USN

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## Internal Assessment Test III – March 2023

| Sub: | Mathematics-I for Computer Science and Engineering Stream | | | | Sub Code: | BMATS101 | |
|---|---|---|---|---|---|---|---|
| Date: | 31/03/2023 | Duration: | 90 mins | Max Marks: 50 | Sem / Sec: | 1 / A to G (PHY CYCLE) | OBE |

**Question 1 is compulsory and answer any SIX questions from the rest.**

| | Question | MARKS | CO | RBT |
|---|---|---|---|---|
| 1. | Find the Orthogonal Trajectories of $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, where $\lambda$ is a parameter. | [08] | CO3 | L3 |
| 2. | Solve : $xyp^2-(x^2+y^2)p+xy=0$ | [07] | CO3 | L3 |
| 3. | Solve $x\frac{dy}{dx}+y=x^3y^6$ | [07] | CO3 | L3 |
| 4. | Solve $x^3+5x+1\equiv 0(mod 27)$ | [07] | CO4 | L3 |
| 5. | Solve the system of linear congruence by using CRT. $x\equiv 3(mod 5)$, $x\equiv 2(mod 6)$, $x\equiv 4(mod 7)$ | [07] | CO4 | L3 |
| 6. | (i) Find the last digit of $7^{2013}$. (ii) Find the remainder when the number $2^{1000}$ is divided by 13. | [07] | CO4 | L3 |
| 7. | Find the general solution of the equation $(px-y)(py+x)=a^2p$ by reducing into Clairaut's form by taking the substitution $X=x^2, Y=y^2$. | [07] | CO3 | L3 |
| 8. | Solve $2x+6y\equiv 1(mod7)$, $4x+3y\equiv 2(mod7)$ | [07] | CO4 | L3 |

Solution - IAT-3 (P-Cycle)

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Q1. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2+\lambda} = 1$ —(1)

$\dfrac{2x}{a^2} + \dfrac{2yy_1}{b^2+\lambda} = 0 \Rightarrow b^2+\lambda = -\dfrac{yy_1a^2}{x}$ —(2)

~~(1) & (2) ⇒ $\dfrac{x^2}{a^2} - \dfrac{xyy_1}{yy_1a^2} = 1 \Rightarrow \dfrac{x^2}{a^2} \dfrac{xyy_1}{a^2}$~~

from (1) & (2) $\dfrac{x^2}{a^2} - \dfrac{y^2x}{yy_1a^2} = 1$

$x^2 - \dfrac{y^2x}{y_1} = a^2$ —(3)

Changing $y_1 \to -1/y_1$ for O.T., eqn (3) becomes

$x^2 + xyy_1 = a^2 \Rightarrow xy\dfrac{dy}{dx} = a^2 - x^2$

$y\,dy = \dfrac{(a^2-x^2)}{x}dx \Rightarrow \int y\,dy = \int \dfrac{a^2dx}{x} - \int x\,dx$

$\dfrac{y^2}{2} = a^2\log x - \dfrac{x^2}{2} + c \Rightarrow \boxed{y^2 - 2a^2\log x + x^2 = C}$

Q9. $xyp^2 - (x^2+y^2)p + xy = 0$

$p = \dfrac{(x^2+y^2) \pm \sqrt{(x^2+y^2)^2 - 4x^2y^2}}{2xy} = \dfrac{(x^2+y^2) \pm (x^2-y^2)}{2xy}$

$p = \dfrac{x}{y}$ ; $p = \dfrac{y}{x}$

$\dfrac{dy}{dx} = \dfrac{x}{y}$ ; $\dfrac{dy}{dx} = y/x$

$y\,dy = x\,dx$ ; $\dfrac{dy}{y} = dx/x$

$\dfrac{y^2}{2} - \dfrac{x^2}{2} - c = 0$ ; $\log y - \log x = \log c$

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The G.S is $(y^2 - x^2 - 2c)(y/x - c) = 0$

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Ques-3 $\quad x\frac{dy}{dx} + y = x^2 y^6$ —— ① Bernoulli's eqn

$\frac{dy}{dx} + \frac{y}{x} = x^2 y^6$

$y^{-6}\frac{dy}{dx} + \frac{y^{-5}}{x} = x^2$ $\qquad$ Let $y^{-5} = t$

$-5y^{-6}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow -\frac{1}{5}\frac{dt}{dx} + \frac{t}{x} = x^2$

$\Rightarrow \frac{dt}{dx} - \frac{5}{x}t = -5x^2$ —— ② L.D.E.

I.F $\Rightarrow e^{-\int \frac{5}{x}dx} = e^{-5\log x} = 1/x^5$

Soln is $\quad \frac{t}{x^5} = \int -5x^2 \cdot \frac{1}{x^5}\, dx + C$

$\frac{t}{x^5} = -5 \cdot \frac{x^{-2}}{-2} + c \Rightarrow \boxed{\frac{1}{x^5 y^5} = \frac{5}{2x^2} + c}$ Ans

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Ques-4 $\quad x^3 + 5x + 1 \equiv 0 \pmod{27}$ —— ①

$\because 27 = 3^3$, Let's check for

$x^3 + 5x + 1 \equiv 0 \pmod 3$

$f(0) = 1 \not\equiv 0 \pmod 3$

$f(1) = 7 \not\equiv 0 \pmod 3$

$f(2) = 19 \not\equiv 0 \pmod 3$

$\Rightarrow$ eqn ① has no solution

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$$\begin{array}{cccccc} a_1 & n_1 & a_2 & n_2 & a_3 & n_3 \end{array}$$

Qus-5 $x \equiv 3 \pmod{5}$ $x \equiv 2 \pmod{6}$ $x \equiv 4 \pmod{7}$

$n = n_1 n_2 n_3 = 5 \times 6 \times 7 = 210$

$N_1 = \frac{n}{n_1} = 42$, $N_2 = \frac{n}{n_2} = 35$, $N_3 = \frac{n}{n_3} = 30$

Now $42 x_1 \equiv 1 \pmod{5}$ $\Rightarrow x_1 = \frac{5k+1}{42} \Rightarrow x_1 = 3$

$35 x_2 \equiv 1 \pmod{6}$ $x_2 = \frac{6k+1}{35} \Rightarrow x_2 = 5$

$30 x_3 \equiv 1 \pmod{7}$ $x_3 = \frac{7k+1}{30} \Rightarrow x_3 = 4$

$\therefore$ $X = a_1 x_1 N_1 + a_2 x_2 N_2 + a_3 x_3 N_3$

$= 3 \times 3 \times 42 + 2 \times 5 \times 35 + 4 \times 4 \times 30$

$= 378 + 350 + 480$

$= 1208$

$\therefore$ $X \equiv x' \pmod{n}$ $\Rightarrow 1208 \equiv x' \pmod{210}$

$1208 \equiv 158 \pmod{210}$ Ans

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Ques-6 (i) $\because$ $7^2 \equiv -1 \pmod{10}$

$\Rightarrow (7^2)^{1006} \equiv (-1)^{1006} \bmod 10$

$7^{2012} \equiv 1 \pmod{10}$ $\Rightarrow 7^{2013} \equiv 7 \pmod{10}$

$\boxed{\text{last digit} = 7}$

(ii) $2^{1000} \equiv x \bmod(13)$

$2^{12} \equiv 1 \bmod 13 \Rightarrow (2^{12})^{80} \equiv (1)^{80} \bmod 13$

$\Rightarrow 2^{960} \equiv 1 \bmod 13$ $\because$ $(12)^{36} \equiv 1 \bmod 13$

$2^{996} \equiv 1 \bmod 13$ $2^4 = 3 \bmod 13$

$2^{1000} \equiv 3 \bmod 13$ $\boxed{\text{remainder} = 3}$

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Ques-7 $p = \frac{dy}{dx} = \frac{dy}{dY} \cdot \frac{dY}{dX} \cdot \frac{dX}{dx}$, let $\frac{dY}{dX} = P$

$X = x^2$. ~~$\rightarrow$ px~~ $\frac{dX}{dx} = 2x$

$Y = y^2$ $\frac{dY}{dy} = 2y$

$p = \frac{1}{2y} \cdot P \cdot 2x$ $\Rightarrow$ $p = \frac{x}{y} P = \frac{\sqrt{X}}{\sqrt{Y}} P$ — (1)

given eqn is $(px - y)(py + x) = a^2 p$ —— (2)

$\Rightarrow$ $\left\{\frac{\sqrt{X}}{\sqrt{Y}} P \cdot \sqrt{X} - \sqrt{Y}\right\}\left\{\frac{\sqrt{X}}{\sqrt{Y}} P \cdot \sqrt{Y} + \sqrt{X}\right\} = a^2 \frac{\sqrt{X}}{\sqrt{Y}} P$

$$\left\{\frac{XP-Y}{\sqrt{Y}}\right\}\left\{(P+1)\sqrt{X}\right\} = \frac{a^2\sqrt{X}}{\sqrt{Y}}P$$

$$(XP-Y)(P+1) = a^2P \Rightarrow Y = PX - \frac{a^2P}{P+1} \quad \text{--- (3)}$$

$\therefore$ The G.S. of (3) is $Y = CX - \dfrac{a^2C}{C+1}$

$\therefore$ The G.S. of (2) is $\boxed{y^2 = cx^2 - \dfrac{a^2c}{c+1}}$ Ans

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Ques-8 $\quad 2x + 6y \equiv 1 \pmod 7$ --- (1)

$\quad 4x + 3y \equiv 2 \pmod 7$ --- (2)

$\gcd(2,6,7) = 1 \quad 1 \mid 4 \Rightarrow$ solⁿ exists

$\gcd(-18, 7) = 1 \Rightarrow$ unique solⁿ.

(1) − (2)×2 $\Rightarrow \quad -6x \equiv -3 \pmod 7$

$2x \equiv 1 \pmod 7$ ~~$\Rightarrow x = 4$~~

$\frac{2x-1}{7} = k \Rightarrow x = \frac{7k+1}{2} \Rightarrow x = 4, \; k = 1$

from (1) $\quad 8 + 6y \equiv 1 \pmod 7 \Rightarrow 6y \equiv -7 \pmod 7$

$6y \equiv 0 \pmod 7 \Rightarrow y = 0$

So the solⁿ of given set $(4, 0)$

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