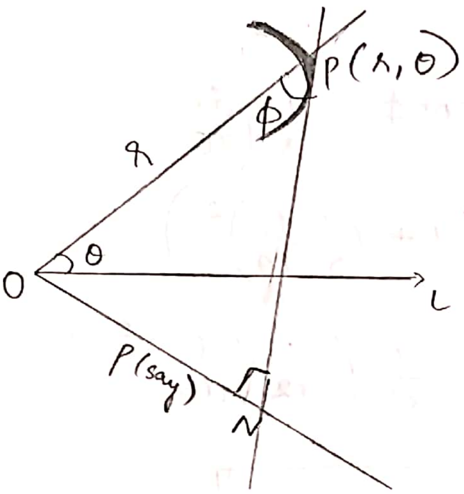
1. **Proof** :



Take the initial line OL.

Let $p(r, \theta)$ be any point on the curve. $r = f(\theta)$
such that $OP = r$ i.e, the radius vector.
and $\angle LOP = \theta$ is the angle between the radius vector and the x-axis.
Let the angle between the radius vector and tangent be ϕ.
Draw a tangent to the curve such that the tangent is perpendicular to the line ON.

$\Rightarrow \quad ON \perp PN \quad \Rightarrow \quad \angle PNO = 90^\circ$.

Now, In $\triangle OPN$,

$$\sin \phi = \frac{p}{r}. \qquad \left(\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} \right)$$

$$\Rightarrow \boxed{p = r \sin \phi}. \quad \text{— (1)}$$

Squaring eq. (1) we get,

$$p^2 = r^2 \sin^2 \phi$$

~~Reposi~~ Reciprocate the above eqn. we get.

$$\frac{1}{p^2} = \frac{1}{r^2 \sin^2 \phi}$$

we know, $\csc^2\theta = 1 + \cot^2\theta$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{r^2}\csc^2\phi$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left(1+\cot^2\phi\right)$$

we know that, $\cot\phi = \frac{1}{r}\cdot\frac{dr}{d\theta}$.

so, $$\frac{1}{p^2} = \frac{1}{r^2}\left(1+\left(\frac{1}{r}\cdot\frac{dr}{d\theta}\right)^2\right)$$

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^2}\left(\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right)$$

$$\Rightarrow \boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2}$$

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2. $r^2 \sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$.

Taking log on both sides,

$\log(r^2 \sin 2\theta) = \log(4)$

$2\log r + \log\sin 2\theta = \log 4$

Differentiating on both sides, w.r.t θ,

$\frac{2}{r}\cdot\frac{dr}{d\theta} + \frac{1}{\sin 2\theta}\cdot\cos 2\theta\cdot 2 = 0$

$\cancel{2}\cdot\frac{1}{r}\frac{dr}{d\theta} = -\cancel{2}\cdot\cot 2\theta$.

we know that, $\frac{1}{r}\cdot\frac{dr}{d\theta} = \cot\phi$

$\Rightarrow \cot\phi_1 = -\cot 2\theta$

$\boxed{\phi_1 = -2\theta}$

Taking log on both sides,

$2\log r = \log 16 + \log\sin 2\theta$.

Differentiating w.r.t θ on both sides

$\frac{\cancel{2}}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\sin 2\theta}\cdot\cos 2\theta\cdot\cancel{2}$

$\frac{1}{r}\frac{dr}{d\theta} = \cot 2\theta$

we know, $\frac{1}{r}\frac{dr}{d\theta} = \cot\phi$

$\Rightarrow \cot\phi_2 = \cot 2\theta$

$\boxed{\phi_2 = 2\theta}$

Angle between two curves is $= |\phi_2 - \phi_1|$

$$= |2\theta - (-2\theta)| = \underline{\underline{4\theta}}$$

Given $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$

$$\Rightarrow r^2 = \frac{4}{\sin 2\theta} \qquad r^2 = 16 \sin 2\theta$$

equating both the equations,

$$\frac{\cancel{4}}{\sin 2\theta} = \cancel{16}^{\,4} \sin 2\theta .$$

$$\sin^2 2\theta = \frac{1}{4}$$

$$\sin 2\theta = \frac{1}{2} .$$

$$2\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{12} .$$

$\therefore$ Angle between two curves is $4\theta = \cancel{4} \times \frac{\pi}{\cancel{12}\,3} = \frac{\pi}{3}$

$$\boxed{|\phi_2 - \phi_1| = \frac{\pi}{3}}$$

---

3. $r^2 = a^2 \sec 2\theta$

Taking log on both sides,

$$2 \log r = \log a^2 + \log \sec 2\theta$$

Differentiating w.r.t θ on both sides

$$\frac{\cancel{2}}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\cancel{\sec 2\theta}} \cdot (\cancel{\sec 2\theta} \tan 2\theta) \cdot \cancel{2}$$

$$\frac{1}{r} \cdot \frac{dr}{d\theta} = \tan 2\theta$$

$\Rightarrow \cot\phi = \tan 2\theta$

$\Rightarrow \cot\phi = \cot\left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow \boxed{\phi = \frac{\pi}{2} - 2\theta}$ — ①

~~Given~~
we know that,

$$p = r\sin\phi$$

Hence, $p = r\sin\left(\frac{\pi}{2} - 2\theta\right)$

$$\boxed{p = r\cos 2\theta} \quad — ②$$

Given $r^2 = a^2 \sec 2\theta$

$$\frac{r^2}{a^2} = \sec 2\theta$$

$$\Rightarrow \boxed{\cos 2\theta = \frac{a^2}{r^2}} \quad — ③$$

Substitute $(\cos 2\theta)$ value in eq. ②, we get

$$p = r\left(\frac{a^2}{r^2}\right)$$

$$\Rightarrow \boxed{p = \frac{a^2}{r}}$$

5. $x + y + z = 6$

$x + 2y + 5z = 10$

$2x + 3y + \lambda z = \mu$

$$[A : B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 5 & 10 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_2 \to R_2 - R_1 \quad , \quad R_3 \to R_3 - 2R_1$

$$\Rightarrow \cancel{\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 5 & 10 \\ 2 & & & \end{array}\right.}$$

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 4 \\ 0 & 1 & \lambda-2 & \mu-12 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & \lambda-6 & \mu-16 \end{array}\right]$$

a) For unique solution, $\rho(A) = \rho(A : B) = r = n$.

Here $\underline{n = 3}$.

$r = 3$

For rank to be 3,

$\lambda - 6 \neq 0$

$\boxed{\lambda \neq 6 \text{ and } \mu \text{ can have any value}}$

b) For infinite solution,

$$\rho(A) = \rho(A : B) = r < n.$$

For this condition to satisfy,

$$\lambda - 6 = 0 \quad \text{and} \quad \mu - 16 \cancel{\neq} 0$$

$$\boxed{\lambda = 6 \quad \text{and} \quad \mu = 16}$$

c) For no solution,

$$\rho(A) \neq \rho(A : B).$$

For this condition to satisfy,

$$\lambda - 6 = 0 \quad \text{and} \quad \mu - 16 \neq 0.$$

$$\boxed{\lambda = 6 \quad \text{and} \quad \mu \neq 16}$$

7. Gauss - Seidel.

$$5x + 2y + z = 12$$

$$x + 4y + 2z = 15$$

$$x + 2y + 5z = 20.$$

Since the given equations are diagonally dominant, we write the equations in terms of x, y, z.

$$x = \frac{1}{5}\left[12 - 2y - z\right] \quad \text{——} (1)$$

$$y = \frac{1}{4}\left[15 - x - 2z\right] \quad \text{——} (2)$$

$$z = \frac{1}{5}\left[20 - x - 2y\right] \quad \text{——} (3)$$

Let us assume, $\boxed{x=1,\ y=0,\ z=3}$

1st iteration

$$x^{(1)} = \frac{1}{5}\left[12-2(0)-3\right] = 1.8$$

$$y^{(1)} = \frac{1}{4}\left[15-1.8-2(3)\right] = 1.8$$

$$z^{(1)} = \frac{1}{5}\left[20-1.8-2(1.8)\right] = 2.92$$

2nd iteration

$$x^{(2)} = \frac{1}{5}\left[12-2(1.8)-2.92\right] = 1.096$$

$$y^{(2)} = \frac{1}{4}\left[15-1.096-2(2.92)\right] = 2.016$$

$$z^{(2)} = \frac{1}{5}\left[20-1.096-2(2.016)\right] = 2.9744$$

3rd iteration

$$x^{(3)} = \frac{1}{5}\left[12-2(2.016)-2.9744\right] = 0.9987$$

$$y^{(3)} = \frac{1}{4}\left[15-0.9987-2(2.9744)\right] = 2.0131$$

$$z^{(3)} = \frac{1}{5}\left[20-0.9987-2(2.0131)\right] = 2.9950$$

Hence, the solution to the given equations is

$$\boxed{x = 0.9987,\ y = 2.0131,\ z = 2.9950}$$

8. $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ $\quad X = \begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix}$

$$AX^{(0)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix} = \begin{bmatrix} 5.6 \\ 5.2 \\ -5.2 \end{bmatrix} = 5.6 \begin{bmatrix} 1 \\ 0.9285 \\ -0.9285 \end{bmatrix}$$

$$AX^{(1)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.9285 \\ -0.9285 \end{bmatrix} = \begin{bmatrix} 5.857 \\ 5.714 \\ -5.714 \end{bmatrix} = 5.857 \begin{bmatrix} 1 \\ 0.9755 \\ -0.9755 \end{bmatrix}$$

$$AX^{(2)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.9755 \\ -0.9755 \end{bmatrix} = \begin{bmatrix} 5.951 \\ 5.902 \\ -5.902 \end{bmatrix} = 5.951 \begin{bmatrix} 1 \\ 0.9917 \\ -0.9917 \end{bmatrix}$$

$$AX^{(3)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.9917 \\ -0.9917 \end{bmatrix} = \begin{bmatrix} 5.9834 \\ 5.9668 \\ -5.9668 \end{bmatrix} = 5.9834 \begin{bmatrix} 1 \\ 0.9972 \\ -0.9972 \end{bmatrix}$$

$$AX^{(4)} = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.9972 \\ -0.9972 \end{bmatrix} = \begin{bmatrix} 5.9944 \\ 5.9888 \\ -5.9888 \end{bmatrix} = 5.9944 \begin{bmatrix} 1 \\ 0.999 \\ -0.999 \end{bmatrix}$$

$\therefore$ The largest Eigen value is ~~5.[illegible]~~ 6

and the corresponding Eigen vector is $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

6. $x + 2y + z = 3$

$2x + 3y + 3z = 10$

$3x - y + 2z = 13$

$$[A : B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{array}\right]$$

$R_1 \to R_1 + 2R_2$, $R_3 \to R_3 - 7R_2$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 3 & 11 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$R_1 \to 8R_1 + 3R_3$, $R_2 \to 8R_2 + R_3$

$$\Rightarrow \left[\begin{array}{ccc|c} 8 & 0 & 0 & 16 \\ 0 & -8 & 0 & 8 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$-8z = -24$ —— ①

$\boxed{z = 3}$

$-8y = 8$ —— ②

$\boxed{y = -1}$

$8x = 16$

$\boxed{x = 2}$ —— ③

Hence the required solution is $x = 2$, $y = -1$, $z = 3$

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