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CMRIT

Internal Assessment Test II – March 2023

Sub:	Engineering Mathematics-I					Sub Code:	22MATS11	
Date:	02/03/2023	Duration:	90 mins	Max Marks:	50	Sem / Sec:	I / A to G (PHY CYCLE)	OBE

Question 1 is compulsory and answer any SIX questions from the rest.

		MARKS	CO	RBT
1.	Expand $\log(1+\cos x)$ by Maclaurin's series up to the term containing x^6	[08]	CO2	L3
2.	Evaluate: (i) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$ (ii) $\lim_{x\to 0}(\cot x)^{1/\log x}$	[07]	CO2	L3
3.	Find the extreme values of the function $f(x,y) = 2(x^2 - y^2) - x^4 + y^4$	[07]	CO2	L3
4.	Show that for the curve $r = a(1-\cos\theta)$ is ρ^2/r = constant.	[07]	CO1	L3

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5.	If $u = \log(\tan x + \tan y + \tan z)$ show that, $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$.	[07]	CO2	L3
6.	Find the radius of curvature of $a^2y = x^3 - a^3$, at the point where **curve meets x-axis**	[07]	CO1	L3
7.	Solve: $\{4x^3y^2 + y\cos(xy)\}dx + \{2x^4y + x\cos(xy)\}dy = 0$	[07]	CO3	L3
8.	If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, then find the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$	[07]	CO2	L3

Internal Assessment Test – II

1) $y(x) = \log(1+\cos x)$ $\qquad y(0) = \log 2$

$y_1(x) = \dfrac{-\sin x}{1+\cos x}$

$= \dfrac{-2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} \Rightarrow -\tan\frac{x}{2}$, $y_1(0) = 0$

$y_2(x) = -\sec^2\frac{x}{2}\cdot\frac{1}{2}$, $y_2(0) = -\frac{1}{2}$

$y_3(x) = -\frac{1}{2}\left(2\sec^2\frac{x}{2}\cdot\tan\frac{x}{2}\right)\cdot\frac{1}{2}$

$= -y_1(x)\cdot y_2(x)$, $y_3(0) = 0$

$y_4(x) = -(y_2(x)\cdot y_2(x) + y_1(x)\cdot y_3(x))$

$= -(y_2^2(x) + y_1(x)\cdot y_3(x))$, $y_4(0) = -\frac{1}{4}$

$y_5(x) = -(2y_2(x)\cdot y_3(x) + y_2(x)\cdot y_3(x) + y_1(x)\cdot y_4(x))$

$= -(3y_2(x)\cdot y_3(x) + y_1(x)\cdot y_4(x))$

$y_5(0) = 0$

$$y_6(x) = -\left(3\left((y_3(x))^2 + y_2(x)\cdot y_4(x)\right) + 4\,\cancel{y_2 x_4}\left(y_2(x)\cdot y_4(x) + y_1(x)\cdot y_5(x)\right)\right)$$

$$y_6(0) = -\left(3\left(-\frac{1}{2}\cdot -\frac{1}{4}\right) + \left(-\frac{1}{2}\cdot -\frac{1}{4}\right)\right)$$

$$= -\left(3\left(\frac{1}{8}\right) + \frac{1}{8}\right)$$

$$\Rightarrow -4\left(\frac{1}{8}\right) = -\frac{1}{2}$$

$$y(x) = y(0) + y_1(0)x + y_2(0)\frac{x^2}{2!} + y_3(0)\frac{x^3}{3!} + y_4(0)\frac{x^4}{4!} + y_5(0)\frac{x^5}{5!} + y_6(0)\frac{x^6}{6!}$$

$$\Rightarrow \log(1+\cos x) = \log 2 + 0 - \frac{1}{2}\cdot\frac{x^2}{2!} + 0 - \frac{1}{4}\cdot\frac{x^4}{4!} + 0 - \frac{1}{2}\cdot\frac{x^6}{6!}$$

$$\Rightarrow \log(1+\cos x) = \log 2 - \frac{x^2}{4} - \frac{x^4}{96} - \frac{x^6}{1440}$$

2)

i) $\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{1/x^2} = (1)^{\infty}$

taking log on both sides,

$\Rightarrow \log K = \lim_{x \to 0} \frac{\log\left(\frac{\sin x}{x}\right)}{x^2} = \frac{0}{0}$ (K is the limit)

LH Rule,

$= \lim_{x \to 0} \frac{\frac{x}{\sin x} \cdot \frac{x\cos x - \sin x}{x^2}}{2x}$

$= \lim_{x \to 0} \frac{x}{\sin x} \cdot \lim_{x \to 0} \frac{x\cos x - \sin x}{2x^3}$

$\left[\lim_{x \to 0} \frac{\sin x}{x} = 1\right]$

$= (1) \cdot \lim_{x \to 0} \frac{x\cos x - \sin x}{2x^3} = \frac{0}{0}$

LH Rule,

$= \lim_{x \to 0} \frac{\cancel{\cos x} - x\sin x \cancel{- \cos x}}{6x^{\cancel{2}}} = \frac{-\sin x}{6x} = \frac{0}{0}$

LH Rule,

$= \cancel{\lim_{x \to 0}} \cancel{\frac{(x\cos x + \sin x)}{}}$

$= \lim_{x \to 0} \frac{-\cos x}{6} = -\frac{1}{6}$

$\Rightarrow \log K = -\frac{1}{6}$

$$K = e^{-\frac{1}{6}} = \frac{1}{e^{1/6}}$$

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iii) $\lim_{x\to 0} (\cot x)^{1/\log x} = (\infty)^0$

taking log on both sides,

(K is the limit)

$\Rightarrow \log K = \frac{1}{\log} \lim_{x\to 0} \frac{1}{\log x} \cdot \log \cot x = \frac{\infty}{\infty}$

LH Rule, $\Rightarrow \lim_{x\to 0} \frac{\frac{-\csc^2 x}{\cot x}}{\frac{1}{x}} = \frac{-x \csc^2 x}{\cot x} = \frac{0}{0}$

LH Rule,

$$= \lim_{x\to 0} \frac{\cancel{-}(x(-2\cancel{\csc^2} x \cdot \cot x) + \cancel{\csc^2 x})}{\cancel{-\csc^2 x}}$$

$$= \lim_{x\to 0} -2x \cdot \cot x + 1$$

$$= \lim_{x\to 0} \frac{-2x}{\tan x} + \lim_{x\to 0} (1) = \frac{0}{0} + 1$$

L-H Rule on one side

$\Rightarrow$

$$= \lim_{x\to 0} \frac{-2}{\sec^2 x} + \lim_{x\to 0} (1)$$

$$= -2 + 1 = -1$$

$\Rightarrow \log K = -1$

$K = e^{-1} = \frac{1}{e}$

4) $r = a(1-\cos\theta)$

taking log on both side

$\log r = \log a + \log(1-\cos\theta)$

diff wrt θ

$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\sin\theta}{1-\cos\theta}$

$= \cot\phi = \frac{\cancel{2}\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{\cancel{2}\sin^2\frac{\theta}{2}} = \cot\frac{\theta}{2}$

$\Rightarrow \phi = \frac{\theta}{2}$

$\Rightarrow p = r\sin\phi$

$\Rightarrow p = r\sin\frac{\theta}{2}$ --- eq (i)

From the question :- $r = a(1-\cos\theta)$

$= (1-\cos\theta) = \frac{r}{a}$

$2\sin^2\frac{\theta}{2} = \frac{r}{a}$

$\sin\frac{\theta}{2} = \sqrt{\frac{r}{2a}}$

Substituting $\sin\frac{\theta}{2}$ in eq 1

$$\Rightarrow \quad p = r \cdot \sqrt{\frac{r}{2a}}$$

$$\Rightarrow \quad p^2 = \frac{r^3}{2a}$$

diff wrt p

$$\Rightarrow \quad 2p = \frac{3r^2}{2a}\frac{dr}{dp}$$

$$\Rightarrow \quad r\frac{dr}{dp} = \frac{4ap}{3r}$$

$$\left[r\frac{dr}{dp} = \rho \right]$$

$$\Rightarrow \quad \rho = \frac{4ap}{3r}$$

$$\left[p = r\sqrt{\frac{r}{2a}} \right]$$

$$= \frac{4a}{3\cancel{r}} \cdot \cancel{r}\sqrt{\frac{r}{2a}}$$

$$= \frac{2 \cdot \sqrt{2} \cdot \cancel{\sqrt{2}} \cdot \cancel{\sqrt{a}} \cdot \sqrt{a}}{3} \cdot \frac{\sqrt{r}}{\cancel{\sqrt{2a}}}$$

$$\rho = \frac{2}{3}\sqrt{2ar}$$

$$\frac{\rho^2}{r} = \frac{\left(\frac{2}{3}\right)^2 \cdot 2a\cancel{r}}{\cancel{r}} \Rightarrow \frac{8a}{9} = \text{constant}$$

$\therefore$ Proved

5) $u = \log(\tan x + \tan y + \tan z)$

$$\frac{\partial u}{\partial x} = \frac{1}{\tan x + \tan y + \tan z} \cdot \sec^2 x$$

$$\frac{\partial u}{\partial y} = \frac{1}{\tan x + \tan y + \tan z} \cdot \sec^2 y$$

$$\frac{\partial u}{\partial z} = \frac{1}{\tan x + \tan y + \tan z} \cdot \sec^2 z$$

$$\Rightarrow \sin 2x \cdot \cancel{\sec^2 x}\, U_x + \sin 2y \cdot U_y + \sin 2z \cdot U_z$$

$$\Rightarrow \frac{\sin 2x \cdot \sec^2 x}{\tan x + \tan y + \tan z} + \frac{\sin 2y \cdot \sec^2 y}{\tan x + \tan y + \tan z} + \frac{\sin 2z \cdot \sin^2 z}{\tan x + \tan y + \tan z}$$

$\Rightarrow \cancel{2\sin\theta}$ $\quad$ $\left[\sin 2\theta = 2\sin\theta \cdot \cos\theta\right.$

$$\Rightarrow \frac{2\sin x \cdot \cancel{\cos x} \cdot \sec^2 x + 2\sin y \cancel{\cos y} \cdot \sec^2 y + 2\sin z \cdot \cancel{\cos z} \cdot \sec^2 z}{\tan x + \tan y + \tan z}$$

$\left[\cos x \cdot \sec x =\right.$

$\left[\sin x \cdot \sec x = \tan x\right.$

$$\Rightarrow \frac{2\tan x + 2\tan y + 2\tan z}{\tan x + \tan y + \tan z}$$

$$\Rightarrow \frac{2\cancel{(\tan x + \tan y + \tan z)}}{\cancel{(\tan x + \tan y + \tan z)}} = 2 \quad \therefore \text{Proved}$$

6) $a^2 y = x^3 - a^3$ ... eq (1)

(x-axis = $(x, 0)$

Substituting $y=0$ in the equation 1

$\Rightarrow \quad 0 = x^3 - a^3$

$x^3 = a^3$

$x = a$

The given curve meets x-axis at $(a, 0)$

$$y = \frac{x^3 - a^3}{a^2} = \frac{x^3}{a^2} - \frac{a^{\cancel{3}}}{\cancel{a^2}} = \frac{x^3}{a^2} - a$$

diff wrt x,

$$y_1 = \frac{3x^2}{a^2} - 0 \quad .... \quad \text{eq (2)}$$

$(x=a)$, $\quad y_1 = \frac{3\cancel{a^2}}{\cancel{a^2}} = 3 \qquad y_1 = 3$

diff eq (2) wrt x

$$\Rightarrow y_2 = \frac{6x}{a^2}$$

$(x=a)$, $\quad y_2 = \frac{6\cancel{a}}{a^{\cancel{2}}} = \frac{6}{a} \qquad y_2 = \frac{6}{a}$

$$\Rightarrow \rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{(1+3^2)^{3/2}}{\frac{6}{a}}$$

$$= \frac{a(10)^{3/2}}{6}$$

$$= \frac{a\,\cancel{10}^{5}\sqrt{10}}{\cancel{6}\,3} = \frac{5a\sqrt{10}}{3}$$

7) $(4x^3y^2 + y\cos(xy))dx + (2x^4y + x\cos(xy))dy = 0$

$\Rightarrow Mdx + Ndy = 0$

$\Rightarrow M = 4x^3y^2 + y\cos(xy)$

$N = 2x^4y + x\cos(xy)$

$\Rightarrow M_y = 8x^3y + \cos(xy) - y\sin(xy).x$

$\Rightarrow N_x = 8x^3y + \cos(xy) - x\sin(xy).y$

$M_y = N_x$ $\therefore$ The equation is an exact differential equation

$$\Rightarrow \int M\,dx + \int N\,dy = C$$

$$\Rightarrow \int (4x^3y^2 + y(\cos(xy)))\,dx + \int 0\,dy = C$$

$$\left[\int x^n dx = \frac{x^{n+1}}{n+1}\right]$$

$$\Rightarrow \left[\frac{\cancel{4}x^4y^2}{\cancel{4}} + \frac{\cancel{y}\sin(xy)}{\cancel{y}}\right] = C$$

$$\Rightarrow x^4y^2 + \sin(xy) = C$$ solution of the equation

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8) $U = \frac{yz}{x}$, $V = \frac{zx}{y}$, $W = \frac{xy}{z}$

$U = \frac{yz}{x}$	$V = \frac{zx}{y}$	$W = \frac{xy}{z}$
$U_x = \frac{-yz}{x^2}$	$V_x = \frac{z}{y}$	$W_x = \frac{y}{z}$
$U_y = \frac{z}{x}$	$V_y = \frac{-zx}{y^2}$	$W_y = \frac{x}{z}$
$U_z = \frac{y}{x}$	$V_z = \frac{x}{y}$	$W_z = -\frac{xy}{z^2}$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$$

$$= \begin{vmatrix} -\frac{yz}{x^2} & \frac{z}{x} & \frac{y}{x} \\ \frac{z}{y} & -\frac{zx}{y^2} & \frac{x}{y} \\ \frac{y}{z} & \frac{x}{z} & -\frac{xy}{z^2} \end{vmatrix}$$

$$\Rightarrow -\frac{yz}{x^2}\left(\frac{\cancel{z}x}{y^{\cancel{2}}}\cdot\frac{x\cancel{y}}{z^{\cancel{2}}} - \frac{x^2}{zy}\right) + \frac{z}{x}\left(\frac{x\cancel{z}}{z} + \frac{x}{z}\right) + \frac{y}{x}\left(\frac{x}{y} + \frac{\cancel{y}}{\cancel{z}}\cdot\frac{\cancel{z}x}{y^{\cancel{2}}}\right)$$

$$\Rightarrow -\frac{yz}{x^2}\left(\frac{x^2}{zy} - \frac{x^2}{zy}\right) + \frac{\cancel{z}}{\cancel{x}}\left(\frac{2\cancel{x}}{\cancel{z}}\right) + \frac{\cancel{y}}{\cancel{x}}\left(\frac{2\cancel{x}}{\cancel{y}}\right)$$

$$\Rightarrow 0 + 2 + 2 \quad = \underline{\underline{4}}$$

As we know $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\log k = -1$

$k = e^{-1} = 1/e$ //

Q3) $f(x,y) = 2(x^2 - y^2) - x^4 + y^4$

Diff $f(x,y)$ w.r.t to x,

$f_x = 2(2x - 0) - 4x^3 + 0$

$= 4x - 4x^3$

Diff $f(x,y)$ w.r.t to y,

$f_y = 2(-2y) + 4y^3$

$= -4y + 4y^3$

Putting f_x & f_y as 0.

$0 = 4x - 4x^3$

$4x(1-x^2) = 0$

$x = 0$ or ± 1

$0 = 4y^3 - 4y$

$0 = 4y(y^2 - 1)$

$y = 0$ or ± 1

Therefore all ~~stai stas staiistory~~ point are

$(0,0), (0,1) (0,-1),$
$(1,0), (1,1), (1,-1)$
$(-1,0), (-1,1), (-1,-1)$

Diff f_x w.r.t x

$A = f_{xx} = 4 - 12x$

Diff f_y w.r.t y

$C = f_{yy} = -4 + 12y$

Diff f_y w.r.t x

$B = 0$

	(0,0)	(0,1)	(0,-1)	(1,0)	(1,1)	(1,-1)	(-1,0)	(-1,1)	(-1,-1)
$A = 4-12x$	4	4	4	-8	-8	-8	16	16	16
$B = 0$	0	0	0	0	0	0	0	0	0
$C = -4+12y$	-4	8	-16	-4	+8	-16	-4	8	-16
$AC - B^2$	-16	32	-64	32	-64	128	-64	128	-256
	Saddle point	Minima	Saddle point	Maxima	Saddle point	Maxima	Saddle point	Minima	Saddle point

Points of Minima = (0,1), (-1,1)

Points of Maxima = (1,0), (1,-1)

Maximum value at (~~0,1~~ 1,0) = ~~…~~ 1

Maximum value at (1,-1) = 0.

Minimum value at (0,1) = -1

Minimum value at (-1,1) = 0.

Maxima is (1,0)

Minima is (0,1) //

Q4) $r = a(1-\cos\theta) \rightarrow (1)$ Show that $\dfrac{\rho^2}{r}$ = constant

Taking log on both sides.

$\log r = \log a + \log(1-\cos\theta)$

Diff w.r.t θ.

$$\frac{1}{r} r_1 = 0 + \frac{+\sin\theta}{1-\cos\theta}$$

$$r_1 = r\frac{(\cancel{2}\,\cancel{\sin\theta/2}\cos\theta/2)}{\cancel{2}\sin^2\theta/2}$$